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The edge version of geometric arithmetic index of nanotubes and nanotori

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The edge version of geometric arithmetic index of graph G, abbreviated as $GA_e(G)$, is defined as $GA_e(G) = \sum_{ef\in E(L(G)} \frac{2\sqrt{d_{L(G)}(e)d_{L(G)}(f)}}{d_{L(G)}(e)+ d_{L(G)}(f)}$, where $d_{L(G)}(e)$ denotes the degrees of an edge e of line graph of G. In this paper, the closed formulas for GA_e index for some nanotubes and nanotorus are given.

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1. Introduction

A single number that can be used to characterize some property of the graph of a molecule is called a topological index for that graph. There are numerous topological descriptors that have found some applications in theoretical chemistry, especially in QSPR/QSAR research [16]. The oldest topological index which introduced by Harold Wiener in 1947 is ordinary (vertex) version of Wiener index [18] which is the sum of all distances between vertices of a graph. Also, the edge version of Wiener index which were based on distance between edges introduced by Iranmanesh et al. in 2008 [10].

One of the most important topological indices is the well-known branching index introduced by Randic [15] which is defined as the sum of certain bond contributions calculated from the vertex degree of the hydrogen suppressed molecular graphs.

Motivated by the definition of Randic connectivity index based on the end-vertex degrees of edges in a connected graph G with the vertex set $V(G)$ and the edge set $E(G)$ [9,11]. Vukicevic and Furtula [17] proposed a topological index named the geometric-arithmetic index (simply GA) as

$$GA(G) = \sum_{uv\in E(G)} \frac{2\sqrt{d_G(u)d_G(v)}}{d_G(u)+d_G(v)}$$

where $d_G(u)$ denotes the degree of the vertex u in G. The reader can find more information about geometric-arithmetic index in [7,17,20].

In [12], the edge version of geometric-arithmetic index was introduced based on the end-vertex degrees of edges in a line graph of G which is a graph such that each vertex of $L(G)$ represents an edge of G; and two vertices of $L(G)$ are adjacent if and only if their corresponding edges share a common endpoint in G, as follows

$$GA_e(G) = \sum_{ef\in E(L(G)} \frac{2\sqrt{d_{L(G)}(e)d_{L(G)}(f)}}{d_{L(G)}(e)+ d_{L(G)}(f)}$$

where $d_{L(G)}(e)$ denotes the degree of the edge e in G. The edge version of GA index of benzenoid graph was studied by Farahaini [6]. The total version of GA index was considered in [13,14].

Carbon nanotubes form an interesting class of carbon nanomaterials. There are three types of nanotubes: armchair, chiral and zigzag structures. Carbon nanotubes show remarkable mechanical properties. Experimental studies have shown that they belong to the stiffest and elastic known materials. Diudea was the first chemist who considered the problem of computing topological indices of nanostructures (see for examples [2-5]). In this paper, we continue this program and compute the edge version of GA index for some families of nanotubes and nanotours.

2. Results and discussion

In this section, we first give the edge version of *GA* index of some standard graphs.

Example 1. Let P_n be a path with n vertices. Then, the edge version of geometric arithmetic index of P_n is

$$GA_e(P_n) = GA(P_{n-1}) = \frac{4\sqrt{2}}{3} + (n-4).$$

Example 2. Let S_n be a star graph with n vertices. Then, the edge version of geometric arithmetic index of S_n is

$$GA_e(S_n) = \binom{n-1}{2}.$$

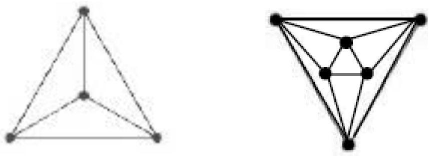
Example 3. Let K_n be a complete graph with n vertices. Then, the edge version of geometric arithmetic index of complete graph, K_n is

$$GA_e(K_n) = GA\big((2n-1) - regular\ graph\big) = \frac{n(n-1)^2}{2}.$$

Example 4. Let C_n be a cycle with n vertices. Then, the edge version of geometric arithmetic index of C_n is

$$GA_e(C_n) = GA(C_n) = n.$$

Example 5. Consider the wheel graph W_4 as depicted in Figure 1. The line graph of W_4, $L(w_4)$ has 12 edges. Then, there are 12 edges with $d_{L(G)}(e) = d_{L(G)}(f) = 4$. Thus, we have $GA_e(W_4) = 12$.



