Ultrasonic study of the elastic properties of superconducting materials

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This work deals with the simulation study superconducting materials elastic properties. The study is based on the simulation of the acoustic signal V (z) received by the acoustic microscope. The simulated signal enabled us to know the variation of reflection coefficient $R(\theta)$ which behaves as a function of incidence angle θ, from the analysis of $R(\theta)$ and V(z) we determines the elastic constant of the material studied.

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1. Introduction

The discovery of the supraconductivity at high critical temperature led to pursue the research toward the use of the acoustic methods in order to understand the mechanism of the supra conductivity. Several studies have been performed to solve the effect of the porosity on the elastic properties of the superconductive materials [1, 2] as well as the porous materials [3]. Concerned with the present field research orientations, we will base our research on the survey on the porosity effect and on the propagation manners and subsequently on the elastic properties in the superconductive materials while comparing our results with those of given in references [1] taken experimentally by micro scanning. The study given by acoustic microscopy of the materials properties requires the understanding of the phenomenon of wave propagation in solids and liquids phases as well as the knowledge of the laws that govern the wave transfers to the interfaces. The knowledge of the reflection coefficient $R(\theta)$, according to the incidence angle θ, proves to be therefore necessary to determine and to analyze the different modes of propagation. It is therefore necessary to know, the amplitude, the direction of propagation and the polarization of the incidental wave, as well as the materials elastic properties.

2. The reflection coefficient of $R(\theta)$

The coefficient $R(\theta)$ expression determination method by using *(.*è) the mechanical balance, continuity of the constraints and displacements to the interface has been developed by Brekhovskikh [4].

The reflection coefficient for the materials massif is given by the expression [4, 5]:

$$R(\theta) = \frac{Z_l \cos^2 2\theta_s + Z_s \sin^2 2\theta_s - Z_0}{Z_l \cos^2 2\theta_s + Z_s \sin^2 2\theta_s + Z_0}$$

Where: Z_0 is the liquid acoustic impedance;
Z_l and Z_s are the acoustic impedances longitudinal and transverse respectively of the solid. While equating the total impedance as follows:

$$Z_{TOT} = Z_l \cos^2 2\theta_s + Z_s \sin^2 2\theta_s$$

We get the relation of the widely-known reflection coefficient in acoustics, either as:

$$R(\theta) = \frac{Z_{TOT} - Z_0}{Z_{TOT} + Z_0}$$

In the most general case and under any impact, the angles of θ_l refraction and θ_s and by continuation impedances acoustic Z_l and Z_s are complex numbers. The reflection function can be expressed in term of a complex shape:

$$R(\theta) = \Gamma \exp(j\xi)$$

Where Γ is the module of $R(\theta)$, and ξ the phase.

One will use this representation to determine the acoustic signature V as a function of (*z*) for the studied materials.

In the case of a massive material one raises variations of phase and amplitude in the neighborhood of the critical angles: longitudinal, transverse and Rayleigh, which allows us to calculate the velocities of the different fashions. The theories concerning the propagation of the mechanical waves in the isotropic materials show that two waves of volume can auto propagate [8, 9], these longitudinal and transversal waves have the respective speeds denoted by V_l and Vs. When the angle of impact of a longitudinal wave (in relation to the plan of the sample) becomes very important and reaches a critical value, three surface waves can be arise: the surface longitudinal wave, the surface transverse wave and the Rayleigh wave. In order to get the elastic constants, it is indispensable a priori to measure V_l and Vs.

The variations more or less important of the module and the phase of $R(\theta)$ some critical angles are going to allow us to interpret the V curves. The simulation of the reflection coefficient of $R(\theta)$ for the studied superconductive material ($DyBa_{2-x}Sr_xCu_3O_{7-\delta}$); ($x \sim 0.3$ and $\delta \sim 0.1$) for different values of the porosity rate and for a frequency of 600 MHz is represented on the Figs. 1 and 2.

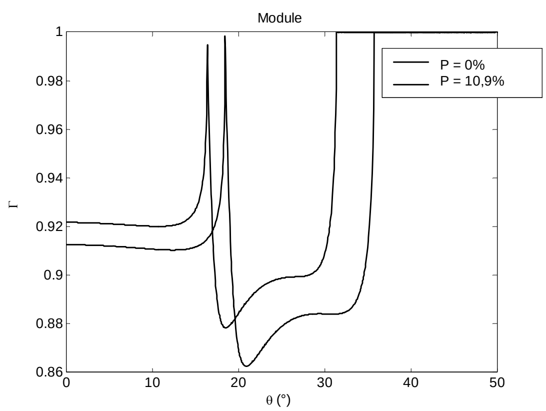

Fig. 1 variation of the module of R (.θ).

