# Study on output characteristics of harmonic mode-locked fiber laser after taking into account the SOA different residual facet reflectivity

G.-Y. JIANG*,Q. WANG, Y.-J. FU, H.-R. XIAO

*College of Automation, Nanchang Hangkong University, Nanchang 330063, China*

After considering the facet residual reflectivity of the semiconductor optical amplifier (SOA), a theoretical model of harmonic mode-locked fiber ring laser based on two SOAs has been established. Using this model, the shape, peak power and pulse width of the pulse output from the harmonic mode-locked fiber ring laser for the different facet residual reflectivity($R_1>R_2$,$R_1=R_2$ and $R_1<R_2$) of the SOA, has been investigated.The results show that the facet residual reflectivity of the SOA affects greatly the mode-locked pulse, even if the facet residual reflectivity of the SOA is very small.

(Received April 20, 2009; accepted May 25, 2009)

*Keywords:* Semiconductor optical amplifier (SOA), Facet residual reflectivity, Harmonic mode-locked fiber laser

## 1. Introduction

Optical sources capable of generating ultra-short pulse trains are key components for the implementation of high speed wavelength division multiplexing (WDM) and optical time division multiplexing (OTDM) transmission systems[1].Among the available laser sources, those that exploit the actively mode-locked fiber laser are the most practical and promising for their capability to generate wavelength tunable, transform-limited, high quality picosecond optical pulses of variable and controllable repetition rates and of relatively high output power, low amplitude noise and timing jitter. The harmonically mode-locked erbium-doped fiber laser (EDFL) is very attractive as a high repetition rate pulse source. More specifically, long cavity lengths are required to provide sufficient gain [2, 3], which results in the instability of the output pulse train, from even small environmental perturbations. For this reason, active mode-locked laser sources can be exploited in system applications of enhanced functionality only if the aforementioned problems arising from environmental sensitivity are resolved. The harmonic mode-locked fiber ring laser based on the SOA (FLSOA)[4], which can generate ultra-short optical pulse with good quality and high repetition rate, has received considerable attention.The function of SOA is used as both the gain and the moduation elements in the cavity.However,we have noticed that, in the relative experimental and theoretical model describing the harmonic mode-locked fiber ring laser based on the SOA, the facet residual reflectivity of which is usually neglected for simplicity. Although the facet reflectivity of the SOA can be reduced by antireflection coating on the facet of the SOA in practical application, the facet residual reflectivity always exists. Therefore, it is necessary to investigate the influence of facet residual reflectivity of the SOA on the pulse output from harmonic mode-locked fiber ring laser. In this paper, after considering the facet residual reflectivity of the SOA, the theoretical model of FLSOA has been established and the influences of different facet residual reflectivity of the SOA on the mode-locked pulse characteristics have been simulated numerically.The results are helpful for designing the harmonic mode-locked fiber ring laser based on the SOA.

## 2. Theoretical model

Fig. 1 shows the schematic diagram of the harmonic mode-locked fiber ring laser (FLSOA) system, which consists of two traveling-wave typed SOAs[5]. An optical signal that is provided by a DFBLD passes though a polarization controller (PC) and optical coupler ($OC_1$) with the ratio of 50:50, then injects into the modulation SOA, where the PC is used to control the optical polarization state. The signal modulation is realized by using the cross-gain modulation (XGM) effect in the modulation SOA, the gain SOA provides the necessary gain and two Faraday optical isolators ensure a unidirectional propagation. In order to obtain harmonic mode-locked pulse, the modulating frequency ($f_1$) of the modulated optical signal should be nearly integral times the fundamental frequency ($f_0$) of the cavity, that is to say, $f_1=Nf_0$ ($N$ is a positive integer).So the nth-order harmonic mode-locking pulse series, whose repetition rate is the same as the modulating frequency, can be achieved and output from the $OC_2$ with ratio of 95:5.

