Structure dependence of resonant tunneling transmission properties in well-in-well structures

JIA GUO-ZHI
Department of Fundamental Subject, Tianjin Institute of Urban Construction, Tianjin 300384, China

A Schrödinger equation has been solved numerically for a well-in-well structure by the transfer matrix technique. Effects of structure parameters on the transmission characteristics of electron are investigated in detail. The relationship between the width of the middle well and the resonant energy levels in well-in-well structure is also studied. It is found that the first and the second quasi-bound energy levels decrease with the increasing of the width of the middle well. The first resonant energy changes more slowly than the second one. The tunneling time has also been investigated. The tunneling time, especially for the structure with the large well width, decreases with the increasing of the confinement energy.

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1. Introduction

The resonant tunneling transmission properties in semiconductor multi-barrier structures have been extensively investigated for increasing interest in quantum physics mechanism and the potential application of high-speed and high-frequency devices [1,2] since the pioneering work of Tsu and Esaki [3]. Although quantum dots structure has been predicted to have more excellent performance than conventional quantum well structure [4], temperature insensitive quantum dots device has not been reported until now due to hot carrier effects [5] and carrier thermal escape [6]. If electrons are introduced directly into the lasing states by tunneling and the tunneling rate is comparable to the stimulated emission rate, the carrier distribution in the active region will remain ‘‘cold’’ and hot carrier effects are minimized [7]. Therefore, the transmission probability and the tunneling time play decisive roles in these devices to implement the perfect performance.

The multi-barrier structure is effective for the reduction of shot noise in resonant-tunneling diodes. Pouyet V et al. [8] showed that shot noise can be suppressed effectively in the triple-well structure. Newaz AKM et al. [9] pointed out that In triple-barrier resonant-tunneling diodes the noise reduction is considerably greater than that predicted by theoretical analysis. The quantum physics mechanism of the electron tunneling through the well-in-well structure has also been investigated extensively. Xu et al. [10] simulated the transmission properties of the double/triple-barrier structure, analysis influence of structure parameter and coupling effects on transmission probability. Furthermore, the transmission probability of electron through the well-in-well structure can be different from the one of the symmetric triple-well structure due to the difference of the confinement energy. In addition, the theoretical [11] and experimental [12,13] work about the dependence of the resonant energy level and tunneling time in well-in-well on the structure parameters remains are scarce. Therefore, a complete understanding of this physics process of electron tunneling through well-in-well structure can be very important to the designing of high performance semiconductor device.

In this paper, we discuss the transmission properties of electron tunneling through the well-in-well structure, the structure dependence of the resonance energies, the tunneling time. This paper is organized as follows. In Section 2, we introduce the structure parameters of well-in-well structure and transfer matrix method. In Section 3, transmission probability characteristics in well-in-well structure are investigated; then numerical calculations are carried out to reveal the elaborate dependence of the tunneling time on structure parameters using the energy uncertainty condition, at the same time, the coupling effect is analyzed. In Section 4, the results of this work are summarized.

2. Theoretical model

The schematic energy diagram of the well-in-well structure under investigation is shown in Fig. 1. A single electron propagating with energy E from left to right is simulated by the transfer matrix method. By solving the Schrödinger equation using effective mass approximation for the well-in-well structure, this process can be described by the one-dimensional Schrödinger equation:

$$\left[-\frac{\hbar^2}{2}\frac{d}{dx}\frac{1}{m^*}\frac{d}{dx}+V(x)\right]\varphi(x)=E\varphi(x) \qquad (1)$$

where $V(x)$ is the potential energy in the heterostructure, the $\hbar$ is the reduced Planck constant, m is the effective mass of the electron, E is the eigenvalue, and φ is the wave function. In each region, the wave function can be

written as:

$$\varphi_j(x) = A_j e^{ik_j x} + B_j e^{-ik_j x} \tag{2}$$

where the amplitude coefficient A_j and B_j are constants determined by the boundary conditions. And the wave vector is:

$$k_j = \sqrt{\frac{2m_j^*(E - V_j)}{\hbar}} \tag{3}$$

m_j^* is the effective mass of the electron in j th region. The boundary condition is:

$$\varphi_j(x)\Big|_{x_0} = \varphi_{j+1}(x)\Big|_{x_0} \tag{4}$$

$$\frac{1}{m_j^*}\frac{d}{dx}\varphi_j(x)\Big|_{x_{0j}} = \frac{1}{m_{j+1}^*}\frac{d}{dx}\varphi_{j+1}(x)\Big|_{x_{0j}} \tag{5}$$

