Thirring optical solitons in birefringent fibers with parabolic law nonlinearity

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This paper obtains dark and singular soliton solutions to Thirring model that is studied with parabolic law nonlinearity. The integration scheme employed is (G'/G)-expansion method. Apart from soliton solutions, singular periodic solutions and plane wave solutions are also obtained as a byproduct. The constraint conditions hold these solitons in place.

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1. Introduction

Optical solitons is one of the most important areas of research in the field of nonlinear optics. There are several aspects of solitons that are exhaustively studied in polarization-preserving fibers [1-10]. However, on the flip side, not many results are reported for birefringent fibers. The governing equation is the nonlinear Schrödinger's equation (NLSE). For birefringent fibers, NLSE splits into two components that lead to an unwanted feature that is known as differential group delay.

This paper focuses on obtaining soliton solutions in birefringent fibers with parabolic law nonlinearity when the self-phase modulation (SPM) is negligible and hence discarded. This results in Thirring solitons. The results on Thirring solitons with parabolic law nonlinearity is being reported for the first time in this paper. The integration scheme adopted here is the (G'/G)-expansion method that retrieves dark and singular solitons only. In addition, singular periodic solutions and plane wave solutions are also obtained as a byproduct of this scheme.

2. The model

The dynamics of solitons in a birefringent fibers with parabolic law medium is governed by coupled NLSE. The dimensionless for of this coupled NLSE is given by:

$$iu_t + a_1 u_{xx} + (b_1 |v|^2 + c_1 |v|^4 + d_1 |u|^2 |v|^2)u = 0 \qquad (1)$$

$$iv_t + a_2 v_{xx} + (b_2 |u|^2 + c_2 |u|^4 + d_2 |v|^2 |u|^2)v = 0 \qquad (2)$$

where $u(x, t)$ and $v(x, t)$ are dependent variables that represent the complex-valued wave profiles. The independent variables are the spatial variable x and the temporal variable t. The coefficients a_j for j = 1, 2 represents group velocity dispersion (GVD). The coefficients of nonlinear terms due to cross-phase modulation are b_j , c_j and d_j . It must be noted that SPM terms are already removed from the model described by (1) and (2).

In order to study this coupled system given by (1) and (2), the wave profiles are split into amplitude and phase components respectively as

$$u(x,t) = U(\xi) e^{i\phi_1} \qquad (3)$$

and

$$v(x,t) = V(\xi) e^{i\phi_2} \qquad (4)$$

where

$$\xi = B(x - vt)$$

The variables $u(x, t)$ and $v(x, t)$ are amplitude components of the wave profiles and the phase factor is given by

$$\phi_j = -\kappa_j x + \omega_j t + \theta_j, \quad for \quad (j = 1, 2) \qquad (5)$$

where κ_j is the frequency of the solitons, ω_j represents the wave number, ϑj is the phase constant. The subsequent section explains the integration scheme. Substituting equations (3) and (4) into equations (1) and (2), and then decomposing into real and imaginary parts leads to

$$\nu = 2\kappa_1 a_1 \tag{6}$$

and

$$\nu = 2\kappa_2 a_2 \tag{7}$$

that follows from the imaginary part. Next, equating the two velocities with each other leads to a constraint relation between the solitonparameters as

$$\kappa_2 a_2 = \kappa_1 a_1 \tag{8}$$

which is a constraint condition for the solitons to exist. The real part equations are discussed in Section-3. The following section reviews the *G*'/G-expansion integration scheme.

3. Overview of the scheme

Suppose that we have a nonlinear evolution equation of the form

$$P(u, u_x, u_t, u_{xx}, u_{xt}, u_{tt}, \cdots) = 0 \tag{9}$$

where u=$u(x, t)$ is an unknown function, P is a polynomial in its arguments, which includes nonlinear terms and highest order derivatives. Let us now review the main steps for solving nonlinear equations using the *G*'/G-expansion scheme [2].

