

Three-dimensional FDTD method for the analysis of optical pulse propagation in photonic crystal fibers

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We propose a three dimensional finite difference time domain method for analyzing the optical pulse propagation in photonic crystal fibers. The propagating pulse is virtually held in the middle of the problem space as simulation continues by adopting the technique of moving problem space to limit the computation domain size. This method is capable to investigate the pulse propagation in photonic crystal fibers as well as spatial phenomena such as transverse mode profile evolution.

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1. Introduction

The finite difference time domain method (FDTD) method has been recognized as one of the most powerful techniques for optics device simulations [1]. It is a competitive candidate to model electromagnetic wave phenomena, such as light propagation, scattering, diffraction and polarization effects. Despite of its strengths, FDTD method is inherently resource demanding especially in three dimensions (3D). A compact 2D finite-difference time-domain method for full-vectorial analysis of photonic crystal has been developed by us [2].

Photonic crystal fibers (PCFs) have attracted a lot of attention in optics research in recent years, and they are finding many promising applications such as in communications and sensing [3]. Highly-Non-Linear (HNL) fibers are an important family member that applies very small core dimensions to provide tight mode confinement to obtain enhanced nonlinearity. By tailoring and engineering the unusual dispersion properties, such fibers can be useful in nonlinear applications such as soliton generation, supercontinuum generation, and ultra-short pulse compression.

Due to the structure complexity of the fiber, the common approach to model and study the pulse propagation in such fibers with nonlinear effects is to use the nonlinear Schrodinger equation [3]. The linear mode properties are obtained by a mode solver. The

nonlinearity, dispersion parameters are then defined in the equation to simulate the nonlinear pulse propagation. Since the approach is based on the slow-varying amplitude approximation, it is restricted to the analyses which satisfy the condition. In addition, it is assumed that modal properties obtained in linear regime are the same in the nonlinear regime. Therefore, any deviation of the mode properties between the two regimes would introduce inaccuracy. For example, in the case of self-focusing effect in a PCF with Kerr-medium, the mode area is decreasing along the propagation direction. The presumption of a constant value for the mode area is thus causing inaccuracy in the nonlinear process. Moreover, the approach of Schrodinger equation neglects the information in fiber transverse plane which in consequence results in its incapability of the studies in the spatial domain. It is therefore an essential and meaningful task to develop some methods to investigate the pulse propagation in the PCF while being able to consider the effects of the structure and to study the spatial phenomena, with affordable computation resources of course. This paper for the first time describes the 3D FDTD method for optical pulse simulation in PCFs. It is shown that the pulse propagation as well as the spatial field evolution in PCFs can be modeled by the truly 3D FDTD simulation. The analysis is carried out in both time domain and frequency domain to illustrate the propagation phenomena as well as spectra behavior. The calculation is done with limited

resource, e.g. a normal PC. Specifically, to limit the computation resource requirement, the optical pulse is being held in the middle of the problem space, with moving background space to simulate relatively long structure [5]. This paper illustrates the 3D FDTD method with the analysis of the optical pulse simulation in a one-ring of air hole PCF. The pulse propagation along the fiber structure and the transverse mode profile evolution are investigated. As an example, the influence of the nonlinearity on the optical pulse propagation is also studied.

2. FDTD method with moving problem space

The time-dependent Maxwell's curls equations for electric flux density \mathbf{D} , electric and magnetic field intensities \mathbf{E} and \mathbf{H} are arranged in a form:

$$\frac{\partial \mathbf{D}}{\partial t} = \nabla \times \mathbf{H} \quad (1)$$

$$\mathbf{D}(\omega) = \varepsilon_0 \varepsilon_r(\omega) \cdot \mathbf{E}(\omega) \quad (2)$$

$$\frac{\partial \mathbf{H}}{\partial t} = -\frac{1}{\mu_0} \nabla \times \mathbf{E} \quad (3)$$

where μ_0 and ε_0 are vacuum permeability and permittivity. $\varepsilon_r(\omega)$ characterizes the material constitution between electric field intensity and electric flux density. Specifically, $\varepsilon_r(\omega)$ is the permittivity distribution of the PCF structure. For dispersive and nonlinear medium, a general expression of the constitutive equation [1, 6] can be used for the update of the electric field. The propagating pulse is introduced in the incident plane of the problem space with an initial amplitude profile, e.g. Gaussian profile, to expedite the establishment of the propagation mode. The mean position of the propagating pulse on the axis in the center of the fiber core is calculated and compared with the position of the half

