Quasi phase matched second harmonic generation using five fold symmetric photonic quasi crystal fiber

SIVACOUMAR RAJALINGAM[a,*], Z. C. ALEX[a]
[a]*School of Electronics Engineering, MEMS & Sensor Division, VIT University, Vellore-632014, Tamilnadu, India*

This study numerically investigated quasi-phase matching condition for Second Harmonic Generation (SGH) in a novel five-fold symmetric Photonic Quasi Crystal fiber (PQF). The optical properties required for the phase matching condition is realized by modifying the air hole size and lattice pitch for the proposed PQF. In this study we have targeted on wavelength mismatch factor, overlap integral of the fundamental and the second order mode and coherence length. Due to strong guidance of the Quasi-lattice structure, we have numerically studied the visible wavelength generation with a relative efficiency of 56.57 % W^{-1} cm^{-2} for the proposed fiber.

(Received July 15, 2015; accepted September 9, 2015)

Keywords: Photonic crystal fibers, Chromatic dispersion, Effective index, Dispersion tolerance

1. Introduction

One of the most typical quadratic nonlinear optical processes is second harmonic generation (SHG). Improving the conversion efficiency and acceptance bandwidth in SHG has attracted considerable attention in recent years. Nonlinear bulk materials conventionally used for frequency doubling optical radiation are quite expensive and difficult to handle, despite of converting the input frequency into the second harmonic with an efficiency of about 50–60%. Second harmonic generation was realized in a one-dimensional photonic crystal with a conversion efficiency hardly reaching about 1% - 4% [1]. Second harmonic generation in isotropic media is difficult to achieve due to the centro-symmetric nature. In spite of the centrosymmetric nature of silica glass material, which generally can exhibit only odd-order nonlinearities. It's interesting to observe the intense optical radiation[4] can bring about a DC electric field induced third order susceptibility originating Second order nonlinearity ($\chi^{(2)}$) in those isotropic medium like silicate glass waveguides [2] [3] [5]. In recent times, there has been keen interest towards SHG employing optical fibers due to size, flexibility and ease of fabrication. Poling of silica requires the insertion of metal electrode wires by making holes in the fiber [3]. An interesting phenomenon during SHG is that the fundamental spectrum can be significantly affected. This fundamental modulation is a Kerr-like effect and can be explained through cascaded quadratic non-linearities[6]. The power dependence of the refractive index is responsible for the Kerr-effect. Depending upon the type of input signal, the Kerr-nonlinearity manifests itself in three different effects such as Self-Phase Modulation (SPM), Cross-Phase Modulation (CPM) and Four-Wave Mixing (FWM) [5].

Efficient SHG requires the relative phase mismatch between fundamental and second harmonic fields, in order to improve phase matching (PM) condition, a number of quasi-phase-matching (QPM) techniques have been developed. We note that the QPM method has been employed in photonic crystals as well as photonic quasi-crystals, relying either on periodic modulation of the generating field strength by periodic static electric field or QPM induced using two counter propagating pulses [7][11].

By tailoring the optical properties like, the group velocity mismatch, the efficiency of SHG can be improved to a drastic level. PCF is a suitable candidate for SGH, since the waveguiding properties could easily be engineered by varying the geometrical parameters, namely, pitch and diameter of the air-holes. Recently Photonic quasi crystal fibers(PQF) have been investigated for various distinct waveguiding properties like very less confinement loss [8], a flat [8] and large dispersion [9], a high nonlinearity [10] and a larger cutoff ratio. These properties are harnessed due to the shattered periodicity in the air hole arrangement in PQF cladding. Special types of PQF's and hybrid cladding structures are made possible by any one of the following methods such as sol–gel, stack-draw, molding technique, extrusion technique, drilling and drawing method,[12-15] etc. The proposed hybrid PQF can be fabricated using sol-gel method, since sol-gel naturally gives well-arranged air holes. As a casting method, the sol-gel technique can fabricate any structure, which can be integrated into a mold. The hole size, shape and spacing may all be adjusted individually.

