Scaling synchronization of hyperchaotic Yang system

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This paper focuses on the problem of scaling synchronization of hyperchaotic Yang system, which is constructed by adding a linear controller to the Yang system with one saddle and two stable node-foci. When the parameters are fully known in advance, we apply the one-way linear coupling approach to synchronize the hyperchaotic Yang system up to a scaling factor. When the parameters are unknown, we utilize the adaptive method to synchronize the uncertain hyperchaotic Yang system up to a scaling factor. Simultaneously, we carry out many numerical simulations to verify the validation of the proposed schemes.

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Keywords: Scaling synchronization, Yang system, Hyperchaotic attractors, One-way linear coupling method, Adaptive method

1. Introduction

High sensitivity to parameter and initial condition perturbations is one of typical approaches for underlying characteristics of chaos [1]. In the last five decades, chaos analysis and control as well as chaotification in dynamical systems have been studied extensively. Hyperchaotic system is usually classified as a chaotic system with more than one positive Lyapunov exponent, which indicates that the chaotic dynamics of the system are expanded in more than one direction giving rise to a more complex attractor [2–9]. At the same time, due to its theoretical and practical applications in many technological fields, such as secure communications, nonlinear circuits, neural networks, control, synchronization, hyperchaos has recently become a central topic in nonlinear sciences fields (see e.g. [10–19] as well as their references).

The first hyperchaotic system was reported by Rössler in 1979 [2]. It might be due to the fact that the hyperchaotic systems are more complex and chaos generation in 4D or more autonomous systems is more difficult than chaotic systems [2-9]. So how to generate a hyperchaotic attractor is always a very attractive and yet technically quite challenging task theoretically. It has wide foreground, important theoretical and practical meanings to carry this research further. Based on the relations of the drive (or master) system and the response (or slave) system, one can classify synchronous behaviors into several different types, such as complete synchronization, anticipation synchronization, phase synchronization, projective synchronization, and generalized synchronization. These definitions are also regarded as different degrees of realization of a universal concept of synchronization. Along this line, chaos synchronization can successfully be realized with the help of various methods such as linear (nonlinear) feedback control, adaptive control, and sliding model control [10-19].

More recently, Yang and Chen proposed an interesting new system with one saddle and two node-foci [20], which make it fascinating mathematical entity itself and a useful candidate in real-world applications, such as security communications. Later on, this new system got its name, Yang system [21]. It is immediately clear that the Yang system will be topologically nonequivalent to the original Lorenz, Chen and all Lorenz-like systems. A more complex local analysis and the existence of homoclinic and heteroclinic orbits of Yang system have been carefully and rigorously studied in [22]. The sufficient and necessary conditions for Lyapunov stability of Yang system was discussed in [23]. The dynamics of Yang system involving fractional calculus was studied in [24]. By introducing a linear feedback controller to the second equation of the Yang system, the authors in [25] obtained the so-called hyperchaotic Yang system, in which they studied some complex dynamical behaviors such as ultimate boundedness, Lyapunov exponents, Poincaré projections, and 4D Hopf bifurcations.

However, how to synchronize hyperchaoticYang system with or without the known parameters is still an open question. This situation motivates us to explore this topic associated with a scaling factor. In other words, we will deal with the problem of scaling synchronization of the hyperchaotic Yang system.

The paper is organized as follows. In Section 2, we formulate the hyperchaoticYang system. In Section 3, we apply one-way linear coupling approach to synchronize the considered system. In Section 4, we design a new adaptive controller with parameter update laws to synchronize the uncertain hyperchaotic Yang system.

2. The hyperchaotic Yang system

The hyperchaotic Yang system is described by the following equations:

$$\begin{cases} \frac{dx}{dt} = a(y - x), \\ \frac{dy}{dt} = cx - xz + \omega, \\ \frac{dz}{dt} = -bz + xy, \\ \frac{d\omega}{dt} = -k_1 x - k_2 y, \end{cases} \tag{2.1}$$

where $a > 0$, $b > 0$, $c > 0$, k_1 and k_2 are two constant parameters, determining the chaotic and hyperchaotic behaviors and bifurcations of system (2.1). For example, when the parameters $(a, b, c, k_1, k_2) = (35, 3, 35, 2, 7.5)$, one have two positive Lyapunov exponents

$$\lambda_1 = 0.2747, \ \lambda_2 = 0.1374,$$

and the other two are

$$\lambda_3 = 0.0000, \lambda_4 = -38.4117.$$

In this situation the hyperchaotic Yang system (2.1) indeed has a hyperchaotic attractor, which is depicted in Fig. 1.

