The broadening of Gauss chirp pulse in optical fiber

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The broadening of Gauss chirp pulse in optical fiber is studied by solving the nonlinear Schrödinger equation through Fourier transform method. The expression of pulse width in terms of root-mean-square and pulse broadening factor of Gauss chirped pulse is given. Meanwhile, the influence of the propagating optical fiber distance on the pulse broadening is given. The influence of the chirped factor on the pulse broadening and the optical fiber dispersion on pulses with different widths is analyzed.

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1. Introduction

Light pulse enter in optical fiber, after passing through a far fetch. The light pulse breadth increases, and light pulse suffer distortion. The fact that optical fiber can broaden light pulse breadth, implies that optical fiber has dispersive capacity. Due to different wavelength of light pulse different transmission speed are obtained. The dispersion reflects pulse broadening of light pulse through optical fiber. The fiber dispersion makes optical fiber bandwidth to become narrow, so the transmission capacity of optical fiber is restricted. At the same time, transmission distance of light signals are restricted, too. In optical fiber digital communication, a series of pulse codes are transmitted. Pulse broadening in optical fiber can bring to the superposition of pulses, namely intercodes interference is engendered, accordingly coming into error codes. On the other hand, pulse breadth of light pulse is becoming large along with transmission distance growing. So, in order to avoid error codes, the transmission distance of optical fiber must be shortened. Especially for the short pulse, the pulse broadening in optical fiber is strong. Therefore, the study of pulse broadening during fiber transmission is very important.

2. The non-linear Schrödinger equation

For pulse width ＞5ps, the non-linear Schrödinger equation that governs the propagation of optical pulses inside single-mode fibers is [1-3]

$$i\frac{\partial A}{\partial z}=-\frac{i\alpha}{2}A+\frac{\beta_2}{2}\frac{\partial^2 A}{\partial T^2}-\gamma\left|A\right|^2 A \qquad (1)$$

In formula (1) $A(z,T)$ is the slowly varying amplitude of the pulse envelope and T is measured in a reference frame moving with the pulse at the group velocity v_g ($T=t-\beta_1 z, \beta_1=1/v_g$), z is transmission distance through optical fiber, β_2 (ps^2/km) is second order nonlinear effect, γ ($W^{-1}km^{-1}$) is nonlinear system parameter, and α (1/km) is loss modulus of the optical fiber. The three terms in the right-hand side of equation (1) govern the effects of fiber losses, the dispersion, and nonlinearity on pulses propagating inside optical fibers, respectively.

We introduce a normalized amplitude *U* as

$$U=A/\sqrt{P_0} \qquad (2)$$

P_0 is peak power of the incident pulse. The equation (1) becomes

$$i\frac{\partial U}{\partial z}=-\frac{i\alpha}{2}U+\frac{\beta_2}{2}\frac{\partial^2 U}{\partial T^2}-\gamma P_0\left|U\right|^2 U \qquad (3)$$

When ignoring the nonlinear effect (namely $\gamma=0$), the equation (3) becomes

$$i\frac{\partial U}{\partial z}=-\frac{i\alpha}{2}U+\frac{\beta_2}{2}\frac{\partial^2 U}{\partial T^2} \qquad (4)$$

It is simpler to solve the equation (4) by using the Fourier transform.

The Fourier transform of the $U(z,\omega)$ is

$$U(z,\omega)=\int_{-\infty}^{\infty}U(z,T)\exp(-i\omega T)dt \qquad (5)$$

According to the Fourier transform relation [2]

$$U(z,T)\leftrightarrow U(z,\omega)$$
$$\frac{\partial^2 U(z,T)}{\partial T^2}\leftrightarrow(i\omega)^2 U(z,\omega) \qquad (6)$$

Formula (4) becomes

$$\frac{\partial U}{\partial z} = -\frac{\alpha}{2}U - \frac{i}{2}\beta_2 (i\omega)^2 U \qquad (7)$$

Solving partial differential equation (7) formula

$$U(z,\omega) = U(0,\omega)\exp(\frac{i}{2}\beta_2\omega^2 - \frac{\alpha}{2})z \qquad (8)$$