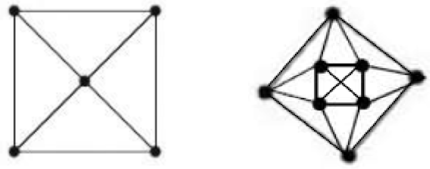
W_4

$L(W_4)$

Fig. 1. Graphs W_4 and $L(W_4)$

Example 6. Consider the wheel graph W_5 as depicted in Fig. 2. The line of W_5, $L(W_5)$ has 18 edges. Then, there are 4 edges with $d_{L(G)}(e) = d_{L(G)}(f) = 4$, 8 edges with type of $d_{L(G)}(e) = 4$, $d_{L(G)}(f) = 5$ and 6 edges with $d_{L(G)}(e) = d_{L(G)}(f) = 5$. Thus we have

$$GA_e(W_5) = 4\left(1 + \frac{8\sqrt{5}}{9} + \frac{3}{2}\right)$$



W_5 $L(W_5)$

Fig. 2. Graphs W_5 and $L(W_5)$

Lemma 1. Let W_n be the wheel graph with n vertices. Then, the edge version of geometric arithmetic index of W_n is

$$GA_e(W_n) = (n-1)\left(1 + \frac{8\sqrt{n}}{4+n} + \frac{n-2}{2}\right)$$

Proof. By continuing an induction argument on *n*, one can check that in general, the line graph of W_n has $3(n-1) + |K_{n-1}|$ edges. On the hand, there are $n-1$ edges with $d_{L(G)}(e) = d_{L(G)}(f) = 4$, $2(n-1)$ edges of type $d_{L(G)}(e) = 4$, $d_{L(G)}(f) = n$, $|K_{n-1}|$ edges with $d_{L(G)}(e) = d_{L(G)}(f) = n$. Since

$$GA_e(G) = \sum_{ef \in E(L(G)} \frac{2\sqrt{d_{L(G)}(e) d_{L(G)}(f)}}{d_{L(G)}(e) + d_{L(G)}(f)}$$

this implies

$$GA_e(W_n) = (n-1)\left(\frac{2\sqrt{4\times 4}}{4+4}\right) + 2(n-1)\left(\frac{2\sqrt{4\times n}}{4+n}\right) + \frac{(n-1)(n-2)}{2}\left(\frac{2\sqrt{n\times n}}{n+n}\right).$$

After an easy simplification, we obtain

$$GA_e(W_n) = (n-1)\left(1 + \frac{8\sqrt{n}}{4+n} + \frac{n-2}{2}\right).$$

Now the proof is complete.

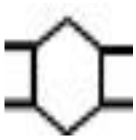
2.1 Edge Version of *GA* Index of $TUC_4C_6C_8[p,q]$ nanotube

We now compute the edge version of *GA* index for $TUC_4C_6C_8[p,q]$ nanotube, with q rows and p column. Firstly, we consider the following examples.

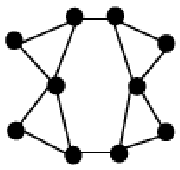
Example 7. Consider the graph of 2-dimensional lattice of $TUC_4C_6C_8[1,1]$ nanotube as depicted in Fig. 4. The line graph of $TUC_4C_6C_8[1,1]$ has 14 edges. If $d_{L(G)}(e)$

and $d_{L(G)}(f)$ be the degree of edge of e, then there are 2 edges with $d_{L(G)}(e) = d_{L(G)}(f) = 3$, 8 edges of type $d_{L(G)}(e) = 3$, $d_{L(G)}(f) = 4$, 4 edges with $d_{L(G)}(e) = d_{L(G)}(f) = 4$. Thus we have

$$GA_e(TUC_4C_6C_8[1,1]) = 18 + 4\left(\frac{8\sqrt{3}}{7} - 3\right)$$



$TUC_4C_6C_8[1,1]$

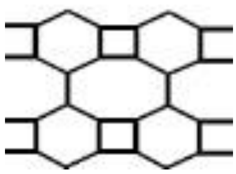


$L(TUC_4C_6C_8[1,1])$

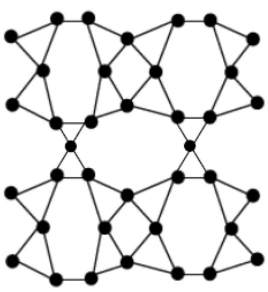
Fig. 4. Graph of 2-dimensional lattice of nanotube $TUC_4C_6C_8[1,1]$ and $L(TUC_4C_6C_8[1,1])$

Example 8. Consider the graph of 2-dimensional lattice of $TUC_4C_6C_8[2,2]$ nanotube as depicted in Fig. 5. The line graph of $TUC_4C_6C_8[2,2]$ has 64 edges. Also, there are 4 edges with $d_{L(G)}(e) = d_{L(G)}(f) = 3$, 16 edges of type $d_{L(G)}(e) = 3$, $d_{L(G)}(f) = 4$, 44 edges with $d_{L(G)}(e) = d_{L(G)}(f) = 4$. Thus, we have

$$GA_e(TUC_4C_6C_8[2,2]) = 72 + 8\left(\frac{8\sqrt{3}}{7} - 3\right)$$



$TUC_4C_6C_8[2,2]$



$L(TUC_4C_6C_8[2,2])$.