So we can get:

$$\begin{bmatrix} A_{j+1} \\ B_{j+1} \end{bmatrix} = M_j \begin{bmatrix} A_j \\ B_j \end{bmatrix} \tag{6}$$

The transfer-matrix is:

$$M_j = \frac{1}{2}\begin{bmatrix} (1+r_j)e^{i(k_j - k_{j+1})x_{0j}} & (1-r_j)e^{-i(k_j + k_{j+1})x_{0j}} \\ (1-r_j)e^{i(k_j + k_{j+1})x_{0j}} & (1+r_j)e^{-i(k_j - k_{j+1})x_{0j}} \end{bmatrix} \tag{7}$$

where

$$r_j = \frac{m_{j+1}^* k_j}{m_j^* k_{j+1}} \tag{8}$$

We relate all the boundary conditions, so we can get:

$$\begin{bmatrix} A_n \\ B_n \end{bmatrix} = M_n M_{n-1} \cdots M_1 \begin{bmatrix} A_1 \\ B_1 \end{bmatrix} = M \begin{bmatrix} A_1 \\ B_1 \end{bmatrix} \tag{9}$$

and the transmission probability can be calculated as:

$$T = \frac{1}{|M_{11}|^2} \tag{10}$$

3. Numerical results and discussion

We have performed numerical calculations of the transmission probabilities for the one-dimensional schematic energy shown in Fig. 1. Here, a, b, c, *L*, e, f are the widths of the barriers and wells. V_1, V_2, V_3, V_4 are the heights of the barriers in the well-in-well structure. It was assumed for all simulation that the structures consist of $Al_xGa_{(1-x)}As$ barriers and GaAs wells. The effective electron mass in $Al_xGa_{(1-x)}As$ is expressed as $(0.067+0.083\,x)\,m_0$ $(0 \leqslant x \leqslant 1)$, m_0 is electron mass in vacuum. The effective electron mass in GaAs well is $0.067\,m_0$. Here, the well-in-well structure (the height of side barriers V_1, V_4 are more high than that of the inner barriers V_2, V_3) can have some difference with the symmetric triple-well structure (the height of barriers are equal).

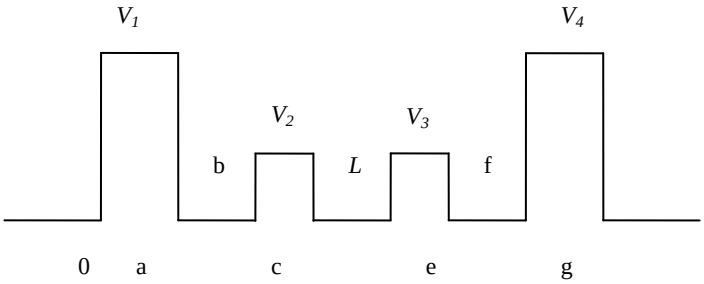

Fig. 1. The figures of the well-in-well structure.

Fig. 2 shows the transmission probability of the three resonance levels in the well-in-well structures with different middle well widths L. The parameters are V_1=V_4=0.7 eV, V_2=V_3=0.4 eV, b=f=5 nm, and a=c=g=e=2 nm. As the middle well width increases, the first quasi-bound energy level shifts to the lower energy regions, the third one to the higher energy regions, and the second one changes only a little. It is well known that the wave function of eigenstates in the three wells can penetrate into each other and strongly couple, forming new quasi-bound energy levels. The coupling effect has also great relation with the width of the barrier and the gap between the energy levels according to the analysis by Xu et al [10]. The coupling between the first and the second quasi-bound energy level decreases, while the coupling between the second and the third quasi-bound energy level increases.

