The effect of hydrostatic pressure on the diamagnetic susceptibility of a magneto-donor in a GaAs cylindrical quantum dot

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The binding energy and diamagnetic susceptibility of shallow hydrogenic impurity in a Cylindrical Quantum Dot (CQD) is calculated using a variational approach within the effective mass approximation. Numerical calculations were performed for a GaAs-based CQD as a function of dot size, under simultaneous influence of hydrostatic pressure and magnetic field. The impurity binding energy increases with decreasing dot size and increasing pressure. The diamagnetic susceptibility increases with hydrostatic pressure and decreasing dot size. The absolute value of diamagnetic susceptibility increases with QD radius or reducing magnetic field strength. Applied magnetic field and hydrostatic pressure strongly affect the diamagnetic susceptibility.

1. Introduction

With the rapid and tremendous progress of nanotechnology in the last decade, making of high quality low-dimensional semiconductor structures is essential for both fudamental study of their novel physical properties and wide range of applications [1–3]. Using the advanced growth techniques, quantum well (QW), quantum wire and quantum dot (QD) structures with well-controlled dimensions and compositions have been successfully fabricated. Their unusual physical properties are greatly affected by quantum confinement and the discreteness of states.

The diamagnetic susceptibility and binding energy in low-dimensional semiconductor systems are subjects of current interest due to promising applications in nanoelectronic and optoelectronic devices. In this regard, study of the effect of external perturbations (such as hydrostatic pressure and magnetic field) on these properties was an important research topic of QDs in the past several years. Recently, Peter and Navaneethakrishnan [4] studied the effect of the pressure and temperature on donors in a GaAlAs/GaAs QW. Many works have been done on the binding energy and diamagnetic susceptibility of QDs under the influence of pressure, temperature, electric, and magnetic field. Gerardin Jayam and Navaneethakrishnan [5] have calculated the effects of electric field and hydrostatic

pressure on the donor binding energy in a spherical QD with parabolic confinement potential. Simultaneous effects of pressure and magnetic field on the donor state (binding energy) in a parabolic GaAs-GaAlAs QD were studied by Perez-Merchancano, Paredes-Gutierrez, and Silva-Valencia [6]. Pressure dependence of the diamagnetic susceptibility of donors in low-dimensional semiconductors was studied by Elangovan and Navaneethakrishnan [7]. Recently, conduction band non-parabolicity effects on the donor states in spherical quantum dots (SQDs) have been investigated by Razei, Doostimotagh and Vaseghi [8], using the variational method. Kilicarslan et al [9] have studied the magnetic field effects on the diamagnetic susceptibility in a $Ga_xIn_{1-x}N_yAs_{1-y}$/GaAs QW and found that the diamagnetic susceptibility and binding energy of the magneto donors increase with Nitrogen mole fraction. Safarpour et al [10] have calculated the binding energy and diamagnetic susceptibility of a hydrogenic donor impurity in a SQD placed at the center of a cylindrical nanowire. Their results show that the binding energy and diamagnetic susceptibility decrease, reach a minimum value and then increase as the nanowire radius increases. El Ghazi, Jorio and Zorkani [11] have studied the dependence of the binding energy as a function of external magnetic field and donor position in a GaN/(Ga,In)N/GaN spherical quantum dot–quantum well (SQDQW). Their results show that the magnetic field effect is more marked in the large layer

than in thin layer and it is more pronounced in the center of spherical layer than in its extremities. Mmadi et al [12, 13] have calculated the effects of an applied magnetic field on the diamagnetic susceptibility of a shallow donor confined to move in a spherical homogeneous quantum dot (HQD) and in a cylindrical quantum dot (CQD). They have found that the diamagnetic susceptibility increases with the magnetic field strength and is more important especialy for larger QD structures. Surajit Saha et al [14] have studied the simultaneous influence of hydrostatic pressure and temperature on diamagnetic susceptibility of impurity doped quantum dots under the aegis of noise. Their results show that the remarkable role played by the interplay between noise, hydrostatic pressure and temperature in controlling the effective confinement imposed on the system which bears unquestionable relevance. The aim of the present paper is to study the simultaneous effects of the hydrostatic pressure and the magnetic field on the binding energy and diamagnetic susceptibility of a donor impurity confined to move in a CQD, within the effective mass approximation, by using the variational approach. The ground state energy and susceptibility are computed as functions of the hydrostatic pressure, quantum size and strength of magnetic field. The quantum confinement with the infinity deep potential well is described.