Fig. 5. Graph 2-dimensional lattice of $TUC_4C_6C_8[2,2]$ nanotube and $L(TUC_4C_6C_8[2,2])$

Example 9. Consider the graph of2-dimensional lattice of $TUC_4C_6C_8[4,5]$ nanotube as depicted in Fig. 6. The line graph of $TUC_4C_6C_8[4,5]$ has 344 edges. Also, there are 8 edges with $d_{L(G)}(e) = d_{L(G)}(f) = 3$, 32 edges of type $d_{L(G)}(e) = 3$, $d_{L(G)}(f) = 4$, 304 edges with $d_{L(G)}(e) = d_{L(G)}(f) = 4$. In other words,

$$GA_e(TUC_4C_6C_8[4,5]) = 360 + 16\left(\frac{8\sqrt{3}}{7} - 3\right)$$

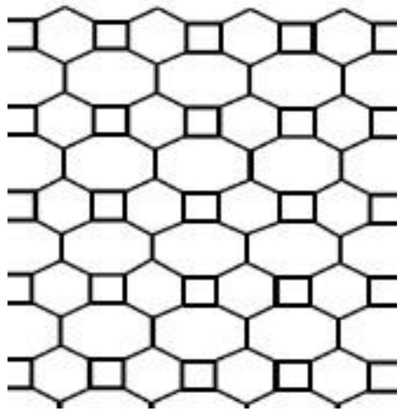


Fig. 6. Graph of 2-dimensional lattice of $TUC_4C_6C_8[4,5]$ nanotube

By continuing this method, one can check that in general, the line graph of $TUC_4C_6C_8[p,q]$ nanotube has $18pq - 4p$ edges. On the hand, there are $2p$ edges with $d_{L(G)}(e) = d_{L(G)}(f) = 3$, $8p$ edges of type $d_{L(G)}(e) = 3$, $d_{L(G)}(f) = 4$, $18pq - 14p$ edges with $d_{L(G)}(e) = d_{L(G)}(f) = 4$.

Now, we can deduce the following result.

Theorem 2. Consider the graph of $TUC_4C_6C_8[p,q]$ nanotube. Then, the GA_e of $TUC_4C_6C_8[p,q]$ nanotube is given as

$$GA_e(TUC_4C_6C_8[p,q]) = 18pq + 4p\left(\frac{8\sqrt{3}}{7} - 3\right).$$

Proof. Let G be the graph of $TUC_4C_6C_8[p,q]$ nanotube. Since

$$GA_e(G) = \sum_{ef\in E(L(G)} \frac{2\sqrt{d_{L(G)}(e)d_{L(G)}(f)}}{d_{L(G)}(e) + d_{L(G)}(f)},$$

this implies that

$$GA_e(TUC_4C_6C_8[p,q) = 2p\left(\frac{2\sqrt{3\times 3}}{6}\right) + 8p\left(\frac{2\sqrt{3\times 4}}{7}\right) + (18pq-14p)\left(\frac{2\sqrt{4\times 4}}{8}\right).$$

After an easy simplification, we obtain

$$GA_e(TUC_4C_6C_8[p,q) = 18pq + 4p\left(\frac{8\sqrt{3}}{7} - 3\right).$$

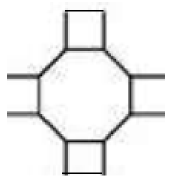
Now the proof is complete.

2.2 Edge Version of *GA* Index of $TUSC_4C_8(S)[m,n]$ nanotube

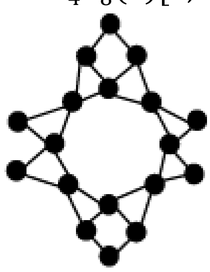
We now compute the edge version of *GA* index for $TUSC_4C_8(S)[m,n]$ nanotube, with *n* rows and *m* column. Firstly, we consider the following examples.

