# Stress analysis and thickness design of casting ceramics shells based on stereolithography prototypes

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In this paper, deformation and stress of casting shell and the resin prototype pattern are analyzed, getting the criterion of non-crack. And solid prototype pattern which is in the diameter of 30mm and hollow prototype pattern which is in the diameter of 30mm with thickness of 1mm are prepared, then coated and sanded, baked to get the casting shell. The results show that criterion of non-crack of casting shell is based on cross sectional ratio $M$ of casting shell, and hollow prototype pattern is more reasonable. When charging temperature is 80 to 100 °C, heating rate is 150 °C/h, the prototype pattern is heated to 950 °C for 30min, then the integrated casting shell is obtained.

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## 1. Introduction

Quick casting (QC) or rapid precision casting is based on rapid prototype which substitutes for wooden or wax pattern in casting. QC technology plays a significant role in enhancing productivity of the single or small lot casting production [1]. Such combination will inevitably bring about an important change in the casting industry or the machinery industry. The stereolithography (SL) pattern is a resin pattern based on the three-dimensional CAD. The Modification of the SL pattern is more convenient than the steel or wooden pattern. When the resin burns and vanishes at high temperature, the dewaxing process can be omitted, the the production period (lasting only a few hours) can be shortened and the cost can be lowered. The SL pattern has shown a good application prospect in many aspects such as development of new product, production of multi-type and small lot casting parts etc. In QC technology, refractory slurry is wrapped outside the SL pattern. The casting shell is obtained by first baking the SL pattern directly and then burning, gasifying. It is used for casting of metallic parts. The Ford Auto Company has achieved satisfactory results in making automobile parts by using QC technology. In the industrial production, QC technology is much better than the traditional investment casting in the productions cycle [2-3].

The material of the SL pattern is UV curable resin. Its expansion coefficient is greater than one of the material of casting shell by one or two orders of magnitude. And the UV curable resin differs from ordinary pattern material in that it can’t soften and can’t melt away. So if the SL pattern substitutes for the wax pattern, the cracking probability of the shell is quite high in the baking process. Hence, crack of casting shell has become the key to popularize the QC technology which is based on SL pattern. Though lots of companies have taken measures, the phenomenon of crack of casting shell is more often than the expected [4-5]. Countries take many efforts to solve the problem on crack of casting shell in the baking process. The unreasonable settings such as technological parameter settings in baking process, preparation of the casting shell, thickness and internal structure of the resin prototype pattern, do affect quality of the casting shell. Hence, it is necessary to make a systematical research on primary factors which impact the crack of the casting shell. Based on the qualitative and quantitative analyses on deformation and stress of casting shell in the baking process, stress between the resin prototype pattern and casting shell has been analyzed, getting the criterion of non-crack of casting shell and the law of thickness of the resin prototype pattern and casting shell. The paper presents a method of using stress analysis, the compound and baking process of shell slurry to manufacture an integrated casting shell. The method verifies the rationality of the criterion of non-crack of casting shell in the baking process.

## 2. Assumptions

The sketch of the resin prototype pattern and casting shell is shown in the Fig. 1. Stress analysis and thickness design of casting ceramics shells are based on the following assumptions:

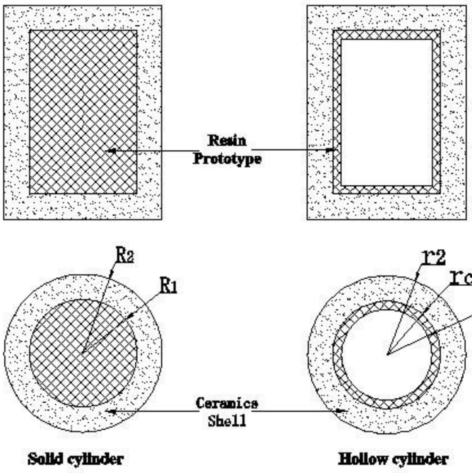

*Fig. 1. Cylindrical resin prototype pattern wrapped with refractory shell*

1) The thickness of the shell is equal, the radius of the resin prototype pattern and casting shell are $R_1$ and $R_2$ respectively, and the axis of the resin prototype pattern and casting shell is coaxial.