Step 1: Seek traveling wave solution of Eq. (9) by taking $u(x,t) = u(\xi)$, $\xi = x - ct$, and transform Eq. (9) into the following ordinary differential equation (ODE)

$$Q(u, u', -cu', u'', -cu'', c^2u'', \cdots) = 0 \tag{10}$$

where c is a constant and primes denote the derivatives with respect to ξ.

Step 2: If possible, integrate Eq. (10) term by term one or more times yields constant(s) of integration. For simplicity the integration constant(s) can be set to zero.

Step 3: Suppose that the solution $u(\xi)$ of ordinary differential Eq. (10) can be expressed as a finite series in the form

$$u(\xi) = \sum_{i=0}^{n} a_i \left(\frac{G'(\xi)}{G(\xi)}\right)^i \tag{11}$$

where a_i are real constants, with $a_n \neq 0$, to be determined, n is a positive integer, which is determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in ordinary differential Eq. (10) and function $G(\xi)$ is the general solution of the auxiliary linear ordinary differential equation

$$G''(\xi) + \lambda G'(\xi) + \mu G(\xi) = 0, \tag{12}$$

where λ and μ are real constants to be determined later.

Substituting Eq. (11) together with Eq. (12) into Eq. (9) yields an algebraic equation involving powers of (G'/G). Equating the coefficients of each power of (G'/G) to zero, to obtain a system of algebraic equations for a_i, λ, μ and c. Then, we solve the system with the aid of a computer algebra system, such as Maple, Mathematica or Matlab to determine these constants. Since the solution of Eq. (10) have been well known for us depending on the sign of the discriminant $\Delta = \lambda^2 - 4\mu$, the exact solutions of the given Eq. (9) can be obtained.

4. Mathematical analysis

Now, in order to seek the optical solitons of Eqs. (1) and (2), We use the transformation defined in Eqs. (3)-(5). The parameters κ, L, ω and υ are undetermined parameters.

$$(-\omega_1 - a_1\kappa_1^2 + b_1V^2 + c_1V^4)U + B^2a_1U'' = 0 \tag{13}$$

$$(-\omega_2 - a_2\kappa_2^2 + b_2U^2 + c_2U^4)V + B^2a_2V'' = 0 \tag{14}$$

Now, we find the solution of Eq. (13) and Eq. (14) by balancing the nonlinear term U with the highest derivative term U'', we get m = 1, and similarly for V, we have n = 1 thus, we can write the solution of the form

$$U(\xi) = \alpha_1\left(\frac{G'}{G}\right) + \alpha_0, \quad \alpha_1 \neq 0 \tag{15}$$

$$V(\xi) = \beta_1\left(\frac{G'}{G}\right) + \beta_0, \quad \beta_1 \neq 0 \tag{16}$$

By using Eqs. (12) we can derive from Eq. (15) and Eq. (16) that

$$U'(\xi) = -\alpha_1\left(\frac{G'}{G}\right)^2 - \alpha_1\lambda\left(\frac{G'}{G}\right) + \alpha_1\mu \tag{17}$$

$$\begin{aligned} U''(\xi) = 2\alpha_1\left(\frac{G'}{G}\right)^3 + 3\alpha_1\lambda\left(\frac{G'}{G}\right)^2 \\ +(2\alpha_1\mu + \alpha_1\lambda^2)\left(\frac{G'}{G}\right) + \alpha_1\lambda\mu \end{aligned} \tag{18}$$

$$U^2(\xi)=\alpha_1^2\left(\frac{G'}{G}\right)^2+2\alpha_0\alpha_1\left(\frac{G'}{G}\right)+\alpha_0^2 \tag{19}$$

$$V^2(\xi)=\beta_1^2\left(\frac{G'}{G}\right)^2+2\beta_0\beta_1\left(\frac{G'}{G}\right)+\beta_0^2 \tag{20}$$

$$\begin{aligned}U^4(\xi)&=\alpha_1^4\left(\frac{G'}{G}\right)^2+\alpha_0^2\alpha_1^2\left(\frac{G'}{G}\right)^2+2\alpha_1^3\alpha_0\left(\frac{G'}{G}\right)^3\\&+\alpha_0^2\alpha_1^2\left(\frac{G'}{G}\right)^2+\alpha_0^4+2\alpha_1\alpha_0^3\left(\frac{G'}{G}\right)+2\alpha_1^3\alpha_0\left(\frac{G'}{G}\right)^3\\&+2\alpha_1\alpha_0^3\left(\frac{G'}{G}\right)+4\alpha_1^2\alpha_0^2\left(\frac{G'}{G}\right)^2\end{aligned} \tag{21}$$