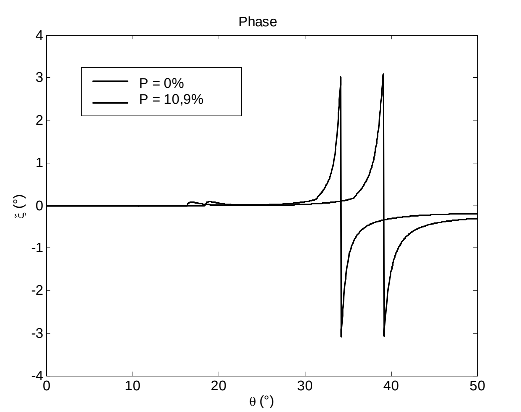

Fig. 2. Variation of the phase of R (.θ).

One can notices an important displacement of the critical angles for all fashions in relation to the relative critical angles to the non porous material (p = 0 %), this displacement is toward the superior angles when the rate of porosity increases, that means a reduction of the velocity of these modes.

3. Signature acoustic *V* (*z*)

The micro characterization of a material passes by the determination of his/her/its signature acoustic *V* (*z*).

The expression of V is given (*z*) by [4-7]:

$$V(z) = \int_0^{\theta_{max}} P^2(\theta) R(\theta) \exp(2jk_0\, z\, coc\theta) \sin\theta \cos\theta\, d\theta$$

Or: $j = \sqrt{-1}$, θ_{max} is the maximal opening of the lens, $P^2(\theta)$ is the function pupil; R *(θ)* is the function of reflection of the specimen and k_0 is the vector of wave in the liquid of coupling.

The acoustic microscope is used in a reflection fashion; the piezoelectric translator transforms the acoustic signal coming from the sample in a V tension *without* losing the relative information to the studied material.

The size of the measured V of the signal is a function of the unfocused (the z distance between the focal plan of the lens and the interface sample - liquid).

This model allows us to calculate the speeds of the different modes from the period Δz as:

$$v_S = \frac{v_0}{\sqrt{1 - \left(1 - \frac{v_0}{2 f \Delta z}\right)^2}}$$

The Fig. 3 shows the signatures acoustic V(*z*) of the studied superconductive material ($DyBa_{2-x}Sr_xCu_3O_{7-\delta}$) for different values of the porosity rate and for a frequency of 600 MHz.

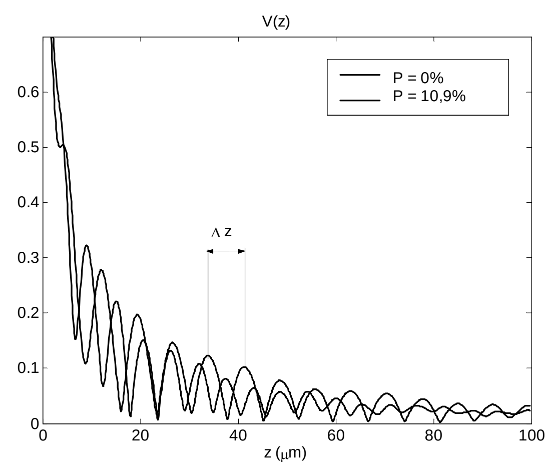

Fig. 3. Signature acoustic V (z).

The period of the pseudo oscillations Δz decreases with the growth of the porosity rate. This qualitative analysis is confirmed by the reduction of the Rayleigh velocity (Table 1).

Table 1. Elastic constants.

	Non porous	Porous (10,9%)
ρ (kg/m3)	6920	6165,7
V_s (m/s)	2874	2683
V_l (m/s)	5274	4860
V_R (m/s)	2669	2378
E (GPa)	147,3	117,6
G (GPa)	116,3	93,9
K (GPa)	57,2	45,5
ν	0.2889	0.2912

4. Treatment of the signature acoustic V(z) by Fourier Transformed (FFT):

It is very difficult to determine the velocity of the different modes directly from the periodicity of the acoustic signature: Indeed, this curve is the superposition of the different modes of surface and the intrinsic answer of the lens.

The V(*z*) signal can be called according to some manners in the goal to exploit it judiciously and in order to extract with precision the velocity of the different fashions to pull of it from the physical properties of the studied material.