Example 10. Consider the graph of 2-dimensional lattice of $TUSC_4C_8(S)[1,1]$ nanotube as depicted in Figure 7. The line graph of $TUSC_4C_8(S)[1,1]$ has 28 edges. Also, there are 4 edges of type $d_{L(G)}(e)=2$, $d_{L(G)}(f)=3$, 8 edges of type $d_{L(G)}(e)=3$, $d_{L(G)}(f)=4$ and 16 edges with $d_{L(G)}(e)=d_{L(G)}(f)=4$. Thus, we obtain

$$GA_e(TUSC_4C_8(S)[1,1]) = 24 + 8\left(\frac{\sqrt{6}}{5} + \frac{4\sqrt{3}}{7} - 1\right)$$



$TUSC_4C_8(S)[1,1]$



$L(TUSC_4C_8(S)[1,1])$

Fig. 7. Graph of 2-dimensional lattice of $TUSC_4C_8(S)[1,1]$ nanotube and $L(TUSC_4C_8(S)[1,1])$

Example 11. Consider the graph of 2-dimensional lattice of $TUSC_4C_8(S)[2,1]$ nanotube as depicted in Fig. 8. The line graph of $TUSC_4C_8(S)[2,1]$ has 56 edges. Also, there are 8 edges of type $d_{L(G)}(e)=2$, $d_{L(G)}(f)=3$, 16 edges of type $d_{L(G)}(e)=3$, $d_{L(G)}(f)=4$ and 32 edges with $d_{L(G)}(e)=d_{L(G)}(f)=4$. Thus, we have

$$GA_e(TUSC_4C_8(S)[2,1]) = 48 + 16\left(\frac{\sqrt{6}}{5} + \frac{4\sqrt{3}}{7} - 1\right)$$

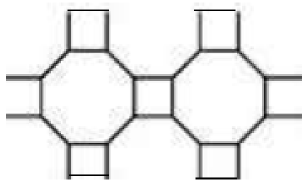


Fig. 8. Graph of 2-dimensional lattice of $TUSC_4C_8(S)[2,1]$ nanotube

Example 12. Consider the graph of 2-dimensional lattice of $TUSC_4C_8(S)[1,2]$ nanotube as depicted in Fig. 9. The line graph of $TUSC_4C_8(S)[1,2]$ has 52 edges. Also, there are 4 edges of type $d_{L(G)}(e)=2$, $d_{L(G)}(f)=3$, 8 edges of type $d_{L(G)}(e)=3$, $d_{L(G)}(f)=4$ and 40 edges with $d_{L(G)}(e)=d_{L(G)}(f)=4$. Thus, we have

$$GA_e(TUSC_4C_8(S)[1,2]) = 48 + 8\left(\frac{\sqrt{6}}{5} + \frac{4\sqrt{3}}{7} - 1\right)$$

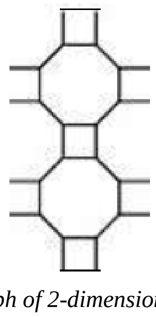


Fig. 9. Graph of 2-dimensional lattice of $TUSC_4C_8(S)[1,2]$ nanotube

Similarly, by continuing the above method in general, the line graph of $TUSC_4C_8(S)[m,n]$ has $24mn+4m$ edges On the hand, there are $4m$ edges of type$d_{L(G)}(e)=2$, $d_{L(G)}(f)=3$, $8m$ edges of type $d_{L(G)}(e)=3$, $d_{L(G)}(f)=4$ and $24mn-8m$ edges with $d_{L(G)}(e)=d_{L(G)}(f)=4$.

Thus, the following result can be obtained.

Theorem 3. Consider the graph of $TUSC_4C_8(S)[m,n]$ nanotube as shown in Fig. 10. Then, the GA_e of $TUSC_4C_8(S)[m,n]$ nanotube is given as

$$GA_e(TUSC_4C_8(S)[m,n]) = 24mn + 8m\left(\frac{\sqrt{6}}{5} + \frac{4\sqrt{3}}{7} - 1\right)$$

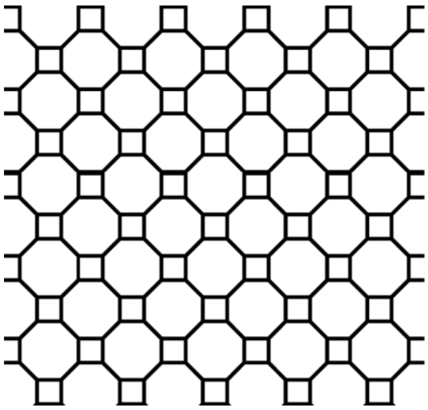


Fig. 10. Graph of 2-dimensional lattice of $TUSC_4C_8(S)[m,n]$ nanotube

Proof. Let G be the graph of $TUSC_4C_8(S)[m,n]$ nanotube. Since

$$GA_e(G) = \sum_{ef \in E(L(G)} \frac{2\sqrt{d_{L(G)}(e) d_{L(G)}(f)}}{d_{L(G)}(e) + d_{L(G)}(f)},$$

this implies that

$$GA_e(TUSC_4C_8(S)[m,n]) = 4m\left(\frac{2\sqrt{2\times 3}}{5}\right) + 8m\left(\frac{2\sqrt{3\times 4}}{7}\right) + (24mn - 8m)\left(\frac{2\sqrt{4\times 4}}{8}\right).$$

After an easy simplification, we obtain

$$GA_e(TUSC_4C_8(S)[m,n]) = 24mn + 8m\left(\frac{\sqrt{6}}{5} + \frac{4\sqrt{3}}{7} - 1\right)$$

Now the proof is complete.