$$\begin{aligned}V^4(\xi)&=\beta_1^4\left(\frac{G'}{G}\right)^2+\beta_0^2\beta_1^2\left(\frac{G'}{G}\right)^2+2\beta_1^3\beta_0\left(\frac{G'}{G}\right)^3\\&+\beta_0^2\beta_1^2\left(\frac{G'}{G}\right)^2+\beta_0^4+2\beta_1\beta_0^3\left(\frac{G'}{G}\right)\\&+2\beta_1^3\beta_0\left(\frac{G'}{G}\right)^3+2\beta_1\beta_0^3\left(\frac{G'}{G}\right)+4\beta_1^2\beta_0^2\left(\frac{G'}{G}\right)^2\end{aligned} \tag{22}$$

$$V'(\xi)=-\beta_1\left(\frac{G'}{G}\right)^2-\beta_1\lambda\left(\frac{G'}{G}\right)+\beta_1\mu \tag{23}$$

$$\begin{aligned}V''(\xi)&=2\beta_1\left(\frac{G'}{G}\right)^3+3\beta_1\lambda\left(\frac{G'}{G}\right)^2\\&+(2\beta_1\mu+\beta_1\lambda^2)\left(\frac{G'}{G}\right)^2+\beta_1\lambda\mu\end{aligned} \tag{24}$$

Substituting Eqs. (15)-(24) into Eq. (13) and Eq. (14) and setting the coefficients of $(G'/G)^i$ $(i=0,1,2,3)$ to zero, we obtain the following system of nonlinear algebraic equations:

$$\left(\frac{G'}{G}\right)^0:-\alpha_0\omega_1-\alpha_0a_1\kappa_1^2+\alpha_0b_1\beta_0^2+\alpha_0c_1\beta_0^4-B^2a_1\alpha_1\lambda\mu=0,$$
$$:-\beta_0\omega_2-\beta_0a_2\kappa_2^2+\beta_0b_2\alpha_0^2+\beta_0c_2\alpha_0^4-B^2a_2\beta_1\lambda\mu=0$$

$$\left(\frac{G'}{G}\right)^1:-\omega_1\alpha_1-\alpha_1a_1\kappa_1^2+\alpha_1b_1\beta_0^2+c_1\beta_0^4\alpha_1+2b_1\alpha_0\beta_1\beta_0$$
$$+2\alpha_0\beta_1^3\beta_0c_1+4\alpha_0\beta_1\beta_0^3c_1+a_1B^2\alpha_1\lambda^2-2B^2a_1\alpha_1\mu=0,$$
$$:-\omega_2\beta_1-\beta_1a_2\kappa_2^2+\beta_1b_2\alpha_0^2+c_2\alpha_0^4\beta_1+2b_2\beta_0\alpha_1\alpha_0$$
$$+2\beta_0c_2\alpha_1^3\alpha_0+4\beta_0\alpha_1\alpha_0^3+B^2a_2\beta_1\lambda^2-2B^2a_2\beta_1\mu=0$$

$$\left(\frac{G'}{G}\right)^2:2b_1\alpha_1\beta_1\beta_0+2c_1\beta_1^3\beta_0\alpha_1+2\beta_1\alpha_1\beta_0^3c_1+2c_1\beta_1\beta_0^3\alpha_1$$
$$+\alpha_0b_1\beta_1^2+\alpha_0c_1\beta_1^4+2\alpha_0c_1\beta_0^2\beta_1^2+4\alpha_0c_1\beta_1^2\beta_0^2$$
$$+2B^2a_1\alpha_1\lambda=0,$$
$$:2b_2\beta_1\alpha_1\alpha_0+2c_2\alpha_1^3\alpha_0\beta_1+2\alpha_1\beta_1\alpha_0^3c_2+2c_2\alpha_1\alpha_0^3\beta_1$$
$$+\beta_0b_2\alpha_1^2+\beta_0c_2\alpha_1^4+2\beta_0c_2\alpha_0^2\alpha_1^2+4\beta_0c_2\alpha_1^2\alpha_0^2$$
$$+2B^2a_2\beta_1\lambda=0$$