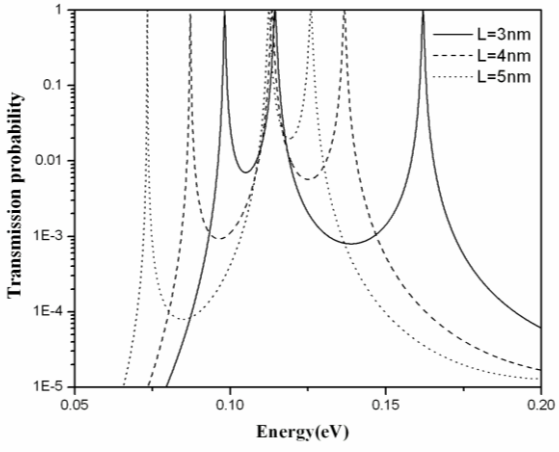

Fig. 2. Transmission probability of the three resonance levels for the well-in-well structures with different middle well widths L.

In order to further analyze the causing, we show in Fig. 3 the variations of three quasi-bound energy levels as

a function of the middle well width for well-in-well structure. As the middle well width increases, the first quasi-bound energy level split to two energy levels, one of them shifts toward the low energy region, the change of the other resonant energy level caused by the middle well width is very small, while the third quasi-bound energy level quickly decreases. It is evident that when L is very small, for example only 1 nm, the well-in-well structure has nearly the same energy levels as the double wells structure; when L is large, for example 6 nm, the third quasi-bound energy level is reduced towards the second one.

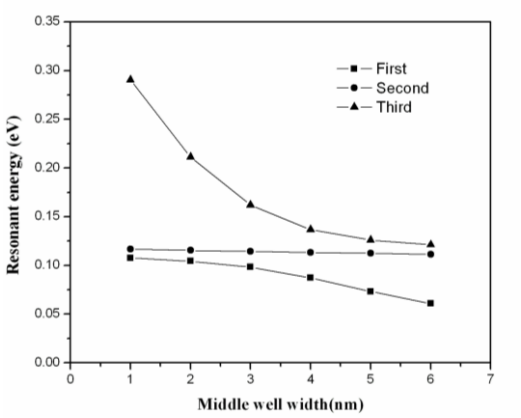

Fig. 3. Dependence of three quasi-bound energy levels on the middle well width for well-in-well structure.

Figs. 4, 5 and 6 show the dependence of quasi-bound levels as a function of the middle well width with Al composition of x =0.4, 0.5 and 0.7, respectively. It is clearly seen that the trend of the resonant energy as a function of the middle well width is similar for the structure with different barrier heights. The resonant energy for the barrier with high Al composition is larger than that for the barrier with low Al composition, which can be ascribed to the increasing of barrier confinement of electron in the well.

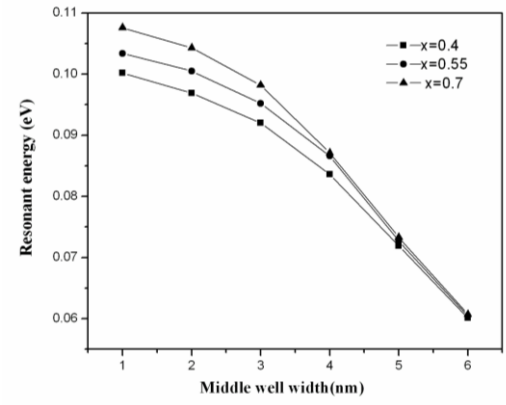

Fig. 4. Dependence of the resonant energy of the first quasi-bound states on the middle well width for different indium concentrations.

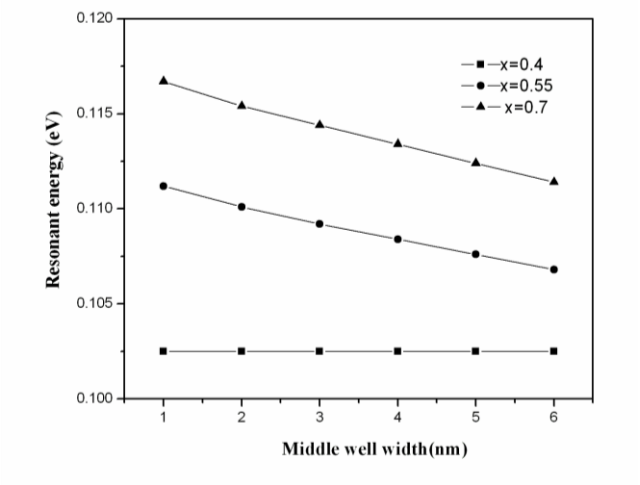

Fig. 5. Dependence of the resonant energy of the second quasi-bound states on the middle well width for different indium concentrations.

