Some topological indices of nanostar dendrimers

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The GA_4 index is a topological index was defined as $GA_4(G)=\sum_{uv\in E}\frac{2\sqrt{\varepsilon(u)\varepsilon(v)}}{\varepsilon(u)+\varepsilon(v)}$, in which eccentricity of vertex *u* denoted by $\varepsilon(u)$. Recently some classes of *GA* index were introduced. In this paper we compute the GA_4 index of nanostar dendrimers.

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1. Introduction

By a graph means a set of vertices and edges which denotes by $V(G)$ and $E(G)$, respectively. If *e* is an edge of *G*, connecting the vertices *u* and *v*, then we write $e = uv$ and say "*u* and *v* are adjacent". A connected graph is a graph such that there is a path between all pairs of vertices. Throughout this paper graph means simple connected graph.

Molecular descriptors play a prominent map in chemistry, pharmacology, etc. Among them, topological indices are very important [1]. Let Σ be the class of finite graphs. A topological index is a function Top from Σ into real numbers with this property that $Top(G) = Top(H)$, if *G* and *H* are isomorphic. Obviously, the number of vertices and the number of edges are topological index. If $x, y \in V(G)$ then the distance $d_G(x,y)$ between *x* and *y* is defined as the length of any shortest path in *G* connecting *x* and *y*. For a vertex *u* of $V(G)$ its eccentricity $\varepsilon(u)$ is the largest distance between *u* and any other vertex *v* of *G*, $\varepsilon(u)=\max_{v\in V(G)} d_G(u,v)$. The maximum eccentricity over all vertices of *G* is called the diameter of *G* and denoted by $D(G)$. The eccentric connectivity index [2-6] $\xi(G)$ of a graph *G* is defined as

$$\xi(G)=\sum_{u\in V(G)}\deg_G(u)\varepsilon(u),$$

where, $\deg_G(u)$ denotes the degree of vertex *u* in *G*, *i. e*., the number of its neighbors in *G*.

The geometric – arithmetic index (*GA*) considered by Vukičević and Furtula [7] as

$$GA(G)=\sum_{uv\in E}\frac{2\sqrt{d_G(u)d_G(v)}}{d_G(u)+d_G(v)}.$$

Fath-Tabar et al. [8] defined the second version of *GA* index as follows:

$$GA_2(G)=\sum_{uv\in E}\frac{2\sqrt{n_u n_v}}{n_u+n_v},$$

where n_u is the number of vertices of *G* lying closer to the vertex *u* than to the vertex *v*. The third member of this class was considered by Zhou et al. [9] as

$$GA_3(G)=\sum_{uv\in E}\frac{2\sqrt{m_u m_v}}{m_u+m_v},$$

where m_u is the number of edges of *G* lying closer to the vertex *u* than to the vertex *v*. The fourth member of this class was considered by A. R. Ashrafi et al. [10] as

$$GA_4(G)=\sum_{uv\in E}\frac{2\sqrt{\varepsilon(u)\varepsilon(v)}}{\varepsilon(u)+\varepsilon(v)},$$

in which eccentricity of vertex *u* denoted by $\varepsilon(u)$. Recently Furtula et al.[11] introduced atom-bond connectivity (*ABC*) index, which it has been applied up until now to study the stability of alkanes and the strain energy of cycloalkanes. This index is defined as follows:

$$ABC(G)=\sum_{e=uv\in E(G)}\sqrt{\frac{d_G(u)+d_G(v)-2}{d_G(u)d_G(v)}}.$$

Through this paper our notations are standard and mainly taken from graph theory book such as [12, 13] and [14 - 41].

2. Main results and discussions

In this section, we compute these topological indices for an infinite family of nanostar dendrimers G_n shown in Fig. 1.