2. General formalism

We consider a donor impurity located at the center of a CQD with radius R and length H (see Fig. 1) in the presence of an applied magnetic field $\vec{B}$ along the z-direction.
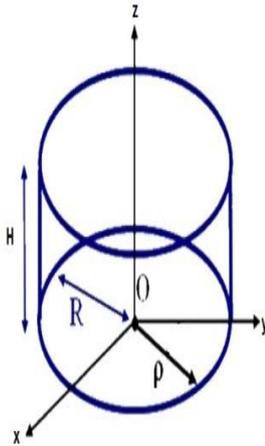
Fig. 1. Schematic representation of cylindrical quantum dot (CQD)

In the effective mass approximation, the Hamiltonian of the donor placed in the center of QD, can be written in cylindrical coordinates in the form [12]:

$$H = -\nabla^2 - \frac{2}{\sqrt{\rho^2 + z^2}} + \frac{\gamma^2}{4}\rho^2 + \gamma L_z + V(\rho, z), \quad (1)$$

where ρ and z are the electron coordinates in the planes perpendicular and along the cylinder axis, respectively; L_z is the z component of the angular momentum operator. The dimensionless parameter γ is given by $\gamma = \hbar\omega_c / R^*$ where $\omega_c = eB/m^*c$ is the cyclotron frequency. We use the effective Bohr radius $a_B^*(P) = \hbar^2\varepsilon(P)/m^*(P)e^2$ and the effective Rydberg energy $R_B(P)^* = m^*(P)e^4 / 2\hbar^2\varepsilon^2(P)$ as the units of length and energy, respectively. $V(\rho, z)$ is the confining potential given by:

$$V(\rho, z) = \begin{cases} 0 & for \ \ \rho < R \ \ and \ \ |z| < \dfrac{H}{2} \\ \infty & for \ \ \rho > R \ \ and \ \ |z| > \dfrac{H}{2} \end{cases} . \quad (2)$$

Since the Schrödinger equation cannot be solved exactly, we follow the Hass variational method for the impurity ground-state and we choose the wave function as [14]:

$$\psi(\rho, z) = \begin{cases} NJ_0\left(\theta_0\dfrac{\rho}{R}\right)\cos\left(\dfrac{\pi z}{H}\right)\exp\left[\left(-\dfrac{\rho^2}{8\alpha^2} + \dfrac{z^2}{8\beta^2}\right)\right] \\ 0 \end{cases} \quad (3)$$

for $\left(\rho \leq R \text{ and } |z| \leq \dfrac{H}{2}\right)$ and $\left(\rho > R \text{ and } |z| > \dfrac{H}{2}\right)$, respectively, where J_0 is the Bessel function of zero order; $\theta_0 = 2.40482$ is its first zero, α and β are variational parameters and N is the normalization constant. The corresponding energy is obtained by minimizing the ground-state energy $\langle H \rangle$ with respect to the variational parameters α and β.

The application of hydrostatic pressure modifies the lattice constants, the effective Rydberg energy, the Bohr radius and the effective masses. We consider a CQD made out of GaAs, for which the variation of dielectric constant and effective mass with pressure is given by [6, 14,15]:

$$\varepsilon(P) = 13.13 - 0.088P,$$
$$m^*(P) = m^*(0)\exp(0.078P),$$

where $m^*(0) = 0.067m_0$ is the effective mass without pressure and m_0 is the free electron mass. The above expression was determined at $T = 300$ K, with pressure P expressed in GPa.

The binding energy of the donor impurity located at the center of a cylindrical quantum dot is given by:

$$\mathrm{E}_b = E_{Sub} - \langle H \rangle_{\min}, \quad (4)$$

where $\langle H \rangle_{\min}$ is the minimum of the Hamiltonian obtained by numerical minimization of the ground-state energy with respect to the parameters α and β.

The diamagnetic susceptibility of a hydrogenic donor, χ_{dia}, is given by [13]:

$$\chi_{dia} = -\frac{e^2}{6m*(P)\varepsilon(P)c^2} < (\vec{r})^2 >, \qquad (5)$$

where c is the speed of light in vacuum and $< (\vec{r})^2 >$ is the mean square distance of the electron from the nucleus.

3. Results and discussion

Let's consider a CQD made out of GaAs, the effective mass $m* = 0.067\ m_0$ and dielectric constant ε_0=13.13 at T=300 K [14]. We obtain then for the effective Bohr radius, $a* = 98.6$ Å and the effective Rydberg energy, $R* = 5.85$ meV.

The binding energy as a function of radius R for different magnetic fields ($\gamma = 0, 1$ and 3) is presented in Fig. 2.