length of the problem space [5]. If the mean position reaches the midpoint, the simulation is halted while all the field values are moved back one cell in the propagating direction. The simulation is resumed until the mean position of the pulse meets the midpoint again. The continuous monitoring of the mean position and the field value reassignment ensure the pulse being virtually propagating along the fiber axis. Since the propagating pulse is being held in the middle of the problem space, the shape of the pulse during the simulation can be stored at any time of interest. While the time domain behavior can be obtained directly from the field updates in the FDTD method, the Fourier Transform of the pulse at its center frequency is necessary to be done in order to investigate the spectra behavior in the frequency domain. In this manner, both temporal and spectral analysis can be carried out for the pulse propagation in PCFs.

3. Analysis of pulse propagation in PCFs

For a typical one-ring of equally spaced air holes PCF structure, the 3D view and the cross-sectional view of the permittivity distribution are shown in Fig. 1, i.e. XY plane is the transverse plane of the PCF, and the permittivity is homogeneous along the longitudinal fiber axis, i.e. Z axis. The hole to hole distance $\Lambda = 6.75 \mu\text{m}$, air hole diameter $d = 5 \mu\text{m}$, propagating mode wavelength $\lambda = 1.4 \mu\text{m}$, the background medium is silica with refractive index $n = 1.45$, and the refractive index of air $n = 1.0$. A Gaussian profile of the amplitude distribution of E-field is initiated in the incident plane. In this work, the initialized E field is polarized to X direction. To minimize the numerical dispersion due to the finite differencing of the spatial and temporal derivatives, the cell size along propagation direction is chosen to be $1/20$ of the wavelength in the material, while the cell size in the transverse plane is chosen to be $1/5$ of the wavelength because the pulse shape changes relatively slowly transversally. The time step is chosen to satisfy the Courant's limit condition. The boundary condition of the

transverse plane is treated with PEC since the tail of the pulse dies out at the edge in the transverse XY plane. The boundary of the propagating direction is left without any treatment since the reference medium is moving to hold the pulse in the midpoint.

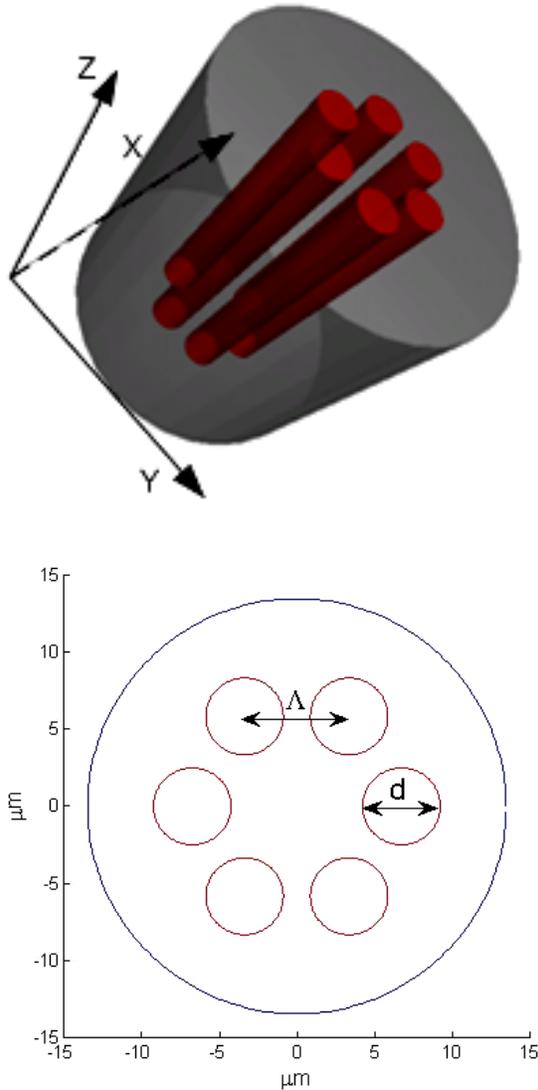


Fig.1: Permittivity distribution of the one-ring of air holes PCF structure.