3. Scaling synchronization via the one-way linear coupling approach

In this section, we focus on the scaling synchronization of hyperchaotic Yang system via the one-way coupling method. More precisely, the corresponding drive-response systems of system (2.1) are expressed, respectively, as

$$\begin{cases} \frac{dx_m}{dt} = a(y_m - x_m), \\ \frac{dy_m}{dt} = cx_m - x_m z_m + \omega_m, \\ \frac{dz_m}{dt} = -bz_m + x_m y_m, \\ \frac{d\omega_m}{dt} = -k_1 x_m - k_2 y_m, \end{cases} \tag{3.1}$$

and

$$\begin{cases} \frac{dx_s}{dt} = a(y_s - x_s) + r_1(x_s - \mu x_m), \\ \frac{dy_s}{dt} = cx_s - x_m z_s + \omega_s + r_2(y_s - \mu y_m) + x_m(z_s - \mu z_m), \\ \frac{dz_s}{dt} = -bz_s + x_m y_s + r_3(z_s - \mu z_m) - x_m(y_s - \mu y_m), \\ \frac{d\omega_s}{dt} = -k_1 x_s - k_2 y_s + r_4(\omega_s - \mu\omega_m), \end{cases} \tag{3.2}$$

where r_i $(i = 1,2,3,4)$ are control parameters such that the two systems can be synchronized.

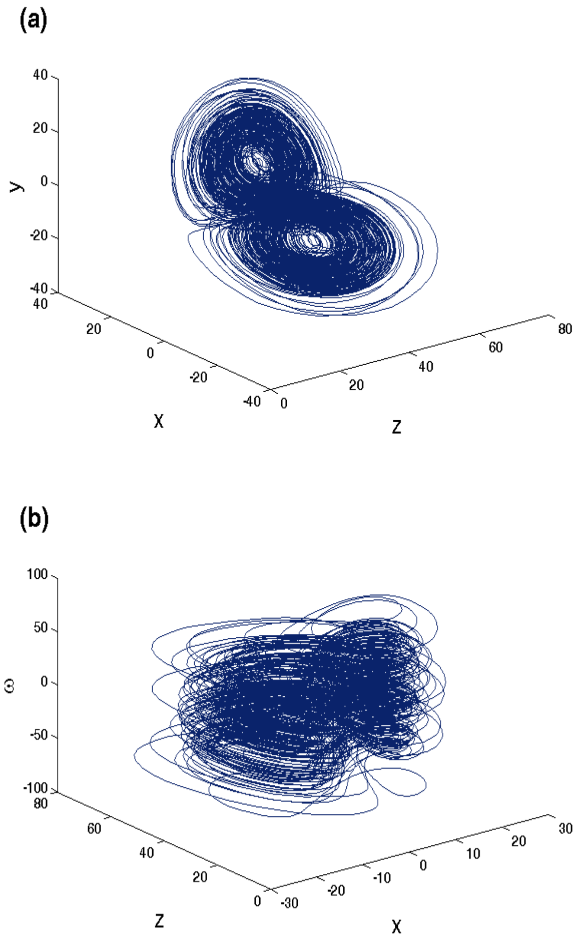

Fig. 1. Hyperchaotic attractor of hyperchaotic Yang system (2.1) with the parameters $(a, b, c, k_1, k_2) = (35, 3, 35, 2, 7.5)$: *(a)* $z - x - y$ *space and (b)* $x - z - \omega$ *space.*

Clearly, if system (3.1) has an attractor $\mathcal{A}$, then the synchronized response system (3.2) has an attractor $\mu\mathcal{A}$.

Let $e_1 = x_s - \mu x_m, e_2 = y_s - \mu y_m, e_3 = z_s - \mu z_m$, $e_4 = \omega_s - \mu\omega_m$, where $\mu \neq 0$ is a scaling factor. Then the error system between system (3.1) and (3.2) reads

$$\begin{cases} \frac{de_1}{dt} = (r_1 - a)e_1 + ae_2, \\ \frac{de_2}{dt} = ce_1 + r_2 e_2 + e_4, \\ \frac{de_3}{dt} = (r_3 - b)e_3, \\ \frac{de_4}{dt} = -k_1 e_1 - k_2 e_2 + r_4 e_4. \end{cases} \tag{3.3}$$

Moreover, it yields the Jacobian matrix

$$J = \begin{pmatrix} r_1 - a & a & 0 & 0 \\ c & r_2 & 0 & 1 \\ 0 & 0 & r_3 - b & 0 \\ -k_1 & -k_2 & 0 & r_4 \end{pmatrix}, \qquad (3.4)$$

and its characteristic equation

$$f(\lambda) = (\lambda - r_3 + b)(\lambda^3 + \theta_1\lambda^2 + \theta_2\lambda + \theta_3),$$

where

$$\begin{cases} \theta_1 = a - r_1 - r_2 - r_4, \\ \theta_2 = (r_1 - a)(r_2 + r_4) + r_2 r_4 - ac + k_2, \\ \theta_3 = -(r_1 - a)(r_2 r_4 + k_2) + a(cr_4 + k_1). \end{cases} \qquad (3.5)$$

According to the Routh–Hurwitz criterion, the real parts of all the roots λ in $f(\lambda) = 0$ are negative if and only if

$$r_3 - b < 0,\ \theta_1 > 0,\ \theta_3 > 0,\ \theta_1\theta_2 - \theta_3 > 0. \qquad (3.6)$$

In other words, (3.6) is the necessary and sufficient condition of the asymptotical stability of error system (3.3).

Based on the above discussion, the following property is verified.