Several treatment techniques of the signature exist, the acoustic V(*z*) one of the most known methods and used extensively for the numeric treatment of V (*z*) is Transformed of Fourier (FFT). This treatment permits to award the different peaks singular of the discreet specter of V(*z*) that correspond to the different fashions of propagation of the acoustic wave. This is how one can determine the velocities of propagation of surface and volume. The signal acoustic V (t) is (z) in fact a transformed of Fourier of a function that one will note Q. If one can find the expression of this Q function as well as his/her/its variable *t*, then Q will be (t) transformed it of inverse Fourier of the signal acoustic V and (*z*) it is her that will give us the specter of V(z) (Fig. 4).

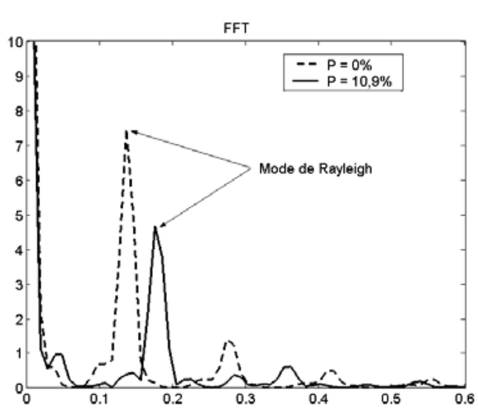

Fig. 4. Transformed of Fourier FFT.

Let's conduct the change of variables following [7]:

$$t = \frac{\cos\theta}{\pi}$$

and

$$u = k_0 z$$

This change drives us to the equalities:

$$2 \cdot k_0 \cdot z \cdot \cos\theta = 2 \cdot \pi \cdot u \cdot t$$

and

$$\sin\theta \cdot \cos\theta \cdot d\theta = -\pi^2 \cdot t \cdot dt$$

Which allow us to write the expression of the V(*z*) under the shape:

$$V(u) = \int_{\cos\theta_{\max}}^{1/\pi} Q(t) \cdot \exp\left(-2j \cdot \pi \cdot u \cdot t\right) \cdot dt$$

$$Q(t) = P^2(t) R(t)$$

It is therefore clear that V(z) is Fourier transformed of the Q(u) function. The V(u) spectrum of will have like t abscissa and therefore it is also the V(z) spectrum with as z abscissa. Now we show that this analysis allows us to discern the modes of surface. For this, let's take the simplest case where only one mode of surface is generated, the one of Rayleigh. In this case, we distinguish two important discontinuities in the Q*(t)* function [5], the first to $t_0 = 1/\pi$, bus above this value, Q(t) annuls itself in a discontinuous way, and the second is to $t_R = \cos\theta/\pi$ bus to this value the phase of R(*t*) changes a discontinuous manner of *2.π*. This last corresponds to the angle critical of Rayleigh, giving the V_R velocity thus.

5. Determination of the elastic properties:

To calculate elastic properties for different velocities will permits us to determine the elastic parameters of the materials. There are four elastic constants [8, 9], wich are:

- The Young modulus:

$$E = \rho V_s^2 \left(\frac{3V_l^2 - 4V_s^2}{V_l^2 - V_s^2} \right)$$

- The shear modulus:

$$G = \rho V_s^2$$

- The compressibility modulus:

$$K = \rho\left(\frac{3V_l^2 - 4V_s^2}{3}\right)$$

- The Poisson coefficient:

$$\nu = \frac{2V_s^2 - V_l^2}{2(V_s^2 - V_l^2)}$$

Another method of calculation consists in getting the velocity from Rayleigh V_R from the graph of the phase of the R*(.θ)* reflection coefficient and the velocity of the mode transverse V_l of the graph of the module of R(.θ). We use the relation of Victorov then to determine the longitudinal velocity to calculate the elastic constants. The relation of Victorov is given by the following [9]:

$$V_R = V_s \frac{0.718 - (V_s/V_l)^2}{0.750 - (V_s/V_l)^2}$$

The values of the elastic constants for the superconductive material: ($DyBa_{2-x}Sr_xCu_3O_{7-\delta}$) porous with a rate of porosity of 10,9% and non porous are summarized Table 1. These results show a strong decrease of all elastic constants with the porosity.

6. Conclusion

The simulation results of the signal received by the acoustic microscope and the analysis of the reflective power simulated according to the model of Brekhovskikh allowed us to characterize the superconductive materials; the obtained results are in perfect concordance with the results of reference [1].

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