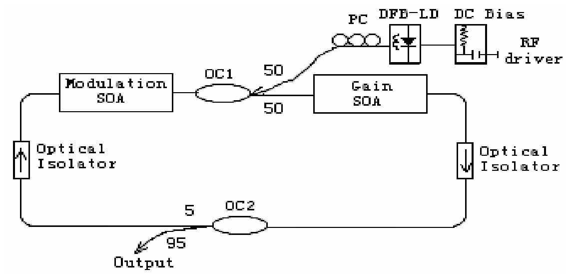



*Fig. 1. Schematic diagram of the harmonic mode-locked based on two SOAs.*

In above model, due to the stimulated radiation, the carrier density varies continuously with both time and space along the direction of the light transmission in the SOA. In order to describe accurately the propagation characteristics of pulse in the SOA,which is spliced into many sections in the simulation. The differential rate equations for the carrier density in the $j$ th section of the SOA, and the propagation equations that describe the time-varied powers of the mode-locked FLSOA and the DFBLD signals, are given by [5,6]

$$\frac{\partial N_j(z,T)}{\partial T}=\frac{I}{qV}-\frac{N_j(z,T)}{\tau_c}-\left(\frac{\Gamma g_{m,j}(z,T)}{h\nu_m A_{cross}}\overline{P_{m,j}}+\frac{\Gamma g_{l,j}(z,T)}{h\nu_s A_{cross}}\overline{P_{l,j}}\right) \quad (1)$$

$$\frac{\partial P_{m,j}(z,T)}{\partial z}=-(\Gamma g_{m,j}(z,T)-\alpha_{\text{int}})P_{m,j}(z,T) \quad (2)$$

$$\frac{\partial P_{l,j}(z,T)}{\partial z}=(\Gamma g_{l,j}(z,T)-\alpha_{\text{int}})P_{l,j}(z,T) \quad (3)$$

$$\frac{\partial \phi_{m,j}}{\partial z}=-\frac{1}{2}\beta_c g_{m,j}(z,T) \quad (4)$$

$$\frac{\partial \phi_{l,j}}{\partial z}=-\frac{1}{2}\beta_c g_{l,j}(z,T) \quad (5)$$

where the subscripts $m$ and $l$ denote the parameters of modulated DFBLD and mode-locked FLSOA, respectively, $N_j$ is the carrier density in the $j$ th section of the SOA, $T$ ($=t-z/v_g$, where $v_g$ is the grope velocity in the SOA) represents the time in a reference frame moving along with the pulse, $I$ is the injection current, $V$ is the volume of the SOA, $q$ denotes the electron charge, h$\nu$ is the photon energy, $\Gamma$ is the confinement factor, Across represents the cross-sectional area of the active layer in the SOA. The carrier lifetime is defined as $\tau_c=1/(A+BN_j+cN_j^2)$, where $A$, $B$ and $C$ (their values usually vary with different SOA and should be determined by comparison with experiments) represent the non-radiative recombination, the spontaneous emission and the Auger recombination process, respectively, $\alpha_{int}$ is the internal loss of the FLSOA cavity, $\overline{P_{mj}}$ and $\overline{P_{l,j}}$ are the average powers of backward-injecting and mode-locking pulse in the $j$ th section of the SOA, respectively, which can be calculated by[6]

$$\overline{P_{m,j}}=\frac{1}{-\Delta L}\int_{(j+1)\Delta L}^{j\Delta L}P_{m,j+1}\exp\{-[\Gamma g_{m,j}(N_j)-\alpha_{\text{int}}]z\}dz=\frac{\exp\{[\Gamma g_{m,j}(N_j)-\alpha_{\text{int}}]\Delta L\}-1}{[\Gamma g_{m,j}(N_j)-\alpha_{\text{int}}]\Delta L}P_{m,j+1} \quad (6a)$$