Considering a Gaussian pulse propagating along the PCF, with the pulse width of $2.8 fs$, modulated by cosine function at $1.4 \mu m$, grid size is $0.193 \mu m$ in XY plane and $0.048 \mu m$ along Z axis, each time step is $0.08 fs$. Fig.2 depicts the pulse propagation in the moving problem space in YZ plane, where Z being the propagation direction, and

X being the polarization direction of the E field. (a) shows the pulse entering the problem space after $9 fs$, (b) shows the pulse reaching the midpoint of the space after $38 fs$, (c) shows the pulse after $77 fs$ and (d) shows the pulse which has propagated $30.7 \mu m$ after $154 fs$. The pulse shape of Fig.2. (d) has changed substantially from its original profile of Fig. 2 (a), which should be attributed to the waveguide dispersion. In order to analyze the pulse propagation behavior in frequency domain, i.e. at the operating wavelength, a running Fourier Transform [7] treatment is carried out throughout the simulation time. The results of the Fourier Transform are mapped to the center axis of the fiber core in the propagating direction. In Fig. 3, the Fourier amplitude of the propagating pulse in YZ plane at wavelength of $1.4 \mu m$ has maintained its Gaussian profile, i.e. the propagating mode is maintained in the propagation path along Z axis. The inset plot shows the Gaussian profile also holds the same beam width. To verify that the guided mode profiles are not varying along the propagation, the transverse mode profiles are also calculated and found to remain the same during the propagation, as shown in Fig. 3 top right plot. Next, we investigate the pulse propagation behavior in the presence of nonlinearity. It has been proven in [8] that PCF with Kerr-medium can support stable solutions as nonlinear modes. [5] also simulated the nonlinear modes in PCF using the C2D-FDTD method. However only the mode profiles are obtained in [6, 8] while the influence of the nonlinear effects on the pulse propagation is not analyzed. In order to have a better understanding of the pulse propagation in such nonlinear PCFs as well as the transverse mode evolution, the current method is utilized to study the same fiber structure, assuming the nonlinearity parameter $n_2 = 2.7 \times 10^{-20} m^2 / W$, the linear mode profile calculated in the previous part is used as the amplitude profile of the pulse to be launched in the transverse incident plane, with the peak amplitude $E_{max} = 0.8 \times 10^9 V / m$ in the center of the core.

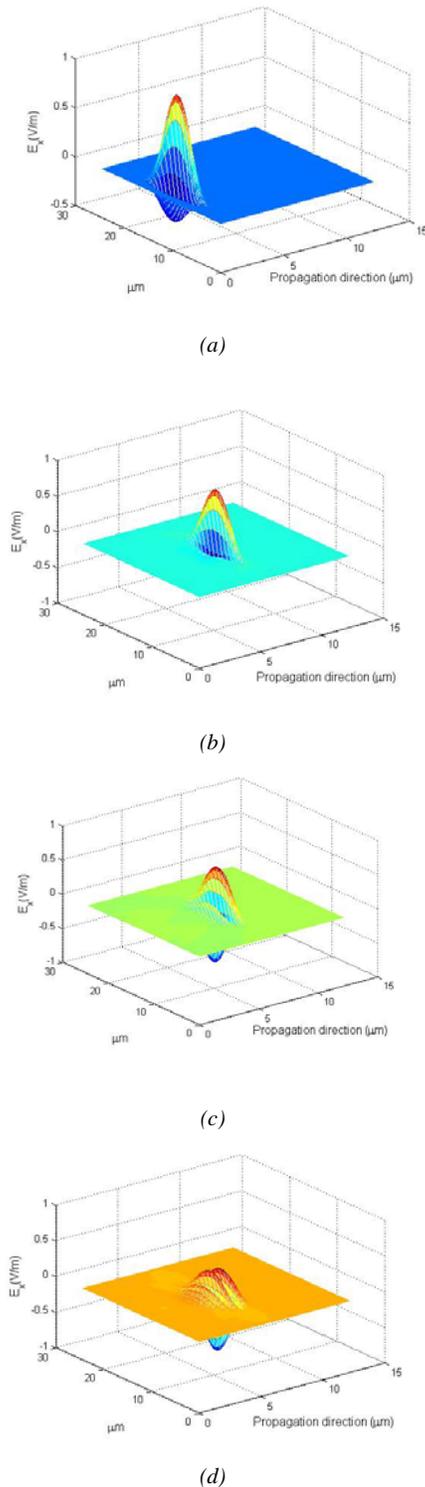


Fig.2: Propagating pulse (a) shows the pulse entering the problem space after 9 fs, (b) shows the pulse reaching the midpoint of the space after 38 fs, (c) shows the pulse which has propagated for 15 μm after 77 fs and (d) shows the pulse which has propagated 30.7 μm after 154 fs.