2) The initial temperature of resin prototype pattern and casting shell are room temperature (25 $^{o}$C), and then their temperature rise to a certain value T.

3) The thermal expansion coefficient and elastic modulus of the resin prototype pattern and casting shell is stable in the heating process. And the tensile elastic modulus of the resin prototype pattern and casting shell is equivalent to that of compression.

4) The length of the resin prototype pattern and casting shell are far greater than their diameter. In brief, only axial expansion of the resin prototype pattern and casting shell is considered while its radial expansion is omitted.

5) Since UV Curable Resin belongs to thermosetting resin, that can’t be soften and not melt away in the baking process. We suppose that only elastic deformation occurs in the resin prototype pattern and casting shell, ignoring its pure plastic deformation.

6) There is no bending deformation on the resin prototype pattern and casting shell in the baking process. And the stress in the system is at the equilibrium and resultant force is zero. The compressive stress of the resin prototype pattern and the tensile stress of casting shell is equal and in opposite directions.

## 3. Analysis of deformation

### 3.1 Deformation analysis of the resin prototype pattern and casting shell

In the heating process, both the resin prototype pattern and casting shell expand. The resin prototype pattern will generate elastic deformation, viscoelastic deformation or elastic-plastic deformation. The casting shell will generate elastic deformation. While expansion coefficient of the resin prototype pattern is greater than that of casting shell by one or two orders of magnitude [6]. Both of the expansion is blocked deformation. The resin prototype pattern and casting shell restrict each other, so that the elastic deformation of the resin prototype pattern is less than its free expansion and the deformation of casting shell is more than its free expansion. As shown in Fig. 2. Finally the expansion (elastic deformation) in the contact surface of the resin prototype pattern and casting shell is equal.

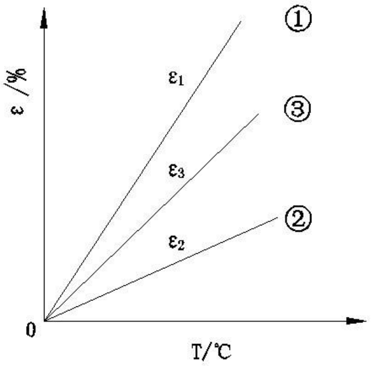

*Fig. 2. Relationship between the expansion and temperature*

In the Fig. 2, line 1 is free expansion strain (expansion rate) of the resin prototype pattern $\varepsilon_1$.Line 2 is free expansion strain (expansion rate) of casting shell $\varepsilon_2$. Line 3 is their actual expansion strain (expansion rate) $\varepsilon_3$. Their corresponding free expansions (elastic deformation) are $\Delta l_1$, $\Delta l_2$, $\Delta l_3$ respectively.

### 3.2 Calculation of deformations

Free expansion (elastic deformation) of the resin prototype pattern is:

$$\Delta l_1 = \alpha_1 l \Delta T \tag{1}$$

Free expansion (plastic deformation) of casting shell is:

$$\Delta l_2 = \alpha_2 l \Delta T \tag{2}$$

Strain (expansion rate) is respectively:

$$\varepsilon_1 = \Delta l_1 / l = \alpha_1 \Delta T \tag{3}$$

$$\varepsilon_2 = \Delta l_2 / l = \alpha_2 \Delta T \tag{4}$$

In Eqs.(1)-(4), $l$ is length of the resin prototype pattern at room temperature, $\alpha_1$, $\alpha_2$ is the thermal expansion coefficients of the resin prototype pattern and casting shell respectively. $\Delta T$ is the temperature difference from room temperature to the measured temperature.