$$\overline{P_{l,j}}=\frac{1}{\Delta L}\int_{(j-1)\Delta L}^{j\Delta L}P_{l,j-1}\exp\{[\Gamma g_{l,j}(N_j)-\alpha_{\text{int}}]z\}dz=\frac{\exp(\{\Gamma g_{l,j}(N_j)-\alpha_{\text{int}}]\Delta L\}-1}{[\Gamma g_{l,j}(N_j)-\alpha_{\text{int}}]\Delta L}P_{l,j-1} \quad (6b)$$

where $\Delta L$ denotes the length of each SOA section, $P_{m,j+1}$ and $P_{l,j-1}$ are the backward-injecting power output from the $(j+1)$th section and mode-locking power output from the $(j-1)$th section, respectively. $g_{m,j}$ and $g_{l,j}$ represent the asymmetric gain of the backward-injection modulation and mode-locked pulse, respectively, and are calculated by[6]

$$g_{m,j}=\frac{a_1(N_j-N_0)-a_2(\lambda_m-\lambda_{Nj})^2+a_3(\lambda_m-\lambda_{Nj})^3}{1+\varepsilon(P_{m,j}+P_{l,j})} \quad (7a)$$

$$g_{l,j}=\frac{a_1(N_j-N_0)-a_2(\lambda_l-\lambda_{Nj})^2+a_3(\lambda_l-\lambda_{Nj})^3}{1+\varepsilon(P_{l,j}+P_{m,j})} \quad (7b)$$

where $a_1$ is the differential gain coefficient, $N_0$ is the transparency carrier density,$a_2$ and $a_3$ are empirically determined constants that characterize the width and asymmetry of the gain profile, $\lambda_{Nj} = \lambda_0 - a_4(N_j - N_0)$ represents the corresponding wavelength for peak gain, and $\lambda_0$ is the gain peak wavelength at transparency, $a_4$ denotes the empirical constant that shows the shift of the gain peak. After considering the facet residual reflectivity of the SOA,the boundary conditions that the optical fields (E)on the end of SOA satisfy, can be expressed by[7]

$$E^f(0,t) = \sqrt{1-R_1}A_l^f(0,t)\exp(i\omega_l t) + \sqrt{R_1}A_m^b(0,t)\exp(-i\omega_m t)$$
$$E^b(L,t) = \sqrt{R_2}A_l^f(L,t)\exp(i\omega_l t) + \sqrt{1-R_2}A_{m0}\exp(-i\omega_m t)$$
$$E_{out}(L,t) = \sqrt{1-R_2}E^f(L,t) + \sqrt{R_2}A_{m0}\exp(i\omega_1 t) \tag{8}$$

where the superscripts $f$ and $b$ denote the forward waves and backward waves, respectively, $R_1$ and $R_2$ are the front and back facet residual reflectivity of the SOA, respectively, $A_{m0}$ is the slowly varying field amplitude of the modulated signal provided by the DFBLD, $L$ is the active region length of the SOA, $\omega$ is the central frequency of the optical pulse.

When the pulse passes through the optical coupler, it is assumed that the effect of OC only makes the energy of pulse attenuate. The transmission of optical pulse in the gain SOA is similar to propagate in the modulation SOA, so it is easy to obtain only setting the terms related with modulated optical pulse to be zero. The nonlinear Schrödinger equation that describes the transmission function of optical pulse in fiber, is given by [9]

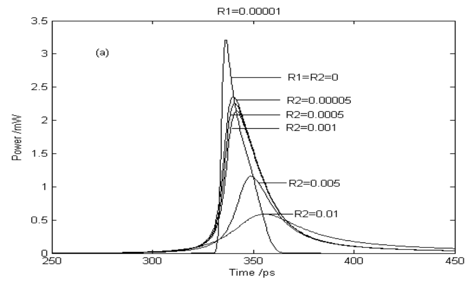
$$\frac{\partial A}{\partial z} + \frac{i}{2}\beta_2\frac{\partial^2 A}{\partial T^2} = i\gamma |A|^2 A \tag{9}$$

where $\beta_2$ is the parameter of the group velocity dispersion (GVD), $\gamma$ is the nonlinear coefficient.