Theorem 1 *The drive system (3.1) can synchronize the response system (3.2) if and only if the condition (3.6) is satisfied up to the scaling factor μ.*

We then carry out some numerical simulations to verify the effectiveness of the proposed synchronization method. We choose the parameters $(a,\ b,\ c,\ k_1,\ k_2) = (35,\ 3,\ 35,\ 2,\ 7.5)$ in all simulations so that the hyperchaotic Yang system exhibits hyperchaotic behaviors if no controls are applied. The initial conditions of both of the drive system (3.1) and the response system (3.2) are chosen as $(1.15,\ 3.5,\ 3.3,\ 1)$. Letting $r_1 = -5,\ r_2 = -30,\ r_3 = 0,\ r_4 = -8,$ then we get

$$\begin{cases} r_3 - b = -3 < 0, \\ \theta_1 = 78 > 0, \\ \theta_3 = 170 > 0, \\ \theta_1\theta_2 - \theta_3 = 42145 > 0. \end{cases}$$

Thus condition (3.6) in Theorem 1 is satisfied, implying that the synchronization between drive system (3.1) and response system (3.2) is achieved.

Next, we can choose any non-zero scaling factor μ, say, $\mu = 2$. It means to be in-phase synchronization and the scaling attractors are showed in Fig. 2. The drive attractor is the smaller one (blue solid line), the response attractor is the bigger one (red dotted line). The evolution of the time series of the drive system (3.1) and the response system (3.2) is shown in Fig. 3. We can see that the response attractor is twice the size of the drive one. The evolution of the error systems for this situation is shown in Fig. 4.

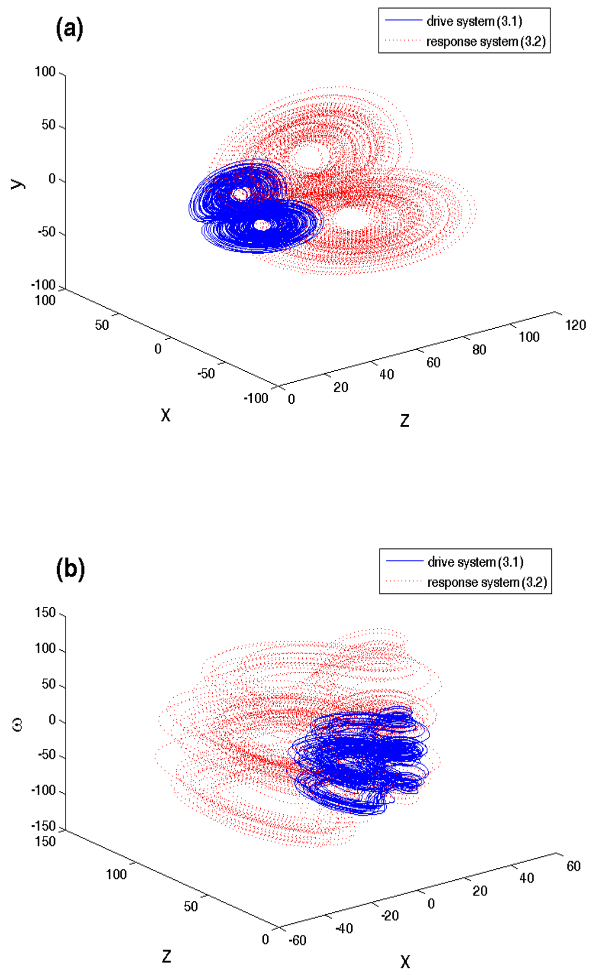

Fig. 2. Scaling attractors of hyperchaotic Yang system with the parameters $(a,\ b,\ c,\ k_1,\ k_2) = (35,\ 3,\ 35,\ 2,\ 7.5)$ and $\mu = 2$: (a) $z - x - y$ space and (b) $x - z - \omega$ space.

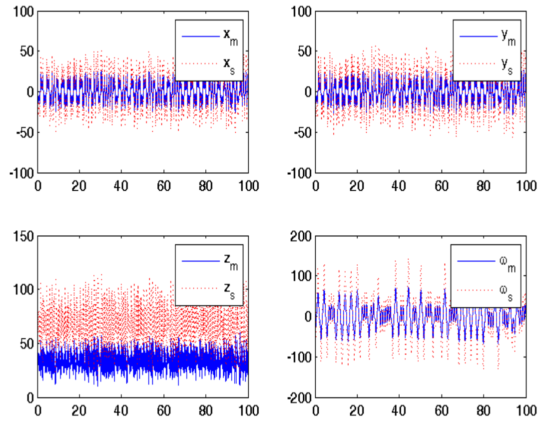

Fig. 3. The time series of drive-response systems with $\mu = 2$.

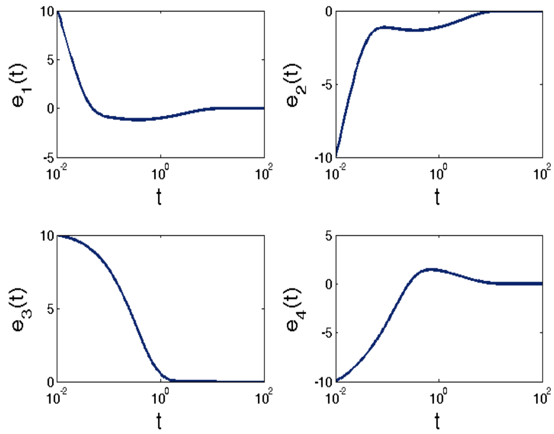

Fig. 4. The evolution of error system (3.3) with $\mu = 2$.