Lemma 1. Consider the nanostar dendrimer G_n Then, for $0 \le i \le 9n-5$ we have

$$\varepsilon(v_i) = 18n - 10 - i\,.$$

Proof. It is easy to see that the diameter of graph G_1 is 8. This value for G_2 is $3\times 8+2$. By induction one can deduce that the diameter of G_n is 8(2n - 1) + (2n - 2)=18n-10. Since $\varepsilon(v_0) = 18n - 10$, then $\varepsilon(v_1) = 18n - 10 - 1$ and so $\varepsilon(v_i) = 18n - 10 - i$ ($0 \le i \le 9n-5$). Now by using the symmetry of graph the proof is completed.

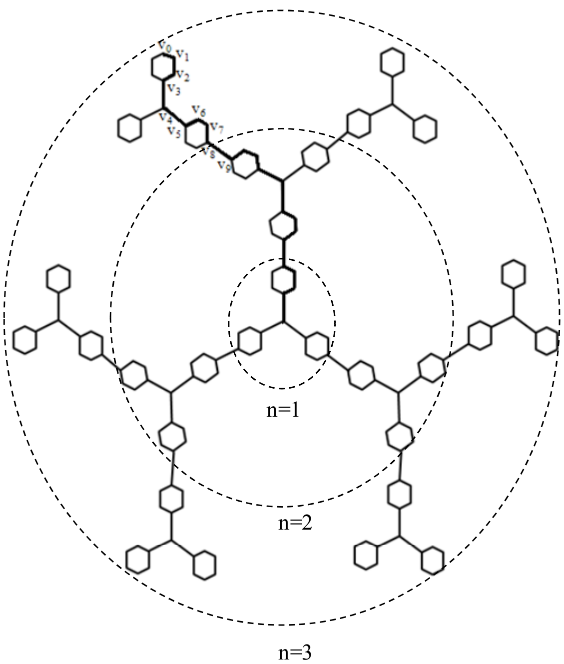

Fig. 1. 2 – D Graph of Nanostar Dendrimer G_n, n = 3.

Theorem 2.

$$\xi(G_n) = \sum_{i=0}^{n-2} 3\times 2^{n-i-2}\Big(6\sum_{j=0,3} A_{i,j} + 8\sum_{j=1,2} A_{i,j} + 3\sum_{j=4,5,8,9} A_{i,j} + 4\sum_{j=6,7} A_{i,j} - 36n + 20\Big) + 405n - 120\,,$$

where $A_{i,j} = 18n - 10 - 10i - j$.

Theorem 3.

$$GA_4(G_n) = \sum_{i=0}^{n-2}(3\times 2^{n-i-2})\left(4\sum_{j=0}^{2} A_{i,j} + 2\sum_{j=3,5,6,7} A_{i,j} + \sum_{j=4,8} A_{i,j}\right) + 6\sum_{i=1}^{3}\frac{2\sqrt{(9n-i)(9n-i-1)}}{18n-2i-1} + 6\frac{\sqrt{(9n-4)(9n-5)}}{18n-9}$$

where

$$A_{i,j} = \frac{2\sqrt{(18n-10-9i-j)(18n-11-9i-j)}}{36n-21-18i-2j}\,.$$

Proof. It should be noted that in the i'th level of graph G_n there exist $3\times 2^{i-2}$ copy of G_1. By substitution values of $\varepsilon(u)$ in Lemma 1 in terms of GA_4 the proof is clear.

Corollary 4.

$$ABC_3(G_n) = \sum_{i=0}^{n-2}(3\times 2^{n-i-2})\left(4\sum_{j=0}^{2} A_{i,j} + 2\sum_{j=3,5,6,7} A_{i,j} + \sum_{j=4,8} A_{i,j}\right) + 6\sum_{i=1}^{3}\frac{\sqrt{18n-2i-3}}{(9n-i)(9n-i-1)} + 3\frac{\sqrt{18n-11}}{(9n-4)(9n-5)},$$

where

$$A_{i,j} = \frac{\sqrt{36n-23-18i-2j}}{(18n-10-9i-j)(18n-11-9i-j)}\,.$$

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