The actual expansion (elastic deformation) $\Delta l_3$ of the resin prototype pattern and casting shell should be equal, which is between the free expansion (plastic deformation) $\Delta l_1$ and $\Delta l_2$. The actual strain (expansion rate) $\varepsilon_3$ of the resin prototype pattern and casting shell is also between its free expansion $\varepsilon_1$ and $\varepsilon_2$.

## 4. Analysis of stress

In the heating process, as the expansion coefficient of the resin prototype pattern is greater than the casting shell by one or two orders of magnitude, both of its expansion belong to blocked deformation which restricts each other, therefore stress occurs in the system between the resin prototype pattern and casting shell. Due to the thermal expansion, compressive stress $F_1(\sigma_1)$ occurs in the resin prototype pattern, tensile stress $F_2(\sigma_2)$ occurs in the casting shell. The system of the resin prototype pattern and casting shell is at the equilibrium. The resultant stress in the contact surface of the resin prototype pattern and casting shell is zero, $\Sigma F=0$. Compressive stress of the resin prototype pattern is equal to tensile stress of casting shell but they are in opposite direction, namely,

$$F_1 = F_2 \tag{5}$$

Because

$$F_1 = \sigma_1 S_1 \tag{6}$$

$$F_2 = \sigma_2 S_2 \tag{7}$$

We can deduce the following relationship,

$$\sigma_1 S_1 = \sigma_2 S_2 \tag{8}$$

In Eq.(8), $S_1$, $S_2$ are cross sectional area in the longitudinal direction of the resin prototype pattern and casting shell respectively.

Suppose that the thermal expansion coefficient and elastic modulus of the resin prototype pattern and casting shell are stable in the baking process and the tensile elastic modulus is equivalent to that of compression. Therefore based on Hook's law, we can obtain the following equalities:

$$\sigma_1 = E_1(\varepsilon_1 - \varepsilon_3) \tag{9}$$

$$\sigma_2 = E_2(\varepsilon_3 - \varepsilon_2) \tag{10}$$

In Eqs.(9) and (10), $E_1$, $E_2$ are elastic modulus of the resin prototype pattern and casting shell respectively.

When putting Eqs.(8)-(10) together, we can obtain:

$$E_1(\varepsilon_1 - \varepsilon_3)S_1 = E_2(\varepsilon_3 - \varepsilon_2)S_2 \tag{11}$$

From Eq.(11), we can deduce the following relationship:

$$\varepsilon_3 = \frac{E_1 S_1 \varepsilon_1 + E_2 S_2 \varepsilon_2}{E_1 S_1 + E_2 S_2} \tag{12}$$

When putting Eq.(12) into Eq.(10), we can obtain tensile stress $\sigma_2$ of casting shell:

$$\sigma_2 = \frac{E_1 E_2 S_1(\varepsilon_1 - \varepsilon_2)}{E_1 S_1 + E_2 S_2} \tag{13}$$

When putting Eq.(3) and (4) into Eq.(13), we can obtain:

$$\sigma_2 = \frac{E_1 E_2 \Delta T(\alpha_1 - \alpha_2)}{E_1 + E_2(\frac{S_2}{S_1})} \tag{14}$$

Order $M=S_2/S_1$, and $M$ is cross-sectional area ratio of the resin prototype pattern and casting shell. We can obtain:

$$\sigma_2 = \frac{E_1 E_2 \Delta T(\alpha_1 - \alpha_2)}{E_1 + E_2 M} \tag{15}$$

From Eq.(14) or Eq.(15), a qualitative and quantitative analysis and evaluation on the stress status of the resin prototype pattern and casting shell can be done.

Analyzing of stress status of casting shell and controlling of its expansion stress and deformation in the baking process is of great value to obtain an integrated and high-precision casting shell.