General solution of equation (4) is

$$U(z,T) = \frac{1}{2\pi}\int_{-\infty}^{\infty} U(0,\omega)\exp[(\frac{i}{2}\beta_2\omega^2 - \frac{\alpha}{2})z - i\omega T]d\omega \qquad (9)$$

In the formula, U(0,ω) is Fourier transform of entering light field at z＝0.

$$U(0,\omega) = \int_{-\infty}^{\infty} U(0,T)\exp(i\omega T)dt \qquad (10)$$

3. Pulse broadening of Gauss Chirp in single mode optical fiber

For Gauss chirp pulse (C is chirp gene in the below formula)

$$U(0,T) = \exp(-\frac{1+iC}{2}\frac{T^2}{T_0^2}) \qquad (11)$$

By taking the formula (11) and introducing in the formula (10), by using the formula $\int_0^{\infty} e^{-x^2}dx = \frac{\sqrt{\pi}}{2}$ one gets:

$$U(0,\omega) = \int_{-\infty}^{\infty}\exp(-\frac{1+iC}{2}\frac{T^2}{T_0^2})\exp(i\omega T)dT = \int_{-\infty}^{\infty}\exp(-\frac{1+iC}{2}\frac{T^2}{T_0^2} + i\omega T)dT$$

$$= \exp[-\frac{T_0^2\omega^2}{2(1+iC)}]\sqrt{\frac{2T_0^2}{1+iC}}_0\sqrt{\pi} = (\frac{2\pi T_0^2}{1+iC})^{1/2}\exp[-\frac{T_0^2\omega^2}{2(1+iC)}] \qquad (12)$$

By using the formula (12) with formula (9), one gets:

$$U(z,T) = \frac{1}{2\pi}\int_{-\infty}^{\infty}(\frac{2\pi T_0^2}{1+iC})^{1/2}\exp[-\frac{T_0^2\omega^2}{2(1+iC)}]\exp[(\frac{i}{2}\beta_2\omega^2 - \frac{\alpha}{2})z - i\omega T]d\omega$$

$$= \frac{1}{2\pi}(\frac{2\pi T_0^2}{1+iC})^{1/2}\exp(-\frac{\alpha}{2}z)\int_{-\infty}^{\infty}\exp[\frac{i}{2}\beta_2\omega^2 z - \frac{T_0^2\omega^2}{2(1+iC)} - i\omega T]d\omega$$

$$= \frac{T_0}{[T_0^2 - i\beta_2 z(1+iC)]^{1/2}}\exp\{-\frac{T^2(1+iC)}{2[T_0^2 - i\beta_2 z(1+iC)]} - \frac{\alpha}{2}z\} \qquad (13)$$

The formula (13) is the amplitude of super Gauss pulse at z in optical fiber direction. Formula (13) describes a Gauss pulse when C＝0.

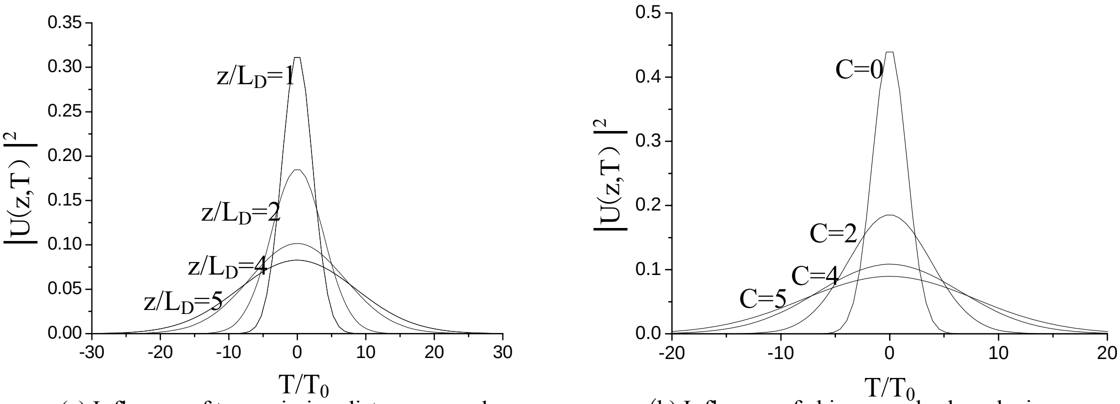
If the absorption in optical fiber is ignored, then

$$U(z,T) = \frac{T_0}{[T_0^2 - i\beta_2 z(1+iC)]^{1/2}}\exp\{-\frac{T^2(1+iC)}{2[T_0^2 - i\beta_2 z(1+iC)]}\} \qquad (14)$$