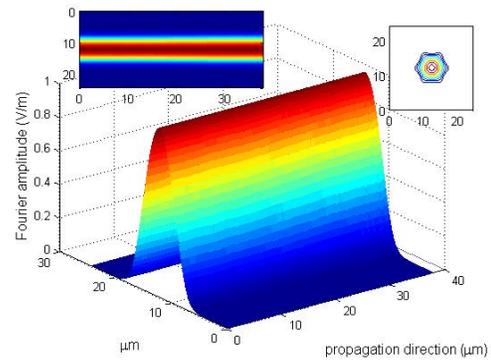


Fig.3: Propagating mode profile along the fiber axis for 37.5 μm at wavelength of 1.4 μm , top left plot is the YZ top view of the Fourier amplitude, top right plot is the linear mode profile at wavelength of 1.4 μm which is the same along the propagation path.

Fig.4 depicts the pulse propagation in the nonlinear PCF structure. (a) shows the pulse entering the problem space after 9 fs, (b) shows the pulse reaching the midpoint of the space after 38 fs, and (c) shows the pulse propagating 14.8 μm after 77 fs and (d) shows the pulse which has propagated 30.2 μm in the PCF after 154 fs. The pulse shape changes substantially from its linear case as shown in Fig. 3. Moreover, the propagating distance in both cases is different. Apparently the nonlinearity has contributed to the pulse distortion and delay. We can also observe that the leading edge of the pulse has denser oscillations. It could be resulted from new frequencies that are introduced in the pulse propagation. To verify this, a spectrum analysis of the pulse is carried out based on the current model. The spectrum of the propagating pulse in the nonlinear one-ring PCF is obtained by the Fourier Transforms of the field values, which are calculated at the core center of the PCF structure while the pulse is propagating along the fiber axis. Fig. 5 illustrates the spectrum of the propagating pulse in the PCF. The solid curve is the spectrum of the incident pulse and the dotted curve is the spectrum of the pulse which has propagated 30 μm . It is observed that the spectrum of the propagating pulse has broadened with new frequency components appearing in the higher frequency region. The resultant change of the spectrum is evidently contributed by the nonlinear effect. The Fourier amplitude of the pulse in YZ plane is shown in Fig. 6 (a). The inset plot is the top view of the amplitude profile. The Gaussian shape of the Fourier amplitude is maintained along the propagation path. However, there is slight amplitude beam narrowing

as indicated in the inset top view plot. This is the self focusing effect due to Kerr nonlinearity.

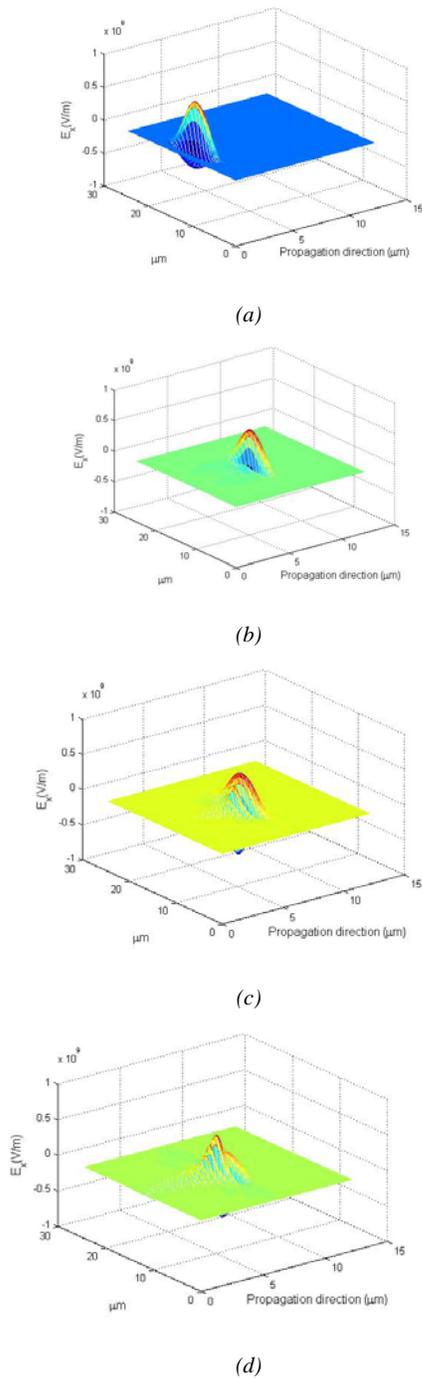


Fig.4: Propagating pulse (a) shows the pulse entering the problem space after 9 fs, (b) shows the pulse reaching the midpoint of the space after 38 fs, (c) shows the pulse propagating 14.8 μm after 77 fs and (d) shows the pulse which has propagated 30.2 μm after 154 fs.