From Eq.(14) or (15), the main factors that affect expansion stress of the casting shell in the baking process include: elastic modulus ($E_1$, $E_2$) of the resin prototype pattern and casting shell, its thermal expansion coefficient($\alpha_1$, $\alpha_2$), its temperature difference $\Delta T$ from room temperature to the measured temperature, its cross sectional area ($S_2$, $S_1$) etc. Besides, heating rate and charging temperature of the casting shell in baking process have great impacts on thermal expansion (value), thermal expansion rate and thermal expansion coefficient of the resin prototype pattern and casting shell, leading to great influence on the expansion stress and expansion stress rate of the casting shell.

## 5. Calculation of the resin prototype pattern and casting shell

To get the integrated casting shell in the baking process, the tensile stress of casting shell must be less than the tensile strength of casting shell, namely,

$$\sigma_2 \le \sigma_0 \tag{16}$$

In Eq.(16), $\sigma_0$ is tensile strength or allowable stress of casting shell.

When putting Eq.(15) into Eq.(16), we can obtain:

$$\frac{E_1E_2\Delta T(\alpha_1-\alpha_2)}{E_1+E_2M}\le\sigma_0 \tag{17}$$

so that we obtain:

$$M\ge\frac{E_1E_2\Delta T(\alpha_1-\alpha_2)-E_1\sigma_0}{E_2\sigma_0} \tag{18}$$

Eq.(18) is criterion of the non-crack of casting shell. And the formula is the cross sectional ratio M of resin prototype pattern and casting shell.

The resin prototype pattern in the experiment is LPR300-1 photosensitive resin made by State Key Lab of Manufacturing System Engineering of Xi'an Jiaotong University, and the properties of the resin are shown in Table 1.

*Table 1. Properties of photocurable resin*

| viscosity(CP) $30^{o}$C | hardness (shore D) | Tensile strength(MPa) | Tensile modulus(MPa) | Elongation at break(%) | Glass transition temperature ($^{o}$C) |
|---|---|---|---|---|---|
| 159.7 | 83.4 | 66.6 | 4517 | 2.2 | 59.2 |

In the experiment, thermal expansion coefficient $\alpha_1$ of the resin prototype is $8.9\times10^{-5}\,{}^{o}C^{-1}$ (actual measured data), and its elastic modulus $E_1$ is 2460 *MP*a. Thermal expansion coefficient $\alpha_2$ of the casting shell is $8.0\times10^{-6}\,{}^{o}C^{-1}$, and its elastic modulus $E_2$ is 8000MPa. $\Delta T$ which rises from room temperature (25 $^{o}$C) to decomposition temperature (270 $^{o}$C) is 245 $^{o}$C. Tensile strength $\sigma_0$ of the casting shell is 1.0-2.0 *MP*a in general, when tensile strength $\sigma_0$ of casting shell is 2.0 *MP*a and putting above-mentioned data into Eq.(18), we can obtain the following result: $M\ge23.801$.

When resin prototype pattern is solid cylindrical prototype, we can obtain:

$$M=\frac{S_2}{S_1}=\frac{R_2^2-R_1^2}{R_1^2}=\frac{R_2^2}{R_1^2}-1 \tag{19}$$

When resin prototype pattern is hollow cylindrical prototype and its thickness is 1mm, we can obtain:

$$M=\frac{S_2}{S_1}=\frac{r_2^2-r_1^2}{r_1^2-(r_1-1)^2}=\frac{r_2^2-r_1^2}{2r_1-1} \tag{20}$$

When tensile strength $\sigma_0$ of the casting shell is 2.0 *MP*a, *M* is 23.801, from Eq.(19) and (20), the calculated value of the integrated casting shell's thickness showed in Table 2. Table 2 shows that, when solid cylindrical prototype is used, the shell's thickness is great. The thickness of the solid cylinder prototype is too great to produce in industry production. Taking the practical experience and actual cost of laboratory into account, solid resin prototype is undesirable, a hollow resin prototype is more reasonable.