2.3 Edge Version of *GA* Index of *H*-Naphtalenic $NPHX[m,n]$ nanotube

We now compute the edge version of *GA* index for *H*-Naphtalenic $NPHX[m,n]$ nanotube. Firstly, we consider the following examples.

Example 13. Consider the graph of 2-dimensional lattice of *H*-Naphtalenic $NPHX[1,1]$ nanotube as depicted in Fig. 11. The line graph of $NPHX[1,1]$ has 22 edges. Also, there are 6 edges with $d_{L(G)}(e) = d_{L(G)}(f) = 3$, 12 edges of type $d_{L(G)}(e) = 3$, $d_{L(G)}(f) = 4$, 4 edges with $d_{L(G)}(e) = d_{L(G)}(f) = 4$. Thus,

$$GA_e(NPHX[1,1]) = 30 + 4\left(\frac{12\sqrt{2}}{3} - 5\right)$$

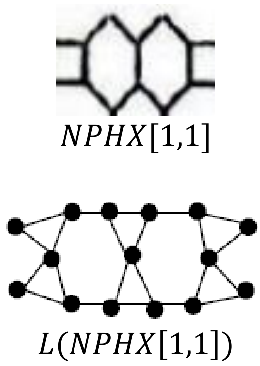


Fig. 11. Graph of 2-dimensional lattice of H-Naphtalenic $NPHX[1,1]$ nanotube and $L(NPHX[1,1])$

Example 14. Consider the graph of 2-dimensional lattice of *H*-Naphtalenic $NPHX[2,2]$ nanotube as depicted in Figure 12. The line graph of $NPHX[2,2]$ has 104 edges. Also, there are 12 edges with $d_{L(G)}(e) = d_{L(G)}(f) = 3$, 24 edges of type $d_{L(G)}(e) = 3$, $d_{L(G)}(f) = 4$, 68 edges with $d_{L(G)}(e) = d_{L(G)}(f) = 4$. Thus,

$$GA_e(NPHX[2,2]) = 120 + 8\left(\frac{12\sqrt{2}}{3} - 5\right)$$

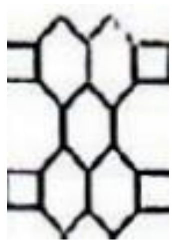


Fig. 12. Graph of 2-dimensional lattice of H-Naphtalenic $NPHX[2,2]$ nanotube

Example 15. Consider the graph of 2-dimensional lattice of *H*-Naphtalenic $NPHX[4,3]$ nanotube as depicted in Fig. 13. The line graph of $NPHX[4,3]$ has 328 edges. Also, there are 24 edges with $d_{L(G)}(e) = d_{L(G)}(f) = 3$, 48 edges of type $d_{L(G)}(e) = 3$, $d_{L(G)}(f) = 4$, 256 edges with $d_{L(G)}(e) = d_{L(G)}(f) = 4$. Thus,

$$GA_e(NPHX[4,3]) = 360 + 16\left(\frac{12\sqrt{2}}{3} - 5\right)$$

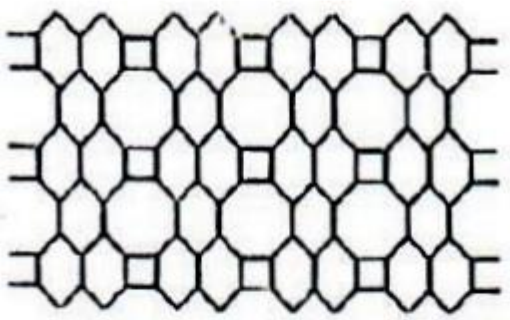


Fig. 13. Graph of 2-dimensional lattice of H-Naphtalenic $NPHX[4,3]$ nanotube

Similarly, by continuing the above method in general, the line graph of $NPHX[m,n]$ has $30mn-8m$ edges. On the hand, there are $6m$ edges with $d_{L(G)}(e)=d_{L(G)}(f)=3$, $12m$ edges of type $d_{L(G)}(e)=3$, $d_{L(G)}(f)=4$, $30mn-26n$ edges with $d_{L(G)}(e)=d_{L(G)}(f)=4$.

The following result can be easily obtained.