Moreover, we let $\mu = -2$, which refers to the anti-phase synchronization and the phase difference between two attractors is π.

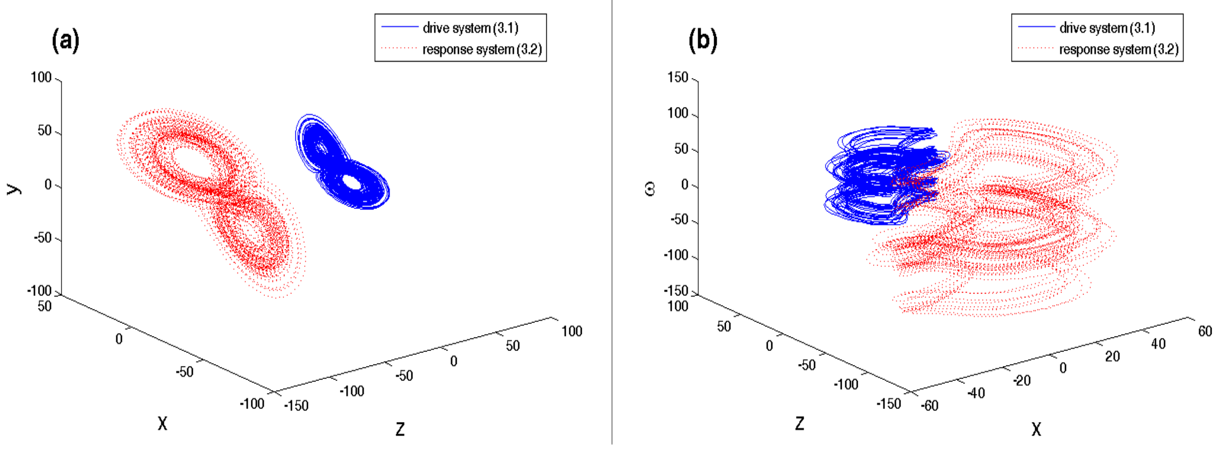

Fig. 5. Scaling attractors of hyperchaotic Yang system with the parameters $(a,\ b,\ c,\ k_1,\ k_2) = (35,\ 3,\ 35,\ 2,\ 7.5)$ *and* $\mu = -2$: *(a)* $z - x - y$ *space and (b)* $x - z - \omega$ *space.*

Herein the scaling attractors are showed in Fig. 5. The drive attractor is the smaller one (blue solid line), the response attractor is the bigger one (red dotted line). The evolution of the time series of the drive system (3.1) and the response system (3.2) is shown in Fig. 6. We can see that the response attractor is twice the size of the drive one, while the phase difference between two attractors is π. The evolution of the error systems for this situation is shown in Fig. 7.

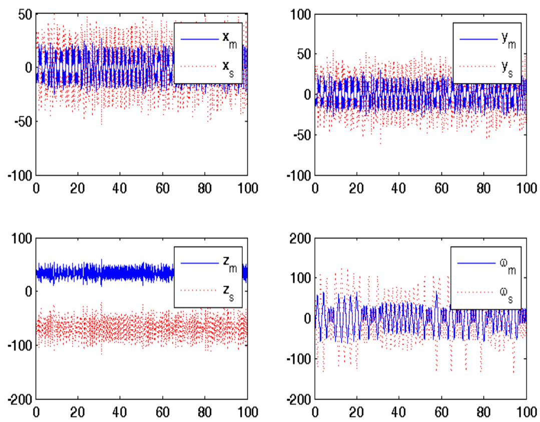

Fig. 6. The time series of drive-response systems with $= -2$.

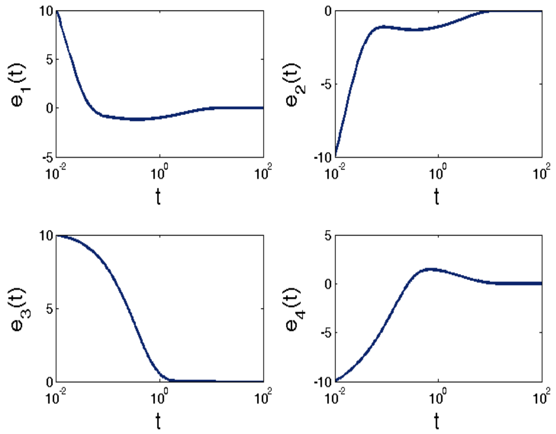

Fig. 7. The evolution of error system (3.3) with $\mu = -2$.

4. Scaling synchronization via the adaptive method

In this section we study the scaling synchronization of hyperchaotic Yang system with fully unknown parameters via adaptive synchronization method. We also choose (3.1) as drive system, and define the response system as

$$\begin{cases} \frac{dx_s}{dt} = a'(y_s - x_s) + u_1, \\ \frac{dy_s}{dt} = c'x_s - x_s z_s + \omega_s + u_2, \\ \frac{dz_s}{dt} = -b'z_s + x_s y_s + u_3, \\ \frac{d\omega_s}{dt} = -k_1' x_s - k_2' y_s + u_4, \end{cases} \tag{4.2}$$

where a', b', c', k_1', k_2' are respectively the parameter estimations of a, b, c, k_1, k_2, and $u_i\ (i = 1,2,3,4)$ are controllers such that the two systems can be synchronized. Clearly, if system (3.1) has an attractor $\mathcal{A}$, then the synchronized response system (4.2) has an attractor $\mu\mathcal{A}$.