From this figure, it can be observed that in the strong confinement case ($R<1.5\ a*$), the binding energy is relatively insensitive to the magnetic field and is identical to that of zero magnetic field case. This explains that fact the main contribution to the binding energy is provided by geometrical confinement and that the spatial confinement of electrons is prevailing over the magnetic field confinement. For a weak confinement ($R>1.5a*$), the different magnetic field curves tend to deviate from each other and reach steady values as the dot radii increase. We can see that in the absence of the magnetic field, the binding energy tends to the bulk semiconductor value ($E_b \rightarrow 1R*$). While, for a given value of the magnetic field strength, the binding energy is larger than in the absence of the magnetic field.
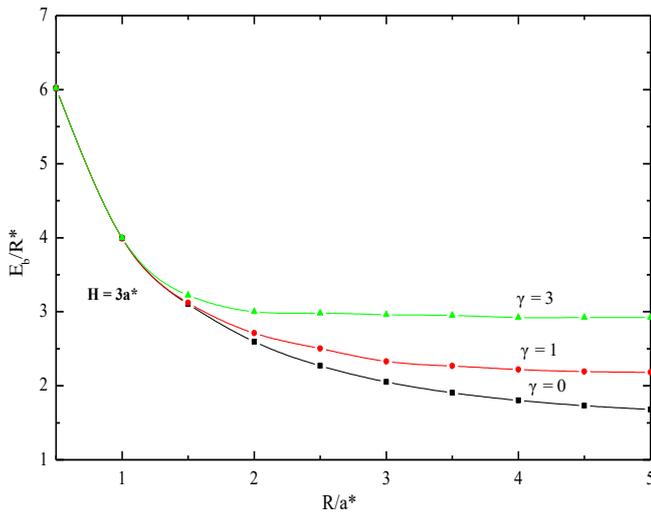
The physical meaning of this is that increasing the strength of the magnetic field shrinks the electron wave function and decreases the cyclotron radius for the electron with respect to the quantum radius and confines the electron closer to the on-center impurity.

Fig. 3 presents the variation of the donor binding energy in a CQD with $H= 1a*$, $R=1a*$, as a function of static perssure P and for three values of the magnetic field strength ($\gamma = 0, 1$ and 2). The binding energy increases with the increment of hydrostatic pressure and decreases for a given dot due to the influence of magnetic field. It is well known that the dielectric constant decreases, while the effective mass of electron increases on increasing pressure [6,15,16].
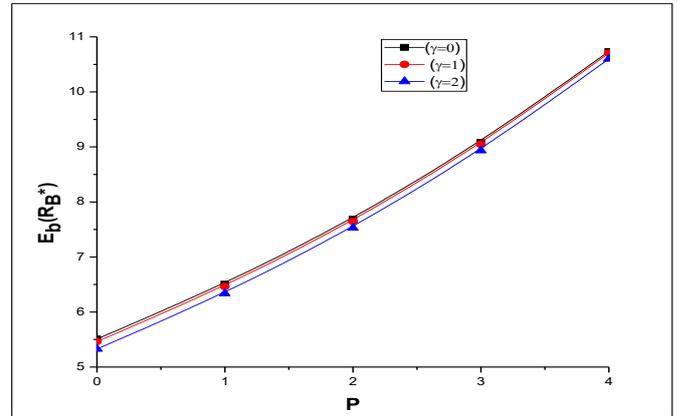
Fig. 3. Variation of donor binding energy as a function of static pressure P for a CQD (H= 1a*and R=1a*)and for three values of magnetic field strength (γ = 0, 1 and 2)

Fig. 4 shows the variation of binding energy as a function of radius R for different values of hydrostatic pressure and magnetic field strength, applied simultaneously. The binding energy decreases as the dot radius increases, as expected. It increases with the magnetic field strength for all dot sizes, when both external perturbations are applied. For a larger dot radius, the binding energy for $\gamma = 0$ approaches that of the bulk case. The result shows that the behavior of binding energies is purely pressure dependent for smaller dots. For the larger dot radius, the effect of both external perturbations seems to take part equally.

In order to investigate the effect of magnetic field, we display in Fig. 5 the diamagnetic susceptibility χ_{dia} as a function of the radius R for different values of the lenght H ($H = 1a*$ and $H=3a*$) with several values of magnetic field strength ($\gamma = 0, 0.5$ and 1).