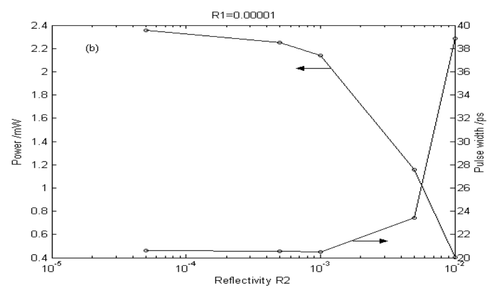
Based on Eqs. (1)-(9), the temporal shape of output pulse for a given input pulse passing though the gain SOA and modulation SOA can be simulated numerically with forth-fifth order Rung-Kutta method. If the pulse can't realize self-reproduction, the obtained optical pulse is taken as the new incident pulse and repeats the above procedure. Only once the pulse realizes self-reproduction, the output optical pulse is the mode-locked optical pulse.

## 3. Results and discussion

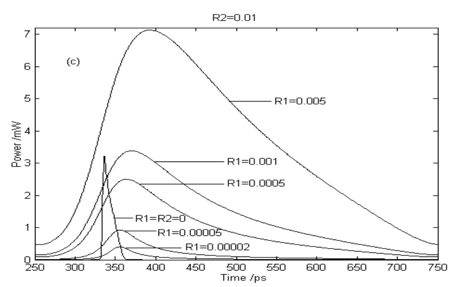
In order to analyze the effect of the facet residual reflectivity on the output characteristics of mode-locked FLSOA, the facet residual reflectivity of the SOA is changed. In the simulation, the SOAs are spliced into 10 sections, and the influence of the fiber on the pulse transmission is neglected, the parameters of the SOA are: $\Gamma = 0.3$, $\alpha_{int} = 20\ cm^{-1}$, $L = 500\mu m$, $S = 0.4\mu m^2$, $a_1 = 2.5\times10^{-20}m^2$, $a_2 = 7.4\times10^{18}m^{-3}$, $a_3 = 3.155\times10^{25}m^{-4}$, $a_4 = 3\times10^{-32}m^4$, $A = 2.5\times10^8 s^{-1}$, $B = 1\times10^{-16}m^3s^{-1}$, $C = 0.94\times10^{-40}m^6s^{-1}$, $N_0 = 1.5\times10^{24}m^{-3}$, $e = 1.6\times10^{-19}C$, $c = 3\times10^8 m/s$, $\lambda_0 = 1605nm$, $\varepsilon = 0.2\ W^{-1}$, the current of the modulation SOA and the gain SOA are 76.5mA and 185mA, respectively, The sinusoidal-wave-modulated provided by the *DFBLD* was injected backward into the modulation SOA. Fig.2 (a) shows the output mode-locked pulse shape for the different $R$ $(R_1<R_2)$. For this graph, it can be seen that, the leading and tailing edges of the mode-locked pulse for the different $R_2$ at a given fixed value of $R_1$ are smoother than these for the $R(R=0)$, the rising time of pulse is obvious less than the falling time [7,8]; Fig. 2 (b) shows the peak power and pulse width of the mode-locked pulses shown in Fig. 2 (a), for a given $R_1(=10^{-5})$,the peak power decreases and its corresponding pulse width is widened gradually when the $R_2$ increases from $5\times10^{-5}$ to $10^{-2}$.The reason is that, with the $R_2$ increasing, the gain of the SOA reduces gradually, meanwhile the threshold of the whole mode-locked fiber laser decreases too, so the pulses in SOA experiencing the different amplification, which leads to the leading edge steeper and trailing edge smoother, fall behind gradually. To compare with Fig.2(a), Fig.2(c) displays the mode-locked pulse shape when the $R_1$ of the SOA are taken the different values form $10^{-5}$ to $5\times10^{-3}$,and the $R_2$ is $10^{-2}$.As the increase of the $R_1$, the leading edge of the mode-locked pulse is much steeper than the tailing edge, the rising time of pulse is obvious less than the falling time because of the gain saturation effect of the SOA;The related peak power of mode-locked pulse is enhanced and its pulse width is widened gradually in Fig. 2 (d).Moreover, we can notice that the peak power in Fig. 2 (c) is higher than that in Fig. 2 (a),and the pulse width increases wider in Fig. 2 (d) than that in Fig. 2 (b).That is, when the $R_1$ increases gradually, the gain characteristics of the SOA makes the pulse is widen slowly, and the peak power is enhanced; On the other hand, when the $R_2$ is $10^{-2}$,the the pulse width and peak power can not obtain the required gain for the different $R_1$,which causes the peak power decrease from 2.35*mW* to 0.40*mW* but the pulse width broaden from 20*ps* to 38*ps*.