$$\left(\frac{G'}{G}\right)^3:\alpha_1b_1\beta_1^2+\alpha_1c_1\beta_1^4+2\alpha_1c_1\beta_0^2\beta_1^2+4\alpha_1c_1\beta_1^2\beta_0^2$$
$$+2\alpha_0c_1\beta_1^3\beta_0+2B^2a_1\alpha_1=0,$$
$$:\beta_1b_2\alpha_1^2+\beta_1c_2\alpha_1^4+2\beta_1c_2\alpha_0^2\alpha_1^2+4\beta_1c_1\alpha_1^2\alpha_0^2$$
$$+2\beta_0c_2\alpha_1^3\alpha_0+2B^2a_2\beta_1=0.$$

Solving the above system with the aid of *Maple 12*, we have the following set of solution

$$\alpha_1=\pm\frac{-\mu b_2+\sqrt{\mu^2b_2^2+8\mu c_2(\omega_2+a_2\kappa_2^2)}}{2B^2\beta_0c_2},\ \alpha_0=0$$
$$\beta_1=-\beta_0\,\frac{\omega_2+a_2\kappa_2^2}{B^2a_2\lambda\mu}$$

The following soliton solutions can be constructed.

<u>Case I</u>: When $\lambda^2-4\mu>0$, we obtain

$$\begin{aligned}u_1(x,t)&=\Big[\pm\frac{-\mu b_2+\sqrt{\mu^2b_2^2+8\mu c_2(\omega_2+a_2\kappa_2^2)}}{2B^2\beta_0c_2}\Big\{-\lambda+\sqrt{\lambda^2-4\mu}\\&\times\Big(\frac{C_1\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2\cos\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}{C_1\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}\Big)\Big\}\Big]\\&\times e^{-\kappa_1x+\omega_1t+\theta_1}\end{aligned} \tag{25}$$

and

$$\begin{aligned}v_1(x,t)&=\beta_0\Big[1-\frac{\omega_2+a_2\kappa_2^2}{B^2a_2\lambda\mu}\Big\{-\lambda+\sqrt{\lambda^2-4\mu}\\&\times\Big(\frac{C_1\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}{C_1\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}\Big)\Big\}\Big]\\&\times e^{-\kappa_2x+\omega_2t+\theta_2}\end{aligned} \tag{26}$$

Where C_1 and C_2 are constants. If C_1 and C_2 assume particular values, soliton solutions are recovered.

(i) If $C_1=0$ but $C_2\neq0$, we obtain the singular soliton solutions as follows

$$u_2(x,t)=\Big[\pm\frac{-\mu b_2+\sqrt{\mu^2 b_2^2+8\mu c_2(\omega_2+a_2\kappa_2^2)}}{2B^2\beta_0 c_2}\times(-\lambda+\sqrt{\lambda^2-4\mu})\times\coth\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi\Big]\times e^{-\kappa_1 x+\omega_1 t+\theta_1} \quad (27)$$

and

$$v_2(x,t)=\beta_0\Big[1-\frac{\omega_2+a_2\kappa_2^2}{B^2 a_2\lambda\mu}\Big\{(-\lambda+\sqrt{\lambda^2-4\mu})\coth\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi\Big\}\Big]\times e^{-\kappa_2 x+\omega_2 t+\theta_2} \quad (28)$$