Let $e_1 = x_s - \mu x_m, e_2 = y_s - \mu y_m, e_3 = z_s - \mu z_m$, $e_4 = \omega_s - \mu\omega_m$, where $\mu \neq 0$ is the scaling factor. Then the error system between system (3.1) and (4.2) reads

$$\begin{cases} \frac{de_1}{dt} = a'(y_s - x_s) - \mu a(y_m - x_m) + u_1, \\ \frac{de_2}{dt} = c'x_s - x_s z_s + \omega_s - \mu(cx_m - x_m z_m + \omega_m) + u_2, \\ \frac{de_3}{dt} = -b'z_s + x_s y_s - \mu(-bz_m + x_m y_m) + u_3, \\ \frac{de_4}{dt} = -k_1' x_s - k_2' y_s - \mu(-k_1 x_m - k_2 y_m) + u_4. \end{cases} \tag{4.3}$$

Thus our objective in what following is to find some effective controllers $u_i\ (i = 1,2,3,4)$ with parameters estimation update laws which can make $\lim_{t\to\infty} e_i = 0$.

Theorem 2 *The hyperchaotic Yang system (3.1) can synchronize the uncertain controlled hyperchaotic Yang system (4.2) up to a scaling factor* μ *by designing the following adaptive controllers*

$$\begin{cases} u_1 = (a' - 1)e_1 - a'e_2, \\ u_2 = -c'e_1 - e_2 - e_4 + x_s z_s - \mu x_m z_m, \\ u_3 = (b' - 1)e_3 - x_s y_s + \mu x_m y_m, \\ u_4 = k_1' e_1 + k_2' e_2 - e_4, \end{cases} \tag{4.4}$$

in which the parameter estimations update laws are

$$\begin{cases} \frac{da'}{dt} = -\mu(y_m - x_m)e_1, \\ \frac{db'}{dt} = \mu z_m e_3, \\ \frac{dc'}{dt} = -\mu x_m e_2, \\ \frac{dk_1'}{dt} = \mu x_m e_4, \\ \frac{dk_2'}{dt} = \mu y_m e_4. \end{cases} \tag{4.5}$$

Proof. Substituting equation (4.4) into equation (4.3) gets

$$\begin{cases} \frac{de_1}{dt} = \mu e_a(y_m - x_m) - e_1, \\ \frac{de_2}{dt} = \mu e_c x_m - e_2, \\ \frac{de_3}{dt} = -\mu e_b z_m - e_3, \\ \frac{de_4}{dt} = -\mu\ e_{k_1} x_m - \mu e_{k_2} e_{k_2} - e_4, \end{cases} \tag{4.6}$$

where the parameter errors are defined such that $e_a = a' - a$, $e_b = b' - b$, $e_c = c' - c$, $e_{k_1} = k_1' - k_1$, $e_{k_2} = k_2' - k_2$. Then one can construct the following Lyapunov function

$$V = \frac{1}{2}\left(\sum_{i=1}^4 e_i^2 + e_a^2 + e_b^2 + e_c^2 + e_{k_1}^2 + e_{k_2}^2\right).$$

Differentiating V with respect to the time and using (4.5) and (4.6), we obtain

$$\frac{dV}{dt} = \sum_{i=1}^4 e_i \frac{de_i}{dt} + e_a \frac{da'}{dt} + e_b \frac{db'}{dt} + e_c \frac{dc'}{dt} + e_{k_1} \frac{dk_1'}{dt} + e_{k_2} \frac{dk_2'}{dt} = -\sum_{i=1}^4 e_i^2 < 0.$$

According to Lyapunov stability theory, we know that error systems (4.3) is asymptotically stable in the neighborhood of the zero solution, which means the synchronization between drive system (3.1) and response system (4.2) is achieved. This completes the proof. ■

We then carry out some numerical simulations to verify the effectiveness of the proposed synchronization method. We choose the parameters $(a,\ b,\ c,\ k_1,\ k_2) = (35,\ 3,\ 35,\ 2,\ 7.5)$ in all simulations so that the hyperchaotic Yang system exhibits hyperchaotic behaviors if no controls are applied. The initial conditions of both of the drive system (3.1) and the response system (4.2) are $(1.15,\ 3.5,\ 3.3,\ 1)$. Moreover, the initial values of the estimated parameters are chosen as $a' = 30$, $b' = 5$, $c' = 30$, $k_1' = 5$, $k_2' = 5$.