2. Proposed five fold symmetric PQF

We have illustrated the transverse cross section of 5-fold PQF in Fig. 1. Quasi-crystals are crystals in which they have long-range translational and orientational orders [17][36]. However, the translational order is not periodic and the structure does not necessarily have crystallographic rotational point symmetry. These structures reveal more significant attributes than the stereotyped PCFs, due to the degree of freedom that has been concealed in aperiodic structures. The proposed PQF structure is formed by two types of rhombic tiles, thin tile with angles of (π/5) & (4π/5), and thick tile with angles of (2π/5) & (3π/5). The resulting connected space-filling packing of unit cells is called Penrose lattice [31]. The inner core and the surrounding air holes are constructed on the base of 2-dimenstional Penrose lattice. In our analysis the refractive index of fused silica as a function λ is approximated using three term sellmeier equation [16][38].

$$n^2(\lambda) = 1 + \sum_{i=1}^{3} \frac{a_i \lambda^2}{\lambda^2 - b_i^2} \tag{1}$$

Where, λ is the wavelength, n the refractive index as a function of λ, a_i the oscillator strength, and b_i is the oscillator resonance frequency. In an index-guiding PCF, the core refractive index is greater than the average index of the cladding, since the air holes are interweaved in the silica matrix, and the fiber can guide the light by total internal reflection as a standard single mode fiber does. That is, the guided light has an effective index n_{eff} that satisfies the condition $n_{core} > n_{eff} = (\beta/k_0)$. As per the theoretical aspects discussed the proposed PQF geometrical layout is formed by thick and thin rhombic units. Here the hole diameter is represented as ‘d’ and the distance between the center of two adjacent air holes, known as pitch or lattice constant, is represented by Λ. In order to optimize the structure, we have designated the lattice constant to be varied between 1 – 19 μm. In order to maintain the single mode operation of the PQF, (d/Λ) ratio is preserved within 0.1 to 0.5 and the fundamental mode and the second harmonic modes are analyzed for different pitch parameter [18-20].

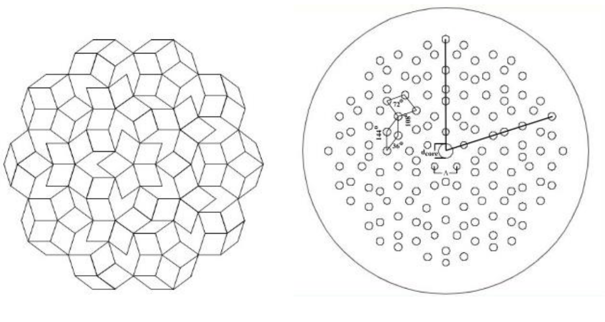

Fig. 1. The geometrical structure of the proposed five-fold symmetric Photonic Quasi crystal fiber

3. SGH - theory

Theoretically it is been investigated that photonic crystal fibers have more freedom for generating second harmonics with superior efficiency. Since, PQFs have even more flexibility in terms exploiting the properties of fiber parameters, SGH efficiency can be few orders greater than the typically poled optical fibers. Efficient SHG requires the relative phase mismatch between fundamental and second harmonic fields to be zero, and fiber dispersion generally prevents cumulative growth of the interacting [21-22]. In general $n_{sh} > n_f$ because of normal dispersion in the materials, so that the fundamental and second harmonic waves travel at different phase velocities. Quasi-phase matching (QPM) is the best suited method which requires periodic poling [23-27]. Periodic poling can be applied to waveguides and a theoretical treatment was given by Somekh and Yariv in 1972 using the Fourier expansion. In QPM, SHG will be at its maximum when the modal propagation constant $\Delta\beta = 0$. The efficiency of the SHG(η) can be expressed as

$$\eta = \frac{P_{2\omega}}{P_\omega} = P_\omega l^2 \frac{(d_{eff})^2}{A_{ovl}} sinc^2\left(\frac{\Delta\beta l}{2}\right)\left(\frac{8\pi^2}{\lambda_f^2 n_f^2 n_{sh} \varepsilon_0 c}\right) \tag{2}$$