Fig. 2. Variation of donor binding energy as a function of R and H=3a* of a CQD for three values of magnetic field (γ = 0, 1 and 3)
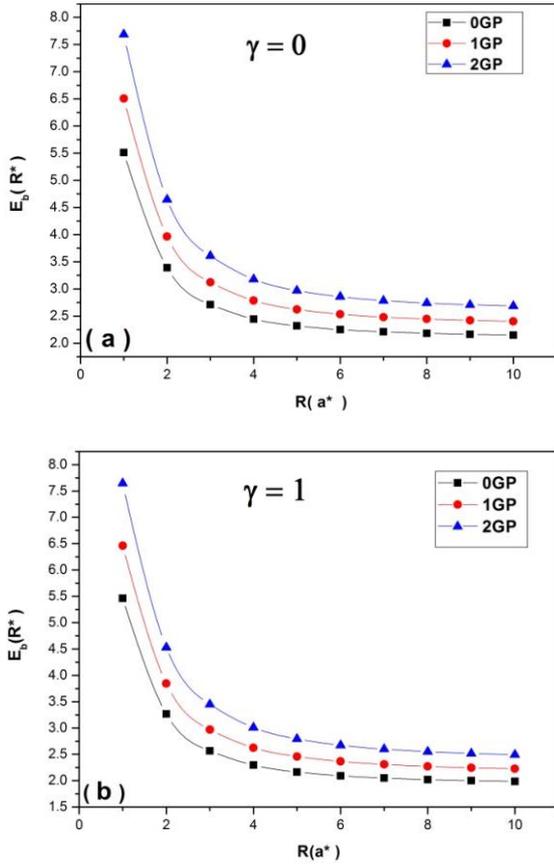
Fig. 4. Variation of donor binding energy as a function of R for a CQD (H= 1a*), for magnetic field strength γ = 0 (a) and γ = 1 (b) and for three values of pressure (P=0,1 and 2 GP)
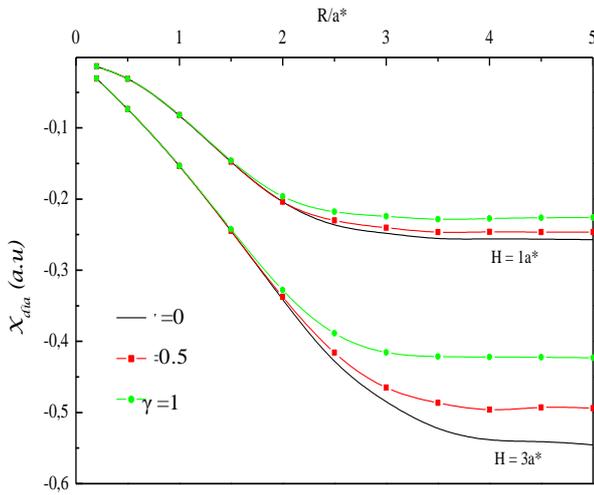
Fig. 5. Variation of the diamagnetic susceptibility χ_dia as a function of R with three values of the magnetic field strength (γ = 0, 0.5 and 1), H= 1 a* and H= 3 a*

There is a competition between the geometric confinement and the magnetic confinement. One can ascertain that for a strong radial confinement (R<1.5a*), the magnetic field effect on the diamagnetic susceptibility is not remarkable.

The diamagnetic susceptibility increases together with magnetic field strength. This increase is due to a shrinking of the charge distribution when an external magnetic field is applied. Furthermore, for given values of R and $γ$, the absolute value of the diamagnetic susceptibility $|χ_{dia}|$ increases for increasing dot length, which reflects the diminution of confinement. These results explain that in the presence of the magnetic field, the diamagnetic susceptibility $χ_{dia}$ remain constant for small quantum dots and dcereases for large dots [17, 18].
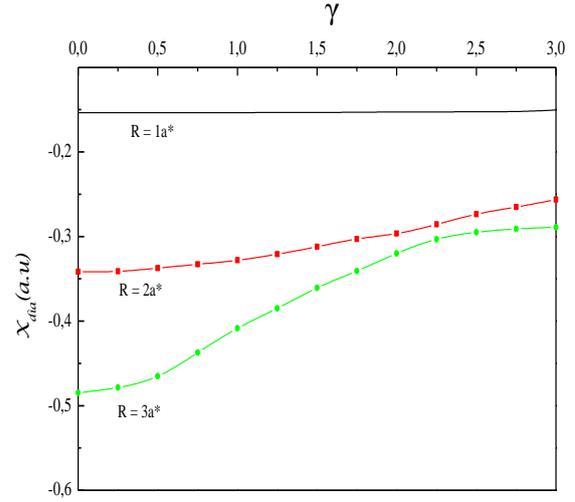
Fig. 6. Variation of the diamagnetic susceptibility χ_dia as a function of magnetic field with three values R (R= 1 a*, 2 a* and 3 a*) and H_c = 3 a*

In Fig. 6, the variation of the diamagnetic susceptibility as a function of the magnetic field strength for fixed QD geometries is presented. This figure reflects correctly the effect of the magnetic field, which confines more the electron and decreases the absolute value of diamagnetic susceptibility $|χ_{dia}|$. We took the variation of the donor diamagnetic susceptibility $|χ_{dia}|$ for three different radius values ($R=1$ $a*$, 2 $a*$ and 3 $a*$) and $H=3$ $a*$.