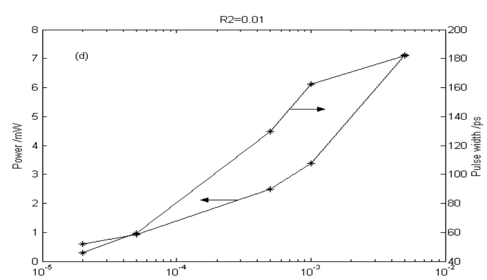

*(a)*




*(b)*



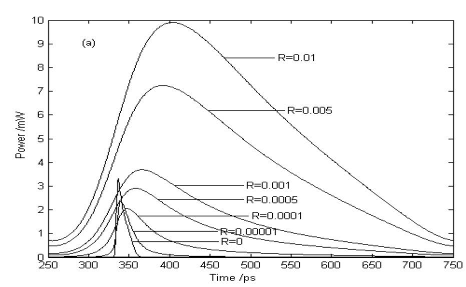

*(c)*



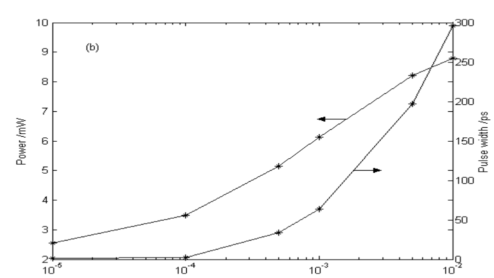

*(d)*

*Fig. 2. The mode-locked pulse shape(a),(c), peak power and pulse width(b), (d) for different $R_2$ when $R_1=10^{-5}$ and different $R_1$ when $R_2=10^{-2}(R_1<R_2)$.*

Fig. 3 shows the output mode-locked pulse shape for the different $R(=R_1=R_2)$. From Fig.3,it can be seen that, the leading edge of the mode-locked pulse is steeper than the tailing edge and the rising time of pulse is obvious less than the falling time, which are induced by the gain saturation effect of the SOA[7,8]; In addition, the influence of the magnitude of the facet residual reflectivity *R* on the mode-locked pulse shape is very great, on the same condition, the mode-locked pulse width obtained after considering the facet residual reflectivity is far broader than the simulated result on the case of traveling wave amplifier *(R=0)*. Hence, the effect of the facet residual reflectivity has to be considered even if the facet residual reflectivity of the SOA is very small ($<10^{-5}$). Fig.3 presents dependence of the peak power and pulse width on the $R$ $(=R_1=R_2)$. From this diagram, it can be seen that, when the *R* of the SOA increases gradually, the peak power of mode-locked pulse is enhanced and its pulse width is widened gradually. The reason is that, on the one hand, the energy entered directly the light route will increase with the increase of R, which results the decrease of the threshold of the whole mode-locked fiber laser; On the other hand, the pulse gain obtained from the SOA will decrease with the increase of *R* [8]. For $10^{-5}<R<10^{-2}$, the influence of former factor dominates mostly, so the small signal gain rounding a circle in the cavity will increase with the increase of *R*, which directly leads to the pulse peak power being enhanced. The increase of the small signal gain will simultaneously lead to the pulse being amplified in a wider range of time, which leads to the increase of the pulse width.