Where P_ω is the input fundamental power, $P_{2\omega}$ is the output second harmonic power, l is the length of the interaction medium. Here the efficiency of the SGH derives to be maximum when sinc function equals 1, i.e period of the fiber exactly compensates for the phase mismatch [22][28]. The term d_{eff} corresponds to effective nonlinear coefficient, $d_{eff} = (d/\pi)$ where d is nonlinearity factor correlates to susceptibility tensor $\chi^{(2)}$. In the above equation A_{ovl} is the effective overlap area between the fundamental and second harmonic modes which can be estimated by [29]

$$A_{ovl} = \frac{1}{I_{ovl}^2} = \left[\iint E_{sh}^* E_f^2 \, dxdy\right]^{-2} \tag{3}$$

Where I_{ovl} is the overlap integral and E_{sh}, E_f are normalized second harmonic and fundamental transverse modes. The efficiency is aggregated for wavelengths that satisfy the QPM condition, but the bandwidth (BW) narrows down as the efficiency increases as length (l) increases since, $sinc^2\left(\Delta\beta l/2\right)$. It is quite obvious to optimize the efficiency without forfeit the bandwidth requirement. In this study we have spotlighted on parameters which influences the efficiency of the SHG. It is noted that the period of the quasi-phase matching has a predominant effect on the efficiency of the SGH [30].

4. Studies on factors influencing efficiency of second harmonic generation

The proposed PQF is an index guiding silica fiber with a matrix of air holes running through the length of the fiber. The lattice has perfect long range periodicity with good rotational symmetry. Here, we have investigated the variation of effective index (n_{eff}) with pitch (Λ) has been analyzed for the first order mode (ω) and the second harmonics (2ω) at 1.064 μm and 0.532 μm respectively. We engaged finite element method (FEM) for calculating the effective indices of fundamental and second harmonics by varying the (d/Λ) ratio. As seen from the Fig. 3(a). when the pitch increases the effective index for fundamental and second harmonics increases and reaches the effective index described by the sellemeier's equation (i.e., n_f = 1.449528, n_{sh} = 1.460680). It is observed from the Fig. 3 (b), when the pitch is at minimum the effective index difference between $HE_{11}(\omega)$ and $HE_{11}(2\omega)$ is at the maximum, as the (d/Λ) ratio increases the relative index differences ceases to minimum. It is noteworthy form the Fig. 3 (a), to indicate $HE_{11}(2\omega)$ moves slowly towards the cutoff when pitch reduces to lower value as compared to the fundamental mode.

The effective index difference has a profound effect on the wave-vector mismatch. The phase matching occurs when a constant phase relationship is maintained between the generated frequency and the propagating frequency [26]. Due to chromatic dispersion, sometimes the wave vector of the second harmonics is greater than the twice of fundamental mode, which can be minimized by choosing different polarization. Here the wave vector mismatch is defined by [28]

$$\Delta k = k_{2\omega} - 2k_{\omega} = \frac{4\pi}{\lambda_f}[n_{sh} - n_f] \quad (4)$$

Where λ_f is the fundamental wavelength. From the eq. 4, we define $\Delta\beta = \beta_2 - 2\beta_1 - 2\pi/G$, where $\Delta\beta$ is the phase mismatch factor and G is the grating period given by $G = 2\pi/(\beta_2 - 2\beta_1)$. Phase modulation along with group velocity mismatch of fundamental and second harmonics can often limit the conversion efficiency [37]. Phase modulation of the fundamental pulse can arise mainly due to the nonlinear index of the propagating medium. When the phase mismatch is sufficiently large, then the power flow between the fundamental mode and the second harmonics will be reversed.

