Two-dimensional Bessel soliton clusters in strongly nonlocal media

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The propagation of two-dimensional Bessel soliton clusters in strongly nonlocal media is investigated analytically. A broad class of exact self-similar solutions to the strongly nonlocal Schrödinger equation has been obtained. We find a Bessel solitary wave solution by using self-similar method. The modulation of the intensity distributions of optical beam under different parameters is discussed in detail.

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1. Introduction

Study of solitary wave in nonlocal media has been the subject of much interest in theory and experiment in these years [1-8]. Nonlocality has also been a phenomenon of intense research over recent years in various nonlinear physical systems [9,10]. Such as nonlocal nonlinear response can suppress modulation instability, prevent the collapse of self-focusing beams [11], support fundamental and vortex solitons [12-16], solitions of Bose–Einstein condensates [17,18], as well as can describe a noncontact bosonic interaction [19]. In nonlinear media, strongly nonlocality means that the characteristic length of the response function is much broader than the width of the optical beam. The phenomena of strongly nonlocality have been found in some experiments in nonlinear media [20,21]. It has been demonstrated that the nonlocality of nonlinear media can be described by a general nonlocal nonlinear Schrödinger equation (NNLSE). Many methods have been applied to solve similar nonlinear problems [22-25]. Snyder et al simplified the NNLSE and proposed a nonlocal linear model to describe the propagation of optical beam in the strongly nonlocal case [5]. Subsequently, some generalized nonlocal models have been proposed and a series of solutions, such as Hermit-Gauss, Laguerre-Gauss, necklace solitons, have been obtained in different dimensional coordinates [3,26,27].

The interest in properties of self-similar waves in complex nonlinear optical systems has grown greatly during recent years. Self-similar solutions have been explored not only in some areas such as plasma physics and nuclear physics, e.g., the light propagation in cold atom gases [28-30], but also in nonlinear optics community [3,31,32]. Studies of self-similar solutions of nonlinear differential equations have been of great value in understanding widely different nonlinear physical phenomena. As an example, exact self-similar solutions in nonlocal Schrödinger equation (NLSE) with distributed coefficients and some solutions in strongly nonlocal nonlinear media [27,31], were extensively investigated. Such self-similar solutions have many features similar to the ideal solitons, so it is also called self-similar solitary wave. In this paper, we will give an exact analytical Bessel type solution for the NLSE in a strongly nonlocal limit. We find a variety of self-similar solitary solutions.

2. Methods

The nonlinear Schrödinger equation of two-dimensional optical beams in a nonlocal media can be written as [3,5,31]

$$i\frac{\partial\psi}{\partial z}+\mu\nabla_{\perp}^{2}\psi+k\frac{\Delta n}{n_0}\psi=0, \tag{1}$$

where $\psi=\psi(r,z,\varphi)$ is a paraxial beam and z is the axis of the light propagation. $\nabla_{\perp}^{2}=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2}$ is the Laplacian operator and μ=1/2k . k is the wave number in the media and Δn is the nonlinear perturbation of the refraction index. In the case of the strongly nonlocality, the Eq.(1) can be normalized in a dimensionless form as [3,31]

$$i\frac{\partial\psi}{\partial z}+\frac{1}{2}\nabla_{\perp}^{2}\psi-sr^2\psi=0. \tag{2}$$

s is the normalized unit corresponding to the beam in the transverse plane. $r=\sqrt{x^2+y^2}$ and x, y are the axises of Cartesian coordinate. By a separation of variables $\psi = u(r,z)\phi(\varphi)$, Eq.(2) can be separated into two functions as follows $\frac{d^2\phi}{d\varphi^2} + m^2\phi = 0,$ (3)

$$i\frac{\partial u}{\partial z} + \frac{1}{2}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{m^2}{r^2}u\right) - sr^2 u = 0. \tag{4}$$