*(a)*




*(b)*

*Fig. 3. The mode-locked pulse shape(a), peak power and pulse width(b) for different $R(R_1=R_2)$.*

The output mode-locked pulse shape, peak power and pulse width have been investigated under conditions of the different values *R* ($R_1<R_2$ and $R_1=R_2$).In fact, when the $R_1$ is $10^{-2}$, the mode-locked pulse changes with the $R_2$, namely for the $R_1>R_2$ shown in Fig. 4, Fig. 4 (a) depicts the impact of the $R_2$ of the SOA on the mode-locked pulse shape. Specially explained here, the mode-locked pulses closely coincide expect for the pulses obviously distinguished for the $R_2$ of $1\times10^{-3}$ and $5\times10^{-3}$ respectively.It is clearly that, the leading and tailing edges of the mode-locked pulse for the *R* ($R_1=R_2=0$) are steeper than these for the various $R_2$ and for the gain saturation effect of the SOA,the decreasing of the rising time of the mode-locked pulses is obvious less than the falling time [8]; Fig. 4 (b) displays the peak power and pulse width with the $R_2$ of the SOA increasing. From this graph, it can be known that, the peak power of mode-locked pulse decreases gradually. Meanwhile, the pulse width is widened, too.These are caused by the reason that the gain of the pulse will reduce compared to the case where the *R* ($R_1=R_2=0$) is not considered and even the decrease of the $R_2$, which slows down the peak power of mode-locked pulse.Moreover,with the increase of the $R_2$,the carrier density alters slowly.Consequently,the pulse width of mode locked pulse lessens gradually. Fig.4(c) shows the variations of the output mode-locked pulses of the FLSOA for the $R_1$ of the SOA varying from $5\times10^{-5}$ to $10^{-2}$ and the $R_2$ of $10^{-5}$.As the increase of the $R_1$, the leading edge of the mode-locked pulse is very steeper and the tailing edge is very slower. Owing to the fact that the high power levels induced by the $R_1$ lead to the carrier depletion and make unsaturated gain region rapidly, and the carrier density begin to recover slowly with the lower power, that is to say, the rising time of pulse is obvious less than the falling time. In addition, the leading edge of pulse experiences unsaturated gain through SOA, while gain saturation limits the trailing edge of pulse.Fig. 4 (d) shows the corresponding peak power and pulse width of the mode-locked pulses shown in Fig. 4 (c).Form the diagram, it can be found that the peak power of mode-locked pulse is enhanced and its pulse width is widened gradually. These results can be clearly indicated that, with the increase of the $R_1$, the carrier density variation and carrier lifetime lead to the gain saturation, and carriers can be recovered to a great extent. Accordingly, the gain is augmented, which make the peak power of mode-locked pulse enhanced.Simultaniously the gain variation characteristics affect seriously mode-locked pulse shape. Therefore, the pulse width suffers a broadening during the $R_1$ changing process.