Theorem 4. Consider the graph of $NPHX[m,n]$ nanotube. Then, the GA_e of $NPHX[m,n]$ nanotube is

$$GA_e(NPHX[m,n])=30mn+4m\left(\frac{12\sqrt{2}}{3}-5\right)$$

Proof. Let *G* be the graph of $NPHX[m,n]$ nanotube. Since

$$GA_e(G)=\sum_{ef\in E(L(G)}\frac{2\sqrt{d_{L(G)}(e)d_{L(G)}(f)}}{d_{L(G)}(e)+d_{L(G)}(f)},$$

this implies that

$$\begin{aligned}GA_e(NPHX[m,n])&=6m\left(\frac{2\sqrt{3\times 3}}{6}\right)\\&+12m\left(\frac{2\sqrt{3\times 4}}{7}\right)\\&+(30mn-26m)\left(\frac{2\sqrt{4\times 4}}{8}\right).\end{aligned}$$

After an easy simplification, we obtain

$$GA_e(NPHX[m,n])=30mn+4m\left(\frac{12\sqrt{2}}{3}-5\right)$$

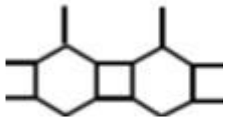
Now the proof is complete.

2.4 Edge Version of *GA* Index of $C_4C_6C_8[p,q]$ nanotori

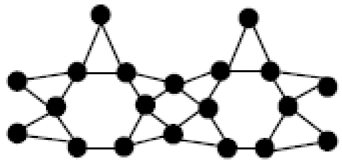
We now compute the edge version of *GA* index for $C_4C_6C_8[p,q]$ nanotori. Firstly, we consider the following examples.

Example 16. Consider the graph of 2-dimensional lattice of $C_4C_6C_8[2,1]$ nanotori as depicted in Figure 14. The line graph of $C_4C_6C_8[2,1]$ has 32 edges. Also, there are 4 edges of type $d_{L(G)}(e)=2$, $d_{L(G)}(f)=4$, 2 edges with $d_{L(G)}(e)=d_{L(G)}(f)=3$, 8 edges of type $d_{L(G)}(e)=3$, $d_{L(G)}(f)=4$, 18 edges with $d_{L(G)}(e)=d_{L(G)}(f)=4$. Thus

$$GA_e(C_4C_6C_8[2,1])=36+8\left(\frac{\sqrt{2}}{3}+\frac{4\sqrt{3}}{7}-2\right).$$



$C_4C_6C_8[2,1]$



$L(C_4C_6C_8[2,1])$

Fig. 14. Graph of 2-dimensional lattice of $C_4C_6C_8[2,1]$ nanotori and $L(C_4C_6C_8[2,1])$

Example 17. Consider the graph of 2-dimensional lattice of $C_4C_6C_8[3,2]$ nanotori as depicted in Fig. 15. The line graph of $C_4C_6C_8[3,2]$ has 102 edges. Also, there are 6 edges of type $d_{L(G)}(e)=2$, $d_{L(G)}(f)=4$, 3 edges with $d_{L(G)}(e)=d_{L(G)}(f)=3$, 12 edges of type $d_{L(G)}(e)=3$, $d_{L(G)}(f)=4$, 81 edges with $d_{L(G)}(e)=d_{L(G)}(f)=4$. Thus,

$$GA_e(C_4C_6C_8[3,2])=108+12\left(\frac{\sqrt{2}}{3}+\frac{4\sqrt{3}}{7}-2\right)$$

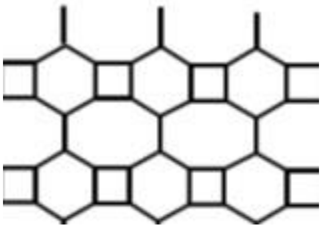


Fig. 15. 2-dimensional lattice of $C_4C_6C_8[3,2]$ *nanotori*

Example 18. Consider the graph of 2-dimensional lattice of $C_4C_6C_8[4,4]$ nanotori as depicted in Fig. 16. The line graph of $C_4C_6C_8[4,4]$ has 280 edges. Also, there are 8 edges of type $d_{L(G)}(e)=2$, $d_{L(G)}(f)=4$, 4 edges with $d_{L(G)}(e)=d_{L(G)}(f)=3$, 16 edges of type $d_{L(G)}(e)=3$, $d_{L(G)}(f)=4$, 252 edges with $d_{L(G)}(e)=d_{L(G)}(f)=4$. Thus,

$$GA_e(C_4C_6C_8[4,4]) = 288 + 16\left(\frac{\sqrt{2}}{3}+\frac{4\sqrt{3}}{7}-2\right)$$