The parameter number m is integers and stands for the physical quantum number of ϕ. The parameter q determines the depth of azimuthal modulation. The solution of Eq.(3) is $\varphi = \cos(m\phi) + iq\sin(m\phi)$. To find the self-similar solutions of Eq.(4), the complex field can be defined as $u(r,z) = A(r,z)e^{iB(z,r)}$ and $A(r,z)$, $B(r,z)$ are real functions. If we substitute the form of $u(r,z)$ into Eq.(4) , we can get the following equations

$$-\frac{\partial B}{\partial z} + \frac{1}{2}\left[\frac{1}{A}\frac{\partial^2 A}{\partial r^2} - \left(\frac{\partial B}{\partial r}\right)^2 + \frac{1}{rA}\frac{\partial A}{\partial r} - \frac{m^2}{r^2}\right] - sr^2 = 0 \tag{5}$$

and
$$\frac{1}{A}\frac{\partial A}{\partial z} + \frac{1}{2}\left[\frac{2}{A}\frac{\partial B}{\partial r}\frac{\partial A}{\partial r} + \frac{\partial^2 B}{\partial r^2} + \frac{1}{r}\frac{\partial B}{\partial r}\right] = 0. \tag{6}$$

To find a self-similar solution, a set of self-similar transformations are introduced [3,31]

$$A(r,z) = \frac{F(\theta)}{w(z)}, \tag{7}$$

$$B(z,r) = a(z) + b(z)r + c(z)r^2, \tag{8}$$

where $w(z)$ is the width of light beam. $F(\theta)$ is the self-similar function and $\theta(r,z) = \frac{r^2}{w^2}$ is the self-similar variable. The parameters $a(z)$, $b(z)$, $c(z)$ respectively stand for the phase offset, the frequency shift and the wave front curvature. As pointed in previous work [3,27,31], after substituting Eq.(7) and Eq.(8) into Eq.(6), we can get $b(z) = 0$ and $c(z) = \frac{1}{2w}\frac{\partial w}{\partial z}$. Substituting these results into Eq.(5), we have

$$-\frac{da}{dz} - \frac{r^2}{2w}\frac{d^2 w}{dz^2} - sr^2 - \frac{2r^2}{w^4} + \frac{2}{Fr^2}[\theta^2\frac{d^2F}{d\theta^2} + \theta\frac{dF}{d\theta} + (\theta^2 - (\frac{m}{\sqrt{2}})^2 F)] = 0. \tag{9}$$

This equation can be rewritten as follows

$$\theta^2\frac{d^2F}{d\theta^2} + \theta\frac{dF}{d\theta} + (\theta^2 - m'^2 F) = 0, \tag{10}$$

$$a = -nz, \tag{11}$$

$$-\frac{1}{2w}\frac{d^2 w}{dz^2} - s' - \frac{2}{w^4} = 0\,. \tag{12}$$

Here the parameters $m' = m/\sqrt{2}$, $s' = s + n$ and n are real numbers. The solution of Eq.(10) is a type of Bessel function and it's solution is $F(\theta) = J_{m'}(\theta)$. In order to find the solution of Eq.(12), we give a transformation $dw/dz = W$. According to the Eq.(12), we obtain $dW/dz = -2s'w - \frac{4}{w^3}$ and $dw/dW = \frac{W}{-2s'w - \frac{4}{w^3}}$. Based on the initial physical conditions: $\begin{cases} w|_{z=0} = w_0 \\ W = dw/dz|_{z=0} = 0 \end{cases}$, we can get the following equation

$$\frac{1}{2}(dw/dz)^2 = -s'w^2 + \frac{2}{w^2}. \tag{13}$$

The solution of Eq.(13) can be easily obtained as

$$w^2 = w_0^2[\cos^2\left(\sqrt{2s'}z\right) + \lambda\sin^2\left(\sqrt{2s'}z\right)]. \tag{14}$$

Where the parameter $\lambda = \frac{1}{w_0^2}\sqrt{\frac{2}{s'}}$. Then the expression of $c(z)$ can be obtained,

$$c(z) = \frac{\sqrt{2s'}w_0^2(\lambda - 1)\sin(2\sqrt{2s'}z)}{2w^{3/2}} \tag{15}$$