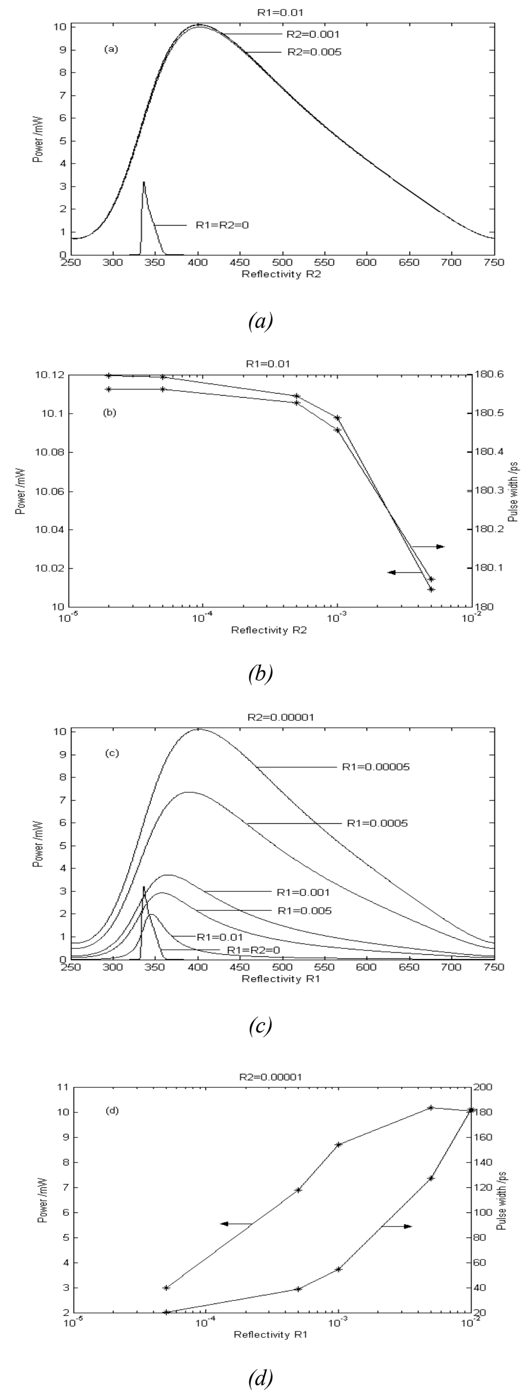



*Fig. 4. The mode-locked pulse shape(a),(c), peak power and pulse width(b),(d) for different $R_2$ when $R_1=10^{-2}$ and different $R_1$ when $R_2=10^{-5}$($R_1>R_2$).*

## 4. Conclusions

In conclusion, after considering the facet residual reflectivity of the SOA, the theoretical model of the harmonic mode-locked fiber ring laser based on SOA has been established and investigated emphatically that the influence of the different facet residual reflectivity of the SOA on output characteristics of the harmonic mode-locked fiber ring laser. The results show that the different facet residual reflectivity of the SOA strongly affects the mode-locked pulse shape, even if the facet residual reflectivity of the SOA is very small; The results obtained here are also different greatly from those in the perfect traveling wave amplifier *(R=0)*.Therefore, in order to simulate accurately the behavior of the harmonic-locked fiber ring laser based on SOA,the influence of the facet residual reflectivity of the SOA must be taken into consideration. The results are helpful for designing the harmonic mode-locked fiber ring laser based on the SOA.

**Acknowledgements**

This work was supported by the Aeronautical Foundation of the Nanchang Hangkong University of PR China (EC20080185).

## References

[1] K. Zoiros,T. Houbavlis, M. Moyssidis, Opt. Commun. **254**, 310 (2005).
[2] S. Yang, Z. Li, C. Zhao, X. Dong, IEEE Photon. Technol. Lett. **15**, 389 (2003).
[3] C. Wu, N. K. Dutta, IEEE J. Quantum Electron. **36**, 145 (2002).
[4] K. Zoiros,T. Stathopoulos, K. Vlachos, A. Hatziefremidis, Opt. Commun. **180**, 301 (2000).
[5] G. R. Lin, Y. S. Liao, G. Q. Xia, Opt. Express. **12**, 2017 (2004).
[6] G. Q. Xia, Z. M. Wu, G. R. Lin, Opt. Commun. **227**, 165 (2003).
[7] G. P. Agrawal, Nonlinear Fiber Optics, Academic Press, California, 1995.
[8] G. P. Agrawal, N. K. Dutta, Semiconductor Lasers,Van Nostrand Reinhold, New York, 1993.

*Corresponding author: jgy579@126.com