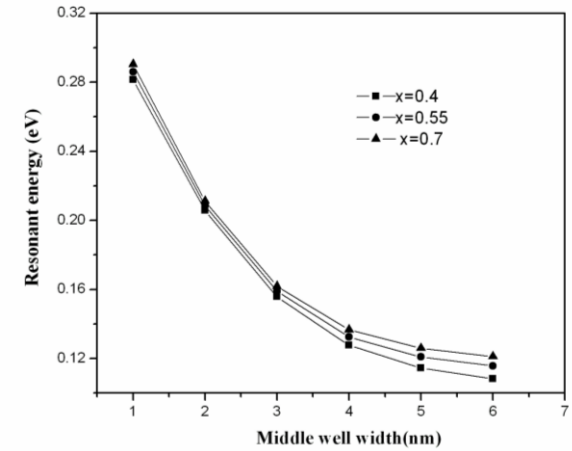

Fig. 6. Dependence of the resonant energy of the third quasi-bound states on the middle well width for different indium concentrations.

Obviously, with the increasing of the middle well width, the gap between the quasi-bound energy levels changes too. The energy level in the middle well is close to the one in the first well with the width of the middle well increasing, which can result in enhancing the coupling between the first and the second well, and weakening the coupling between the first and the third well. Choe et al pointed out that when the states between the adjacent wells are distinct, a doublet forms due to the resonant between the energy levels [14]. Thus, the resonant tunneling time of the well-in-well structure can be calculated as:

$$\tau = \frac{\hbar}{E_2 - E_1} \tag{11}$$

According to formula (11), Figs. 7 and 8 present the tunneling time between different energy levels as a function of the middle well width for the barriers with different Al composition in the well-in-well structure. It can be noticed that the tunneling time τ_1 decreases rapidly

with the increasing of the middle well width, and that the tunneling time for the structure with x=0.55 and 0.7 barrier is smaller than that for the structure with x=0.4 barrier. The tunneling time decreases with barrier height increasing, which can be due to the further increasing of the gap between the quasi-bound energy levels with confinement energy increasing in the barriers.

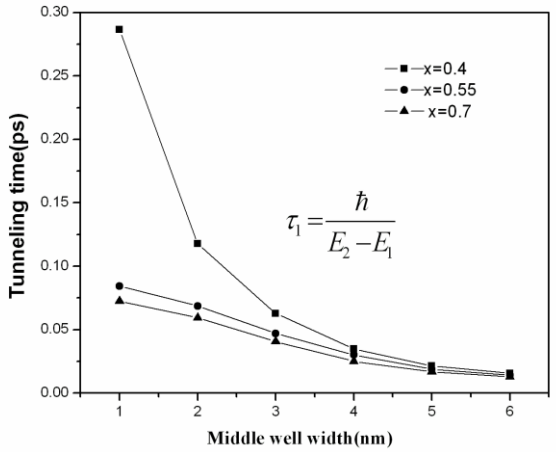

Fig. 7. The tunneling time between the energy level E_1 *and* E_2 *as a function of middle well width.*

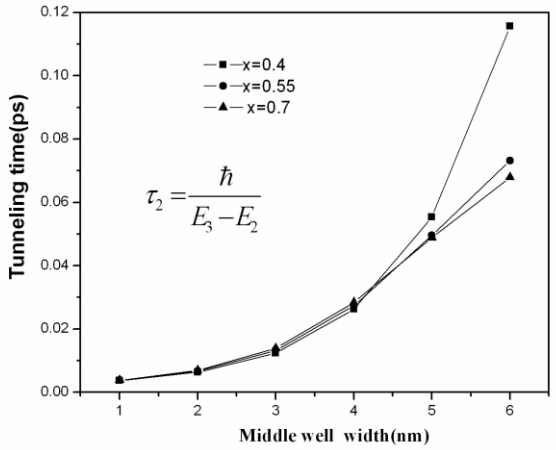

Fig. 8. The tunneling time between the energy level E_2 *and* E_3 *as a function of middle well width.*

4. Conclusions

We have theoretically studied transmission prosperities of electron resonant tunneling through the well-in-well structure by the means of the transfer-matrix technique, and compared it with transmission prosperties of the symmetrical tripl-well structure. With the increasing of the confinement energy, the tunneling time is shorten, which can be used for designing the high-speed device.

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* Corresponding author: dip-coating@163.com