It is evident from the Fig. 4, as the pitch increases, the air hole distance also increases and ultimately the core radius also. Since effective index difference is directly proportional to Δk, the wave vector mismatch found to decrease as the pitch increases. It can be inferred as when pitch increases both the fundamental mode and the second harmonic mode group velocities changes and their respective effective indices also increases. From the Fig. 3(a), it may be noted that the fundamental mode refractive index takes a swift change resulting in a lower index difference contributing for lower wave-vector mismatch. Here the range of d/Λ ratio analysis is made from 0.1 to 0.5 and it is indicative from the Fig. 4 the wave vector mismatch is diminishing as the ratio reduces to 0.1 and beyond this the confinement of the fundamental mode is difficult to achieve. From literature the cut off range for PCF is estimated to be 0.4 and the range for PQF is extending beyond 0.5 from our analysis.

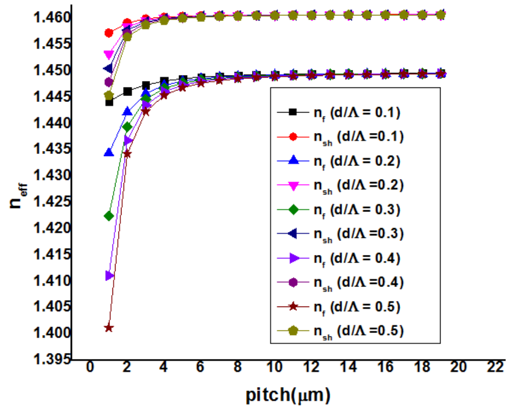

Fig. 3.(a) Change in effective refractive index for fundamental mode and second harmonic mode with reference to increasing pitch for d/Λ ranging from 0.1 to 0.5

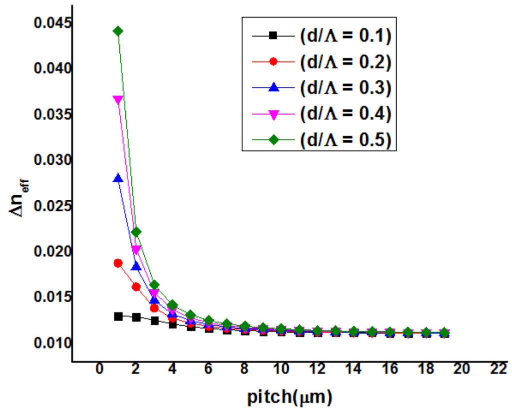

Fig. 3.(b) Variation of effective index difference between fundamental mode and second harmonic mode for various pitch for d/Λ ranging from 0.1 to 0.5

Inherently, if the fundamental mode propagates at a different phase velocity than the harmonic field, after a certain propagation distance the fundamental mode will be exactly out of phase with the corresponding co-propagating second harmonic field. At this point, the generated second harmonic wave will destructively interfere and if the phase mismatch is $\Delta k = 0$, the generated harmonic field will be maximum, otherwise the strength of the field will oscillate sinusoidally with the distance. The periodicity of this oscillation is twice the coherence length given by [32]

$$L_c = \frac{\pi}{\Delta k} \quad (5)$$

where, L_c describes the propagation distance between two locations of generated harmonic waves that are exactly out of phase. In our analysis the variation of L_c with respect to the d/Λ ratio spreading from 0.1 to 0.5 is shown in the Fig. 5. The coherence length for 0.5(d/Λ) is numerically computed to be at 6.01 μm and steadily increases to 23.68

μm. The magnitude of coherence length achieved for the proposed design is much higher than the previously reported for the photonic crystal fibers.

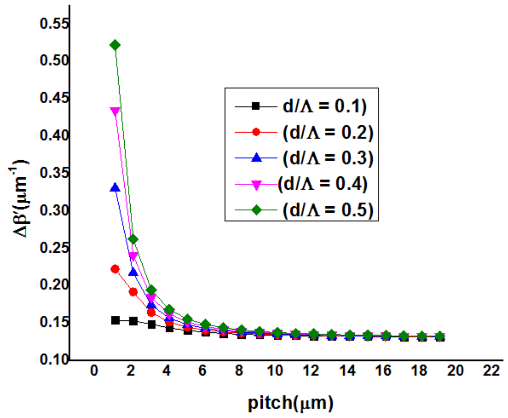

Fig. 4. Variation of wave-vector mismatch for various pitch with for d/Λ ranging from 0.1 to 0.5