Finally, we obtain the exact self-similar soliton solution of Eq.(2):

$$\psi(r,z,\varphi) = \frac{J_{m'}(\theta)}{w}[\cos(\sqrt{2}m'\varphi) + iq\sin(\sqrt{2}m'\varphi)]e^{i(-nz + c(z))}. \tag{16}$$

3. Results and discussion

In this paragraph, we will discuss the intensity distribution of our analytical solutions. In Fig. 1, a comparison of intensity distributions of self-similar solitary waves between analytical solutions and numerical simulations with different m is presented. Here the initial parameters are chosen as $w_0 = 1$, $s' = 1$, $z = \pi/3$ and $q = 0$. The middle figure is our theoretical analysis result of Eq. (16). It is clearly seen that the intensities of solitary waves have a symmetric distribution. As shown in Fig. 1

(a), the intensity distribution of soliton waves at the tangent plane of the symmetric axis clearly shows this symmetric distribution. This phenomenon comes from the additional strongly nonlocal condition [12]. We also get the numerical simulation results by using Fourier transform method. Numerical solution of Eq. (2) is performed to ascertain the stability of soliton clusters and to compare with the analytical solution. In order to test the stability of soliton wave, we add appropriate white noise to the simulation. As expected, no collapse is seen, and excellent agreement with the analytical solution is obtained. It will be noted that the solution becomes to be a usual Bessel type when $m = 0$, and the solitary wave will not be affected by phase modulation. With the increase of m, there is a necklace type distribution around the central point for the facula points and the number of facular points is a double value of m.

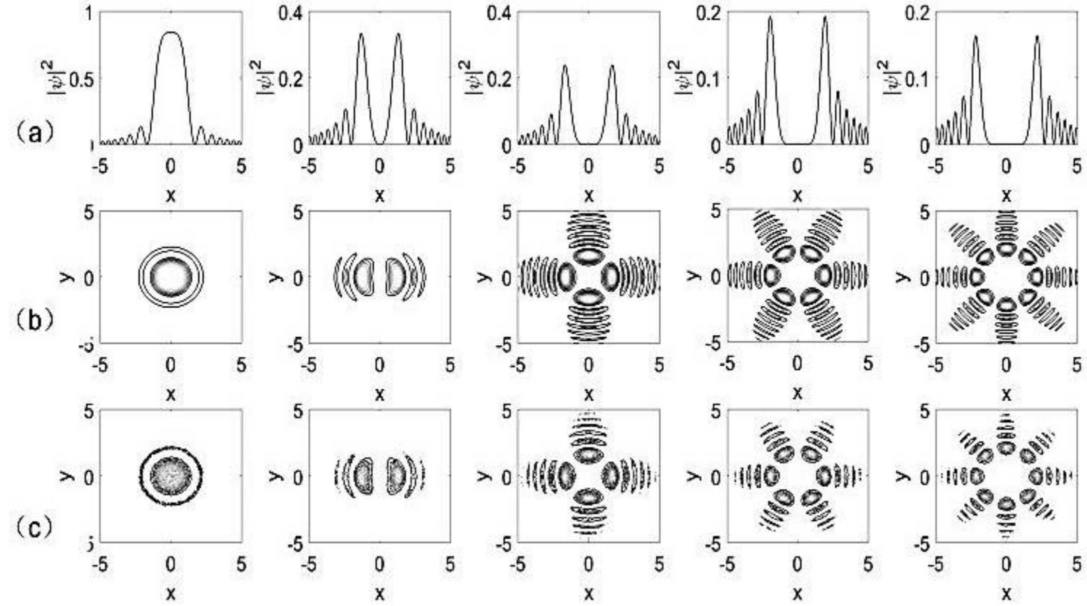

Fig. 1. Comparison of the analytical solution for intensity with the numerical simulation, for different m, when $q = 0$.
(a) The intensity distribution of soliton waves at the tangent plane of the symmetric axis. (b) Analytical solution of equation (16).
(c) Numerical simulation of equation (2).