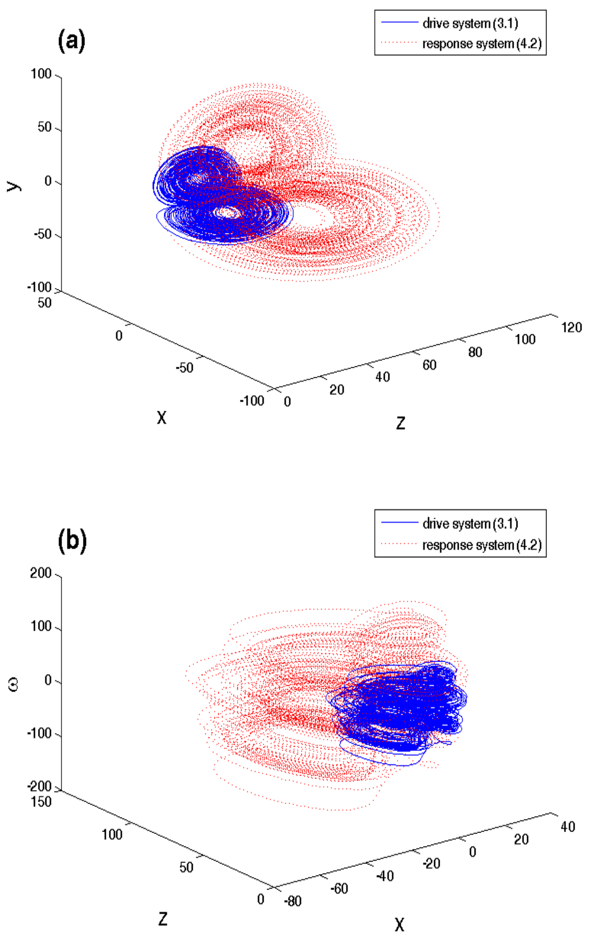

Fig. 8. Scaling attractors of hyperchaotic Yang system with the parameters $(a,\ b,\ c,\ k_1,\ k_2) = (35,\ 3,\ 35,\ 2,\ 7.5)$ *and* $\mu = 2$: *(a)* $z - x - y$ *space and (b)* $x - z - \omega$ *space.*

Next, we can choose any non-zero scaling factor α, say, $\mu = 2$. It means to be in-phase synchronization and the scaling attractors are showed in Fig. 8. The drive attractor is the smaller one (blue solid line), the response attractor is the bigger one (red dotted line). The evolution of the time series of the drive system (3.1) and the response system (4.2) is shown in Fig. 9. The evolution of the error systems for this situation is shown in Fig. 10. The evolutions of parameter estimators are shown in Fig. 11.

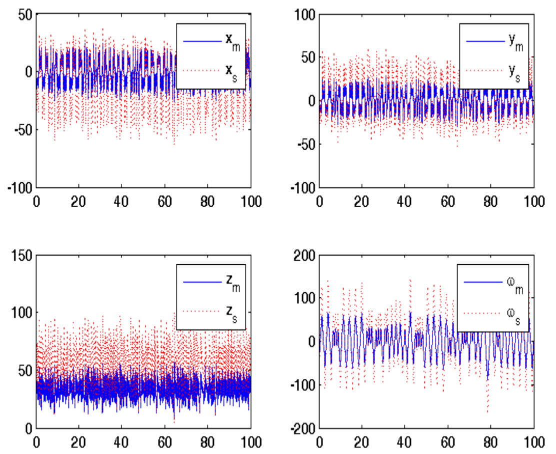

Fig. 9. The time series of drive-response systems with $\mu = 2$.

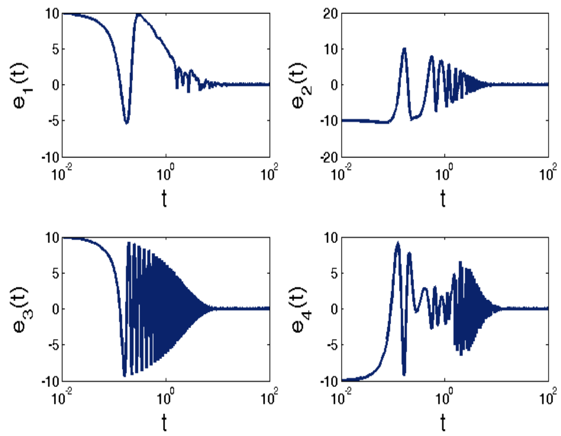

Fig. 10. The evolution of error system (4.3) with $\mu = 2$.