If $C_1\neq 0$ but $C_2=0$, we obtain dark solitons as follows

$$u_3(x,t)=\pm\frac{-\mu b_2+\sqrt{\mu^2 b_2^2+8\mu c_2(\omega_2+a_2\kappa_2^2)}}{2B^2\beta_0 c_2}\times\Big[(-\lambda+\sqrt{\lambda^2-4\mu})\tanh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi\Big]\times e^{-\kappa_1 x+\omega_1 t+\theta_1} \quad (29)$$

and

$$v_3(x,t)=\beta_0\Big[1-\frac{\omega_2+a_2\kappa_2^2}{B^2 a_2\lambda\mu}\times\Big\{(-\lambda+\sqrt{\lambda^2-4\mu})\tanh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi\Big\}\Big]\times e^{-\kappa_2 x+\omega_2 t+\theta_2} \quad (30)$$

(ii) If $C_1\neq 0$, $C_1>C_2$, we also obtain

$$u_4(x,t)=\Big[\pm\frac{-\mu b_2+\sqrt{\mu^2 b_2^2+8\mu c_2(\omega_2+a_2\kappa_2^2)}}{2B^2\beta_0 c_2}\times\Big\{(-\lambda+\sqrt{\lambda^2-4\mu})\tanh\Big(\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+\xi_0\Big)\Big\}\Big]\times e^{-\kappa_1 x+\omega_1 t+\theta_1} \quad (31)$$

and

$$v_4(x,t)=\beta_0\Big[1-\frac{\omega_2+a_2\kappa_2^2}{B^2 a_2\lambda\mu}\times\Big\{(-\lambda+\sqrt{\lambda^2-4\mu})\tanh\Big(\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+\xi_0\Big)\Big\}\Big]\times e^{-\kappa_2 x+\omega_2 t+\theta_2}. \quad (32)$$

<u>Case II</u>: When $\lambda^2-4\mu<0$, we obtain

$$u_5(x,t)=\pm\frac{-\mu b_2+\sqrt{\mu^2 b_2^2+8\mu c_2(\omega_2+a_2\kappa_2^2)}}{2\beta_0 c_2}\Big[(-\lambda+\sqrt{4\mu-\lambda^2})\times\Big(\frac{-C_1\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi+C_2\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}{C_1\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi+C_2\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}\Big)\Big]\times e^{-\kappa_1 x+\omega_1 t+\theta_1} \quad (33)$$

and

$$v_5(\xi)=\beta_0\Big[1-\frac{\omega_2+a_2\kappa_2^2}{B^2 a_2\lambda\mu}\Big\{-\lambda+\sqrt{4\mu-\lambda^2}\times\Big(\frac{-C_1\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi+C_2\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}{C_1\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi+C_2\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}\Big)\Big\}\Big]\times e^{-\kappa_2 x+\omega_2 t+\theta_2} \quad (34)$$

where C_1 and C_2 are constants. If C_1 and C_2 assume specific values, singular periodic solutions are obtained:

(i) If $C_1=0$ but $C_2\neq 0$, we obtain trigonometric traveling wave solution as follows

$$u_6(x,t)=\pm\frac{-\mu b_2+\sqrt{\mu^2 b_2^2+8\mu c_2(\omega_2+a_2\kappa_2^2)}}{2B^2\beta_0 c_2}\times\Big[(-\lambda+\sqrt{4\mu-\lambda^2})\cot\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi\Big]e^{-\kappa_1 x+\omega_1 t+\theta_1} \quad (35)$$

and

$$v_6(x,t)=\beta_0\Big[1-\frac{\omega_2+a_2\kappa_2^2}{B^2 a_2\lambda\mu}\times\Big\{(-\lambda+\sqrt{4\mu-\lambda^2})\cot\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi\Big\}\Big]e^{-\kappa_2 x+\omega_2 t+\theta_2} \quad (36)$$

(ii) If $C_1\neq 0$ but $C_2\neq 0$, we obtain

$$u_7(x,t)=\pm\frac{-\mu b_2+\sqrt{\mu^2 b_2^2+8\mu c_2(\omega_2+a_2\kappa_2^2)}}{2B^2\beta_0 c_2}\times\Big[(-\lambda+\sqrt{4\mu-\lambda^2})\tan\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi\Big]e^{-\kappa_1 x+\omega_1 t+\theta_1} \quad (37)$$

and

$$v_7(\xi)=\beta_0\Big[1-\frac{\omega_2+a_2\kappa_2^2}{B^2 a_2\lambda\mu}\times\Big\{(-\lambda+\sqrt{4\mu-\lambda^2})\tan\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi\Big\}\Big]e^{-\kappa_2 x+\omega_2 t+\theta_2}. \quad (38)$$