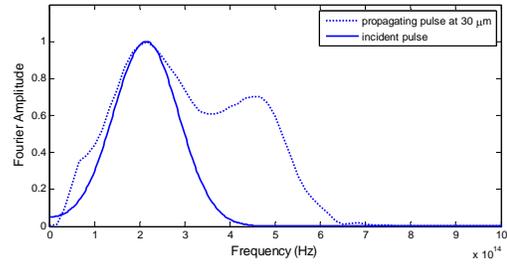


Fig.5: Frequency spectra of the incident pulse (solid curve), and the pulse which has propagated 34.2 μm in the nonlinear one-ring PCF.

The effective area is defined as Eq. 4, and it is calculated along the propagation path and shown in Fig. 6 (b).

$$A_{eff} = \frac{(\iint_S I(x, y) dx dy)^2}{\iint_S I(x, y)^2 dx dy} \quad (4)$$

The decreasing curve confirms the self-focusing effect due to Kerr nonlinearity. The transverse mode profiles in XY planes at 3 μm, 15 μm and 30 μm are shown in Fig.6 (b). A beam propagation method (BPM) has been used to calculate the mode profiles in the same PCF.

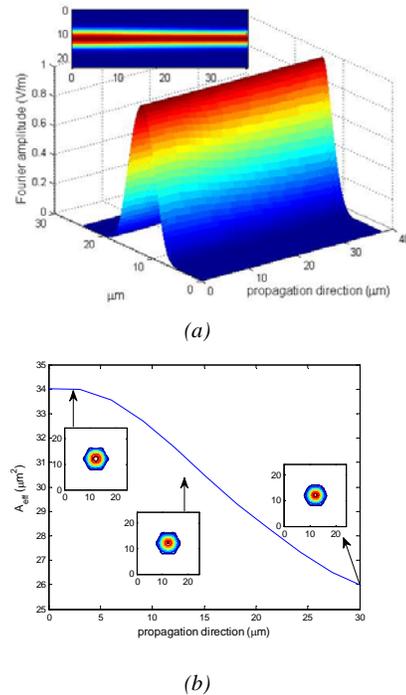


Fig.6: (a) Nonlinear mode profile in YZ plane at wavelength of 1.4 μm, with self-focusing effect along propagation direction, (b) The effective area evolution along the propagation path, with mode profiles in transverse plane (XY plane) at 3 μm, 15 μm and 30 μm in the propagation direction.

Apparently when the electric field amplitude reaches the order of 1×10^9 V/m, the silica material has broken

down. However, this paper is aimed at demonstrating the capability of the 3D FDTD method for optical pulse simulation in the PCF. The electric field amplitude is set ultra high to allow the nonlinear effects paramount within a short length for the ease of demonstration. For HNL PCF, with materials like Chalcogenide with much larger nonlinearity parameter [9], the nonlinear effects can be made orders higher than silica PCF, or nano fibers with the waveguide dimension in subwavelength range has the nonlinearity length in the order of mm [10], the 3D FDTD method can be used to investigate the pulse propagation behavior with limited computing resources.

4. Conclusions

In this paper, we present the three dimensional finite difference time domain method with moving problem space technique to investigate the optical pulse propagation in PCF for the first time. By adopting the technique of moving problem space, the propagating pulse is virtually being held in the middle of the problem space as simulation continues. The computation resource requirement is therefore substantially eased. The pulse propagation behavior in the time domain as well as the transverse mode profile in the frequency domain can be investigated using this method. This method overcomes certain limitations of the other approaches to investigate the pulse propagation behavior in fiber structures, and therefore provides an alternative approach.

The optical pulse simulation in a one-ring of air hole PCF is investigated using the 3D FDTD method. The temporal pulse propagation along the fiber structure and the spectral mode profiles and spectrums are investigated. The influence of the nonlinearity on the optical pulse propagation in such a fiber is also studied.

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