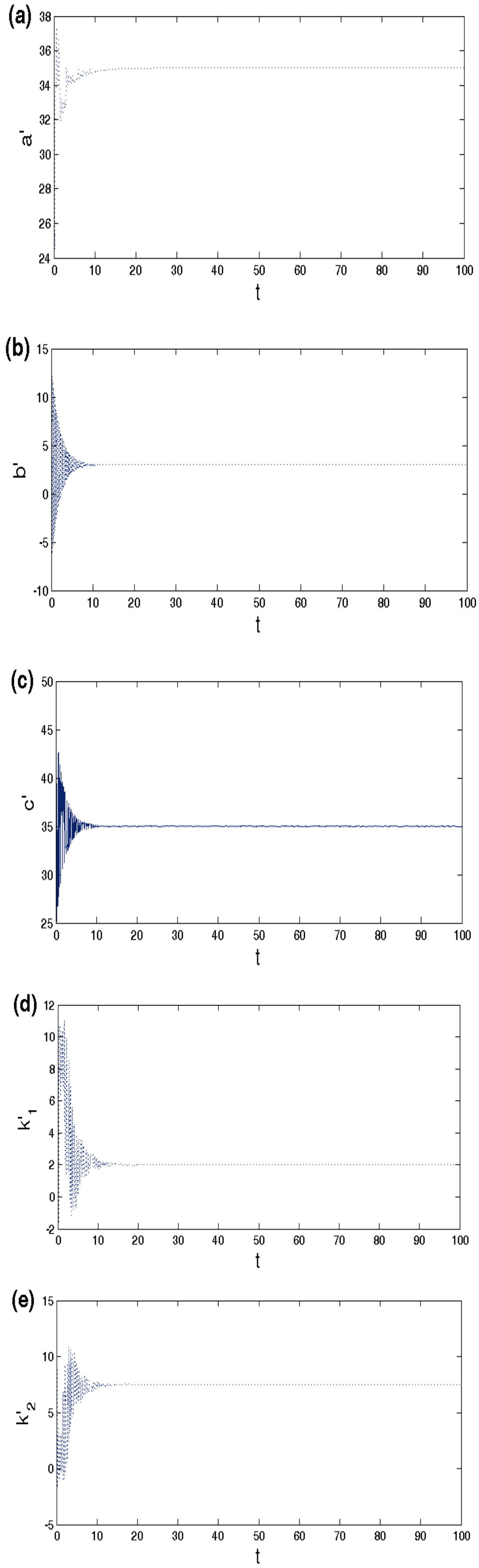

Fig. 11. The evolution of parameter estimators $\mu = 2$.

Moreover, we let $\mu = -2$, which refers to the anti-phase synchronization and the phase difference between two attractors is π. Herein the scaling attractors are showed in Fig. 12. The drive attractor is the smaller one (blue solid line), the response attractor is the bigger one (red dotted line). The evolution of the time series of the drive system (3.1) and the response system (4.2) is shown in Fig. 13. The evolution of the error systems for this situation is shown in Fig. 14. The evolutions of parameter estimators are shown in Fig. 15.

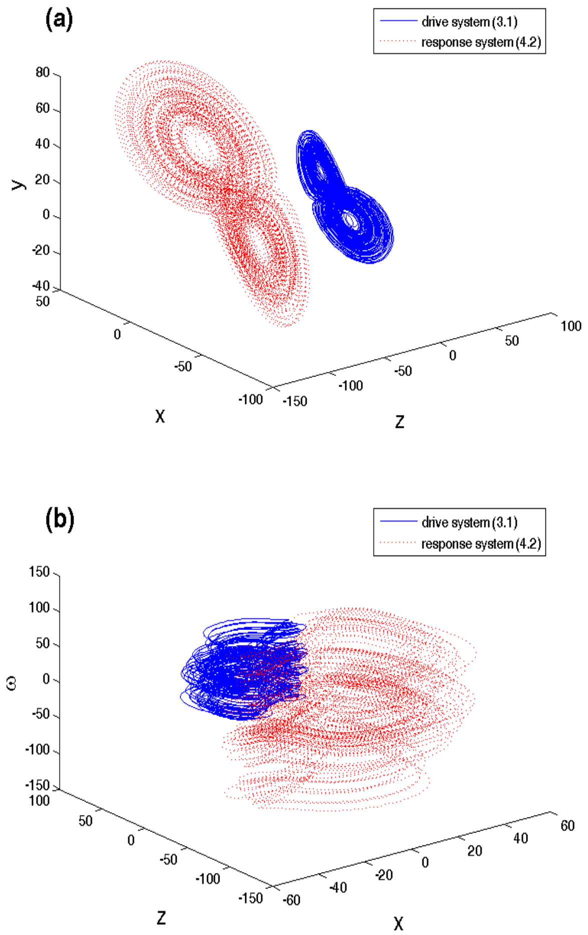

Fig. 12. Scaling attractors of hyperchaotic Yang system with the parameters $(a,\ b,\ c,\ k_1,\ k_2) = (35,\ 3,\ 35,\ 2,\ 7.5)$ *and* $\mu = -2$: *(a)* $z - x - y$ *space and (b)* $x - z - \omega$ *space.*

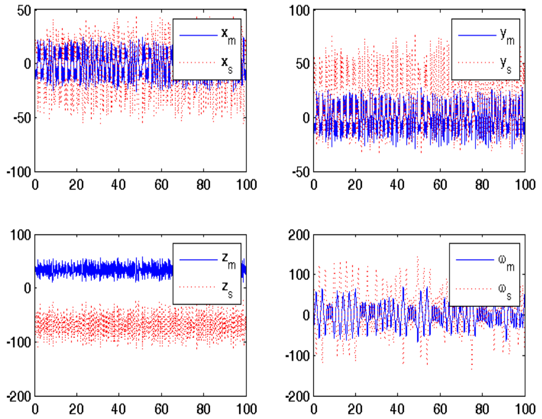

Fig. 13. The time series of drive-response systems with $\mu = -2$.