*Table 2. Calculated value of the integrated casting shell's thickness*

| Solid cylindrical prototype | | Hollow cylindrical prototype | | |
|---|---|---|---|---|
| Diameter ($2R_1/mm$) | Calculated thickness of the casting shell ($(R_2-R_1)/mm$) | External diameter ($2r_1/mm$) | Thickness of prototype(*mm*) | Calculated thickness of the casting shell ($(r_2-r_1)/mm$) |
| 5 | 7.5 | 5 | 1.0 | 5.3 |
| 10 | 20.0 | 10 | 1.0 | 6.8 |
| 20 | 40.0 | 20 | 1.0 | 8.3 |
| 30 | 60.0 | 30 | 1.0 | 9.1 |
| 40 | 80.0 | 40 | 1.0 | 9.5 |
| 50 | 100.0 | 50 | 1.0 | 9.9 |

## 6. Preparation and baking of casting shell

### 6.1 Design of casting shell

Firstly, the solid resin prototype pattern which is in diameter of 30mm and hollow cylindrical prototype pattern which is in the diameter of 30mm with thickness of 1mmare prepared. Then the slurry is coated by the corresponding values of the thickness which are showed in the table 2. Casting shells was dried and hardened in the same condition and then baked. Lastly, the baking results are observed.

### 6.2 Compound of slurry and coating of casting shell

The casting shell is used for cast titanium alloy, so the slurry is composed of stabilized zirconia powders in the diameter of 0.05mm. Zirconia powder is used as refractory material and zirconium acetate in the content of 24% which is used as binder. After mixing two materials into uniform slurry, the resin prototype pattern is coated in the uniform slurry and then sanded. Slurry of the inner double layers is made in powder-liquid ratio of 3.3:1 and its sanding material is stabilized zirconia powders in 0.15mm. Slurry of the outer layers is made in powder-liquid ratio of 3.0:1 with 0.59 to 0.84mm stabilized zirconia powders. The casting shell is coated and sanded until the thickness of shell is 10mm (thickness of the shell in theoretical calculation with no cracking is 9.9mm). Figs.3 and 4 is the resin prototype pattern and the coated, dried and hardened shell resin prototype pattern respectively.

### 6.3 Determination of baking process for casting shell

The baking process of the casting shell is based on the following factors such as charging temperature, heating rate, the thermal expansion coefficient of the resin prototype pattern and casting shell etc. Controlling of these factors makes the resin prototype pattern expand less and slower, the stress which occurs on the casting shell is minimal to zero. And to ensure no crack occurs in the casting shell, the design of the thickness of resin prototype pattern and casting shell is based on the Eq.(18).

Therefore, charging temperature of casting shell should be the temperature when the resin have little thermal expansion, low expansion rate and resin haven’t soften. Thus the stress value of the casting shell in the baking process can be reduced. According to TGA curves and thermal expansion test of photocurable resin, it is more reasonable that the Charging temperature is 80 to 100 $^{o}$C, the heating rate is 150 $^{o}$C/h and the temperatureis heated to 950 $^{o}$C and kept warm for 30 min [7]. The shell’s baking process shows that, no casting shells made in hollow cylindrical prototype cracked while every casting shells made in solid cylindrical prototype cracked. Fig.5 shows the non-crack shell after baking at 950 $^{o}$C. A large number of tests verify the rationality of Eq.(18) so as to guide the design of shell thickness.

*Fig. 3. Resin prototype*

*Fig. 4. Shell with resin prototype*

*Fig. 5. Baked shell*

## 7. Conclusion

Deformation and stress of the resin prototype pattern and casting shell in the baking process are analyzed in this paper. Then according to the experiment results of solid cylindrical and hollow cylindrical of the resin prototype pattern, hollow cylindrical resin prototype is proved to be reasonable. In addition, based on the setting parameters of the baking process included preparation of slurry and shell etc., the integrated casting shell is made in the experiment. Therefore, the law on thickness of the resin prototype pattern and casting shell is verified, and it’s meaningful to guide the actual production.

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