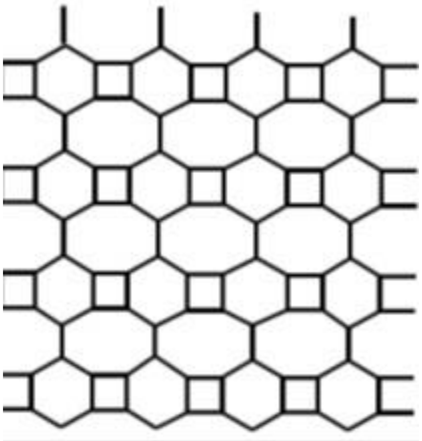


Fig. 16. Graph of 2-dimensional lattice of $C_4C_6C_8[4,4]$ *nanotori*

Similarly, by continuing the above method in general, this line graph has $18pq-2p$ edges. On the hand, there are $2p$ edges of type $d_{L(G)}(e)=2$, $d_{L(G)}(f)=4$, p edges with $d_{L(G)}(e)=d_{L(G)}(f)=3$, $4p$ edges of type $d_{L(G)}(e)=3$, $d_{L(G)}(f)=4$, $18pq-9p$ edges with $d_{L(G)}(e)=d_{L(G)}(f)=4$.

We can now deduce the following result.

Theorem 5. Consider the graph of $C_4C_6C_8[p,q]$ nanotori. Then the GA_e of $C_4C_6C_8[p,q]$ nanotube is given as

$$GA_e(TUC_4C_6C_8[p,q]) = 18pq + 4p\left(\frac{\sqrt{2}}{3}+\frac{4\sqrt{3}}{7}-2\right)$$

Proof. Let *G* be the graph of $C_4C_6C_8[p,q]$ nanotori. Since

$$GA_e(G) = \sum_{ef\in E(L(G)} \frac{2\sqrt{d_{L(G)}(e)d_{L(G)}(f)}}{d_{L(G)}(e)+d_{L(G)}(f)},$$

this implies that

$$\begin{aligned} GA_e(C_4C_6C_8[p,q]) &= 2p\left(\frac{2\sqrt{2\times 4}}{6}\right)+p\left(\frac{2\sqrt{3\times 3}}{6}\right) \\ &+ 4p\left(\frac{2\sqrt{3\times 4}}{7}\right) \\ &+ (18pq-9p)\left(\frac{2\sqrt{4\times 4}}{8}\right) \end{aligned}$$

After an easy simplification, we obtain

$$GA_e(C_4C_6C_8[p,q]) = 18pq + 4p\left(\frac{\sqrt{2}}{3}+\frac{4\sqrt{3}}{7}-2\right)$$

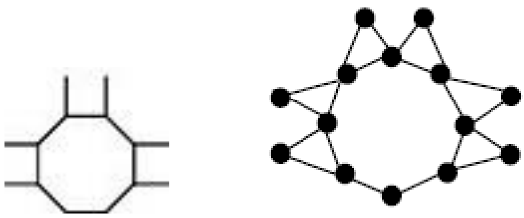
Now the proof is complete.

2.5 Edge Version of *GA* Index of $TC_4C_8(S)\ [p,q]$ nanotori

We now compute the edge version of *GA* index for $TC_4C_8(S)\,[p,q]$ nanotori**.** Firstly, we consider the following examples.

Example 19. Consider the graph of 2-dimensional lattice of $TC_4C_8(S)\ [1,1]$ nanotori as depicted in Figure 17. The line graph of $TC_4C_8(S)\ [1,1]$ has 20 edges. Also, there are 2 edges of type $d_{L(G)}(e)=2$, $d_{L(G)}(f)=3$, 4 edges of type $d_{L(G)}(e)=2$, $d_{L(G)}(f)=4$, 4 edges of type $d_{L(G)}(e)=3$, $d_{L(G)}(f)=4$, 10 edges with $d_{L(G)}(e)=d_{L(G)}(f)=4$. Thus,

$$GA_e(TC_4C_8(S)\ [1,1]) = 24 + 4\left(\frac{\sqrt{6}}{5}+\frac{2\sqrt{3}}{3}+\frac{4\sqrt{3}}{7}-\frac{14}{4}\right)$$



$TC_4C_8(S)\ [1,1]$ $L(TC_4C_8(S)\ [1,1])$

Fig. 17. Graph of 2-dimensional lattice of $TC_4C_8(S)\ [1,1]$ *nanotori and* $L(TC_4C_8(S)\ [1,1])$

Example 20. Consider the graph of 2-dimensional lattice of $TC_4C_8(S)$ [2,2] nanotori as depicted in Fig. 18. This line graph has 88 edges. Also, there are 4 edges of type $d_{L(G)}(e)=2$, $d_{L(G)}(f)=3$, 8 edges of type $d_{L(G)}(e)=2$, $d_{L(G)}(f)=4$, 8 edges of type $d_{L(G)}(e)=3$, $d_{L(G)}(f)=4$, 68 edges with $d_{L(G)}(e)=d_{L(G)}(f)=4$. Thus,

$$GA_e(TC_4C_8(S)\ [2,2]) = 96+8\left(\frac{\sqrt{6}}{5}+\frac{2\sqrt{3}}{3}+\frac{4\sqrt{3}}{7}-\frac{14}{4}\right)$$