If $C_1\neq 0$, $C_1>C_2$, we obtain

$$u_8(x,t) = \pm \frac{-\mu b_2 + \sqrt{\mu^2 b_2^2 + 8\mu c_2(\omega_2 + a_2\kappa_2^2)}}{2B^2\beta_0 c_2} \times \Big[(-\lambda + \sqrt{\lambda^2 - 4\mu}) \tanh\Big(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + \xi_0\Big)\Big] \times e^{-\kappa_1 x + \omega_1 t + \theta_1} \tag{39}$$

And

$$v_8(x,t) = \beta_0\Big[1 - \frac{\omega_2 + a_2\kappa_2^2}{B^2 a_2 \lambda\mu} \times \Big\{(-\lambda + \sqrt{4\mu - \lambda^2}) \tan\Big(\frac{1}{2}\sqrt{4\mu - \lambda^2}\xi + \xi_0\Big)\Big\}\Big] e^{-\kappa_2 x + \omega_2 t + \theta_2} \tag{40}$$

Case III: When $\lambda^2 - 4\mu = 0$, we obtain the plane wave solutions.

$$u_9(x,t) = \pm \frac{-\mu b_2 + \sqrt{\mu^2 b_2^2 + 8\mu c_2(\omega_2 + a_2\kappa_2^2)}}{2B^2\beta_0 c_2} \times \Big[\frac{-\lambda}{2} + \frac{C_2}{C_1 + C_2\xi}\Big] e^{-\kappa_1 x + \omega_1 t + \theta_1} \tag{41}$$

and

$$v_9(x,t) = \beta_0\Big[1 - \frac{\omega_2 + a_2\kappa_2^2}{B^2 a_2 \lambda\mu}\Big\{\frac{-\lambda}{2} + \frac{C_2}{C_1 + C_2\xi}\Big\}\Big] \times e^{-\kappa_2 x + \omega_2 t + \theta_2} \tag{42}$$

5. Conclusions

In this paper, (G'/G)-expansion method was employed to secure Thirring dark and singular optical soliton solutions to birefringent fibers with parabolic law nonlinearity. These soliton solutions appear with their respective constraints that guarantee the existence of these solitons. Additionally, singular periodic solitons and plane wave solutions are also retrieved as a byproduct of this method. The results of the application of the integration scheme is highly promising. Later, this method will be applied to other situations in nonlinear optics that will lead to promising results which will be reported elsewhere.

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References

[1] A. Biswas, A. H. Bhrawy, A. A. Alshaery, E. M. Hilal, Optik, **125**(17), 4946 (2014).
[2] A. Biswas, M. Mirzazadeh, Optik, **125**(17), 4589 (2014).
[3] J. Vega-Guzman, Q. Zhou, A. A. Alshaery, E. M. Hilal, A. H. Bhrawd, A. Biswas, J. Optoelectron. Adv. M. **17**, 165 (2015).
[4] J-V. Guzman, E. M. Hilal, A. A. Alshaery, A. H. Bhrawy, M. F. Mahmood, L. Moraru, A. Biswas, Proceedings of the Romanian Academy, Series A, **16**(1), 41 (2015).
[5] M. Younis, H. Ur Rehman, M. Iftikhar, Applied Mathematics & Computation, **249**, 81 (2014).
[6] M. Younis, Mathematical Science Letters, **3**(3), 193 (2014).
[7] M. Younis, S. Ali, to be appeared in Open Engineering.
[8] M. Younis, M. Iftikhar, H. Ur Rehman, Journal of Advanced Physics, **3**(1), 77 (2014).
[9] M. Younis, S. Ali, Applied Mathematics & Computation. **246**, 460 (2014).
[10] Q. Zhou, Q. Zhu, Y. Liu, H. Yu, P. Yao, A. Biswas, Laser Physics, **25**(1), 015402 (2015).

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