By defining the dispersion length $L_D = T_0^2/|\beta_2|$, from formula (14) is obtained:

$$|U(z,T)|^2 = \frac{1}{\sqrt{(1+Cz/L_D)^2 + (z/L_D)^2}}\exp[-\frac{(T/T_0)^2}{(1+Cz/L_D)^2 + (z/L_D)^2}] \qquad (15)$$

The curve of $|U(z,T)|^2 \sim T/T_0$ with z and C different values show in Fig 1.



(a) Influence of transmission distance on pulse broadening (C=2)

(b) Influence of chirp on pulse broadening (z/L_D=2)

Fig. 1. Pulse broadening curve of Gauss pulse in fiber.

From Fig. 1(a) it is remarked that Gauss pulse width becomes larger as optical fiber becomes longer. Fig. 1(b) shows that chirp gene is strongly affected by pulse broadening. The pulse broadening becomes larger when the chirp gene becomes higher.

4. Influence of chirp gene(C) on pulse broadening

In order to visualize the relation between pulse broadening and transmission distance, it was defined the root-mean-square impulse width [4,5]:

$$\sigma = [<T^2> - <T>^2]^{1/2}$$

Then

$$<T^n> = \frac{\int_{-\infty}^{\infty} T^n |U(z,t)|^2 \, dT}{\int_{-\infty}^{\infty} |U(z,t)|^2 \, dT} \tag{16}$$

When the chirp Gauss pulse is introduced:

$$<T> = \frac{\int_{-\infty}^{\infty} T|U(z,t)|^2 \, dT}{\int_{-\infty}^{\infty} |U(z,t)|^2 \, dT} = \frac{\int_{-\infty}^{\infty} T \exp[-\frac{(T/T_0)^2}{(1+Cz/L_D)^2+(z/L_D)^2}]dT}{\int_{-\infty}^{\infty} \exp[-\frac{(T/T_0)^2}{(1+Cz/L_D)^2+(z/L_D)^2}]dT} = 0 \tag{17}$$

$$<T^2> = \frac{\int_{-\infty}^{\infty} T^2|U(z,t)|^2 \, dT}{\int_{-\infty}^{\infty} |U(z,t)|^2 \, dT} = \frac{\int_{-\infty}^{\infty} T^2 \exp[-\frac{(T/T_0)^2}{(1+Cz/L_D)^2+(z/L_D)^2}]dT}{\int_{-\infty}^{\infty} \exp[-\frac{(T/T_0)^2}{(1+Cz/L_D)^2+(z/L_D)^2}]dT}$$

$$= \frac{T_0^2[(1+Cz/L_D)^2+(z/L_D)^2]}{2} \tag{18}$$

The root-mean-square pulse width is

$$\sigma = [<T^2> - <T>^2]^{1/2} = \frac{T_0}{\sqrt{2}}[(1+Cz/L_D)^2+(z/L_D)^2]^{1/2} \tag{19}$$

If $z=0$, $\sigma_0 = \frac{T_0}{\sqrt{2}}$, the formula (19) becomes

$$\frac{\sigma}{\sigma_0} = [(1+Cz/L_D)^2+(z/L_D)^2]^{1/2} \tag{20}$$

Formula (20) is the relation between pulse broadening gene and transmission distance. The relation curve of chirp gene and distance of chirp Gauss pulse [6], is shown in Fig. 2. From Fig. 2 it is seen that the pulse broadening becomes more large with the increase of fiber length. For different chirp gene, the pulse broadening degree is different too. The curve gradient is higher when chirp gene is higher, the pulse broadening becomes rapid when the transmission distance becomes longer. In this way, the relation between pulse broadening gene and transmission distance is definitely established.