We can remark that the diamagnetic susceptibility $|χ_{dia}|$ decreases as CQD radius R decreases. The diamagnetic susceptibility is completely insensitive to the increase in the magnetic field strength for small CQD ($R=1$ $a*$). For large CQD ($R>2$ $a*$), the variation of the diamagnetic susceptibility is much more pronounced, due to the stronger confinement effect of the magnetic field. From the two forms of the dot, it is clearly seen that the absalute value of diamagnetic susceptibility $|χ_{dia}|$ increases upon increasing QD radius or decreasing magnetic field strength.

Fig. 7 shows that the variation of the diamagnetic susceptibility $χ_{dia}$ as a function of radius R with $H=1$ $a*$ and for three values of pressure ($P= 0, 1$ and 2 GPa) in presence and in absence of the magnetic field ($γ= 0$ and 1).

We observe that the hydrostatic pressure exerts an influence on the diamagnetic susceptibility. Its effect is

more important for large QDs. For small QDs, the diamagnetic susceptibility χ_{dia} is insensitive to the hydrostatic pressure. For large dots, its effect becomes more pronouced, when the pressure increases, the absolute value of the diamagnetic $|\chi_{dia}|$ decreases.
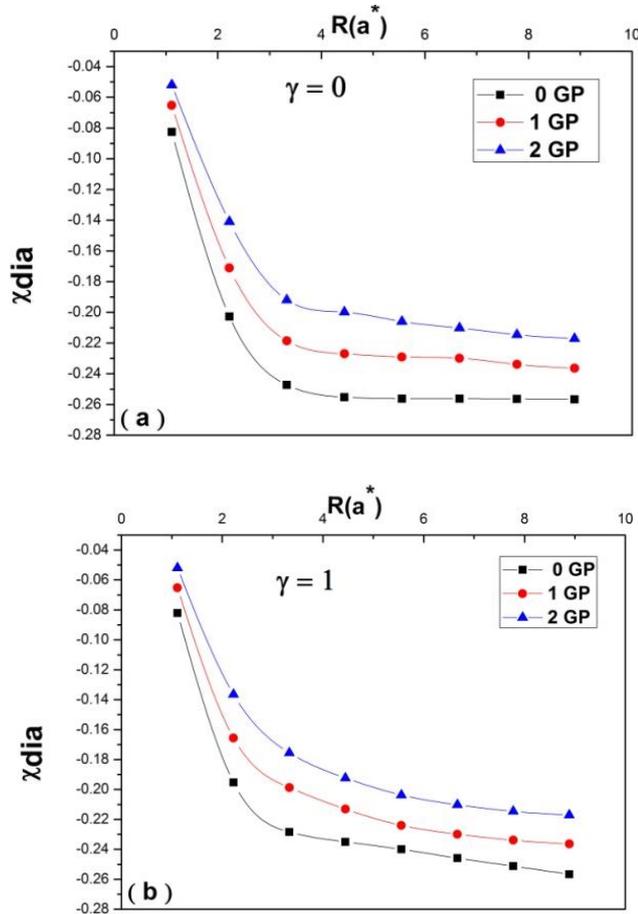
Fig. 7. Variation of the diamagnetic susceptibility χ_{dia} as a function of R with H= 1 a* for three values of perssure (P= 0, 1 and 2 GPa) and for two values of magnetic field strength [$\gamma = 0$ (a), $\gamma = 1$(b)]

4. Conclusion

In the present work, we have reported the study of hydrostatic pressure and the magnetic field effects on the diamagnetic susceptibility of a hydrogenic donor placed in cylindrical quantum dot, by using the effective mass approximation. The diamagnetic susceptibility is dramatically dependent on the size of the cylindre and increases with dot radius.

The absalute value of the diamagnetic susceptibility $|\chi_{dia}|$ increases as the QD radius increases or the magnetic field strength decreases. The simultaneous presence of a magnetic field and a hydrostatic pressure influences the diamagnetic susceptibility, especialy for nanostructures with a wide range of QD sizes.

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