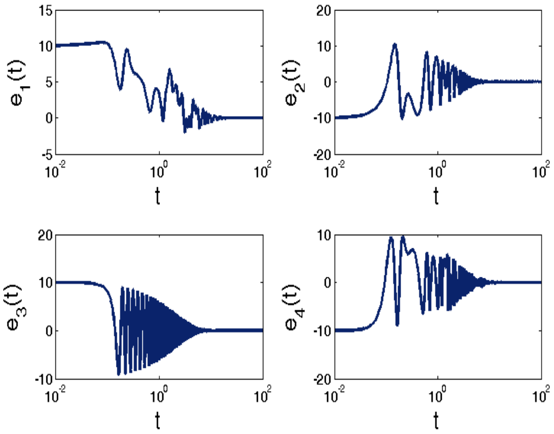

Fig. 14. The evolution of error system (4.3) with $\mu = -2$

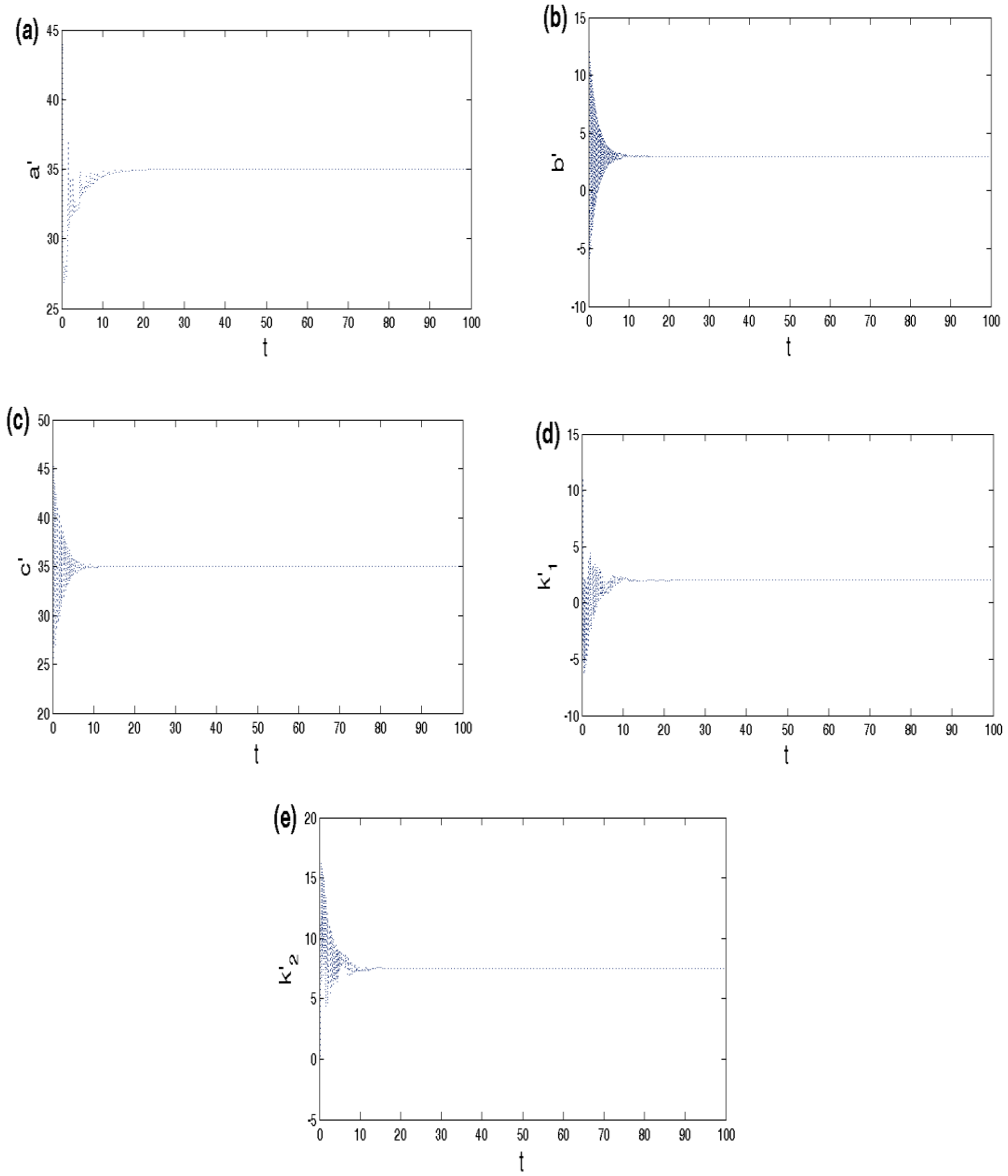

Fig. 15. The evolution of parameter estimators $\mu = -2$.

5. Conclusion

In this paper we have studied the scaling synchronization of hyperchaotic Yang system via the one-way linear coupling approach and the adaptive method, respectively. For the former, we apply the Routh-Hurwitz criterion to obtain the necessary and sufficient condition. For the latter, we utilize the Lyapunov stability theorem to get a sufficient condition. Moreover, we carried out many numerical simulations to verify our proposed synchronization approaches up to a scaling factor. Interestingly, the scaling factor is a free parameter, which makes it very useful in security communications.

Last but not least, we point out the scaling factor can be chosen as different constants for each equation of the considered systems. This situation refers to mixed synchronization, in which some state variables are in-phase synchronization and others are anti-phase synchronization. From the above discussion, our proposed approaches in this paper are also valid for mixed synchronization. From this point of view, the scaling synchronization has potential application in processing industry if one intends to enhance or reduce the concentration and even remove another component in catalytic reactions to obtain a desired final product.

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