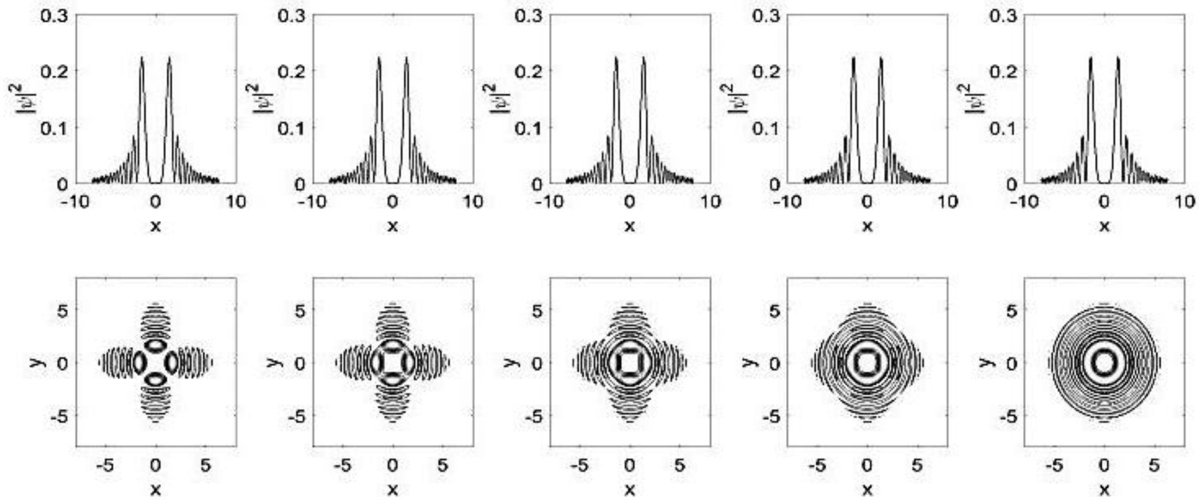

Fig. 2. Intensity distributions of solitons with different values of q. The parameter $m = 2$.

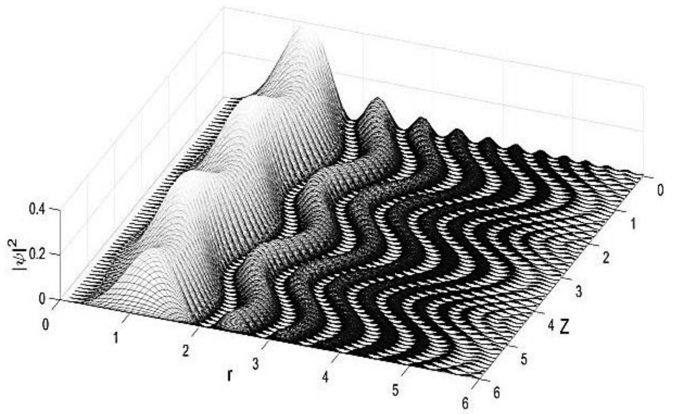

Fig. 3. The intensity distributions of the soliton varies with z *. The parameters* $q=0$ *and* $m=1$ *.*

Fig. 2 shows the intensity distribution of self-similar solitary waves with different azimuthal modulation parameter q. The necklace will become to be more and more obscure with the increase of q azimuthal modulation untill it becomes a Bessel ring at $q=1$. Fig. 3 shows the variation of soliton wave with transmission distance. Obviously, the soliton wave changes periodically with the transmission distance. Although the intensity of soliton wave varies with the transmission distance, there is no loss in the transmission process.

4. Conclusions

In summary, the exact solutions of a nonlocal Schrödinger equation in the strong nonlocal case have been studied. A two-dimensional solution of Bessel solitary waves has been obtained. The intensity distributions of optical beam have been discussed in detail. It is found the solitary waves have a necklace type symmetric distribution around the central point and the number of facular points is a double value of m. The stability of self-similar soliton wave is verified by direct numerical simulation. The variation of soliton wave with transmission distance and modulation parameters is also discussed.

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