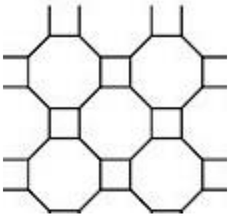


Fig. 18. 2-dimensional lattice of $TC_4C_8(S)$ [2,2] *nanotori*

Example 21. Consider the graph of 2-dimensional lattice of $TC_4C_8(S)$ [5,3] nanotori as depited in Fig. 19. This line graph has 340 edges. Also, there are 10 edges of type $d_{L(G)}(e)=2$, $d_{L(G)}(f)=3$, 20 edges of type $d_{L(G)}(e)=2$, $d_{L(G)}(f)=4$, 20 edges of type $d_{L(G)}(e)=3$, $d_{L(G)}(f)=4$, 290 edges with $d_{L(G)}(e)=d_{L(G)}(f)=4$. In other words,

$$GA_e(TC_4C_8(S)\ [5,3]) = 360 + 20\left(\frac{\sqrt{6}}{5}+\frac{2\sqrt{3}}{3}+\frac{4\sqrt{3}}{7}-\frac{14}{4}\right)$$

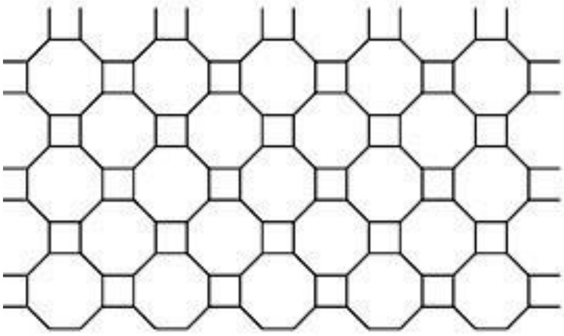


Fig. 19. 2-dimensional lattice of $TC_4C_8(S)$ [5,3] *nanotori*

By continuing this method, one can see that in generally, the line graph of $TC_4C_8(S)\ [p,q]$ has $24pq-4p$ edges. On the hand there are $2p$ edges of type $d_{L(G)}(e)=2$, $d_{L(G)}(f)=3$, $4p$ edges of type $d_{L(G)}(e)=2$, $d_{L(G)}(f)=4$, $4p$ edges of type $d_{L(G)}(e)=3$, $d_{L(G)}(f)=4$, $24pq-14p$ edges with $d_{L(G)}(e)=d_{L(G)}(f)=4$.

We can now deduce the following result.

Theorem 6. Consider the graph of $TC_4C_8(S)\ [p,q]$ nanotori. Then, the GA_e of $TC_4C_8(S)\ [p,q]$ nanotorus is

$$GA_e(TC_4C_8(S)\ [p,q]) = 24pq+4p\left(\frac{\sqrt{6}}{5}+\frac{2\sqrt{3}}{3}+\frac{4\sqrt{3}}{7}-\frac{14}{4}\right)$$

Proof. Let G be the graph of $TC_4C_8(S)\ [p,q]$ nanotori. Since

$$GA_e(G)=\sum_{ef\in E(L(G)}\frac{2\sqrt{d_{L(G)}(e)d_{L(G)}(f)}}{d_{L(G)}(e)+d_{L(G)}(f)},$$

this implies that

$$\begin{aligned}GA_e(TC_4C_8(S)\ [p,q]) &= 2p\left(\frac{2\sqrt{2\times 3}}{5}\right)+4p\left(\frac{2\sqrt{2\times 4}}{6}\right)\\ &+4p\left(\frac{2\sqrt{3\times 4}}{7}\right)\\ &+(24pq-14p)\left(\frac{2\sqrt{4\times 4}}{8}\right)\end{aligned}$$

After an easy simplification, we obtain

$$GA_e(TC_4C_8(S)\ [p,q]) = 24pq + 4p\left(\frac{\sqrt{6}}{5}+\frac{2\sqrt{3}}{3}+\frac{4\sqrt{3}}{7}-\frac{14}{4}\right)$$

Now the proof is complete.

3. Conclusion

In Theoritical Chemistry, the topological indices and molecular structure descriptors are used for modelling physic-chemical, toxicologic, biological and other properties of chemical compounds. In recent years, some researchers are interested to study the topological indices of certain nanotubes and nanotori, for example see [1,8,12]. In this paper, we have investigated the new version of geometric arithmetic index of some families of nanotubes and nanotori.

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