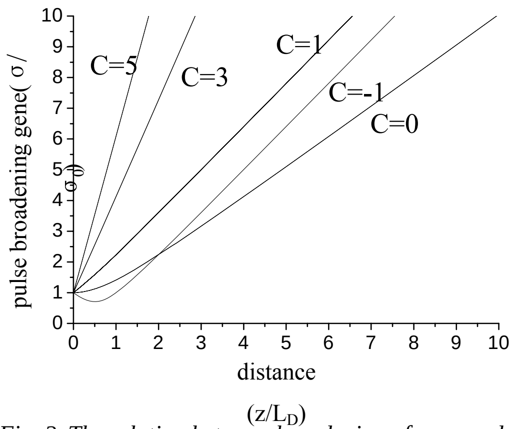


Fig. 2. The relation between broadening of gene and transmission distance of Gauss chirp pulse.

5. Influence of pulse width on pulse broadening

Supposing an entering pulse is a chirp Gauss pulse [1], $U(0,T) = \exp(-\frac{1+iC}{2}\frac{T^2}{T_0^2})$. From formula (20) we can get the relation between root-mean-square pulse broadening and optical fiber length in single mode optical fiber. Due to $L_D = T_0^2/|\beta_2|$, β_2 is a changeless value in certain optical fiber, so dispersion length L_D is related to entering pulse width T_0. The L_D is bigger if the entering pulse breadth is bigger, In the same time the broadening gene is smaller. When, entering pulse breadth is narrower, L_D is smaller, and broadening gene is bigger. The dispersion length is smaller for short pulses, the degree of pulse broadening is higher for the transmission of the fiber with the same length. The relation between broadening gene of Gauss pulse and transmission length in optical fiber is shown in Fig. 3 for $T_0 = 5ps, 10ps, 15ps, 20ps, 50ps$, $\beta_2 = -20ps^2/km$.

When pulse chirp gene is stated (here C=2), one gets from Fig. 3 that the pulse broadening of 5ps width is more grisliness. For 20ps, 50ps, the pulse broadening vanishes. So, for the short pulses and super-short pulses, it is necessary as effective step to insure high quality transmission of optical fiber.

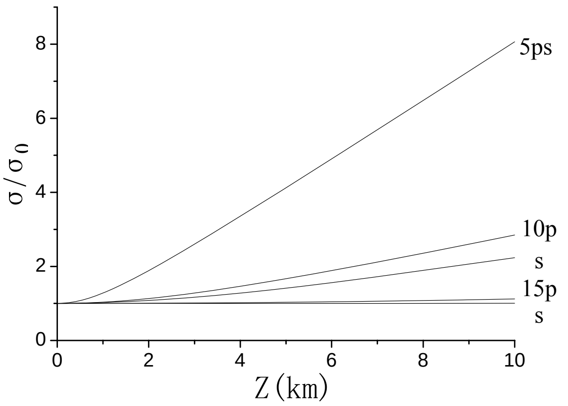


Fig. 3. The change of broadening gene dependent on fiber length for different pulse width (C =2).

The pulse broadening are listed in Table 1 for the pulse transmission at 5 kilometers in the optical fiber of

$$\beta_2 = -20ps^2/km.$$

In the Table 1, various pulse broadening are distinctly indicated when pulse transmission is at 5 kilometer in the same optical fiber. The pulse broadening degree decreases when the pulse width increases. Pulse width of 5 ps is broadening near 10 times when it transmission 5 kilometer in optical fiber. However, here we just consider the influence of dispersion and chirp, so, other factor [7,8] must be considered in another occasion. Therefore, pulse broadening must be regarded in communication of narrow-band width and bulk capacity.

Table 1. Pulse broadening characteristics.

. T_0 (ps)	5	10	15	20	50	100
$L_D = T_0^2/\lvert\beta_2\rvert$	1.25×10^3	5×10^3	11.25×10^3	20×10^3	125×10^3	500×10^3
σ/σ_0	9.85	3.16	1.94	1.52	1.08	1.02

6. Conclusions

The pulse broadening expression of non-linear dispersion was obtained in single mode optical fiber transmission, by using Fourier transform, the root-mean-square pulse width and broadening of chirp Gauss pulse. The relation between Gaussian pulse broadening and broadening dependent on transmission distance are described by drawing method. The influence of chirp gene on pulse broadening is discussed. The pulse broadening in optical fiber transmission was studied, The results are useful for the research of transmission in fibers.

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