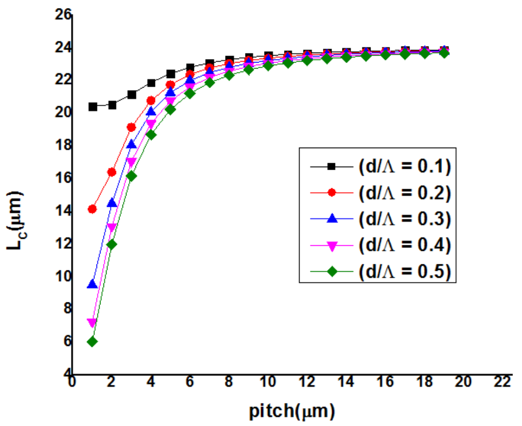

Fig. 5. Coherence length as a function of varying pitch for d/Λ ranging from 0.1 to 0.5

The direction of power flow between the fundamental mode and the second harmonic wave depends on the relative phase. So, the +/-sign is altered at every coherence length by periodic poling, but in silica fibers the nonlinearity appears due to the frozen electric field which cannot be easily reversed. These can be interpreted by the effective nonlinear coefficient d_{eff}. To overcome this problem, QPM technique employs changing the sign of the nonlinear susceptibility $\chi^{(2}$ at every coherence length, the phase of the polarization wave is shifted by π, effectively rephrasing the interaction and leading to monotonic power flow into the second harmonic wave. The thickness of each pair of positive and negative $\chi^{(2)}$ is defined as the quasi-phase matching length l_{qpm} given by $l_{qpm} = 2|L_c|$. As noted in the Fig. 6 the quasi-phase matching length increases as the pitch increases and saturates approximately around 46 μm - 47 μm when the pitch reaches 12 μm. From the plot, we would confer, as the pitch increases the quasi-phase matching length increases with a good poling.

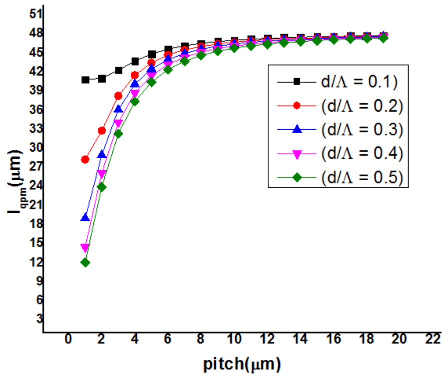

Fig. 6. Quasi-phase matching length as a function of varying pitch for d/Λ ranging from 0.1 to 0.5

The overlap integral is another predominant factor in deciding the efficiency of the second harmonic generation. The overlap integral ($I_{ovl} = \left[\iint E_{sh}^{*} E_f^{2} dxdy\right] = 1/[A_{ovl}]^{1/2}$) between the fundamental mode and the generated second harmonics directly relates to the efficiency of power transfer between these modes [33]. For an instance, when d increases the equivalent index of the cladding decreases, increasing the index contrast between core and cladding, which in turn increases the confinement of the mode in PQF. Reducing the pitch makes the fundamental and the second harmonics to be more confined which reduces the mismatch of their effective areas leading to an increase in the overlap integral. From Fig. 7, it is evident that as the d/Λ various from 0.3 to 0.5 the overlap area changes drastically due to the core area. In our analysis it is clear from the Fig. 6 when the d/Λ=0.5 the A_{ovl} is very minimum compared to the conventional PCF.

The Fig. 8 depicts the conclusive relation between the wavevector mismatch Δβ\`(=$\beta_2$-2$\beta_1$) to that of overlap area ($A_{ovl}$). As the analysis speculates the wavevector mismatch Δβ\` for d/Λ=0.5 declines as the pitch increases but at the same time the overlap area (A_{ovl}) gains an increase. From our analysis its noteworthy to find overlap area is at minimum for d//Λ=0.5.

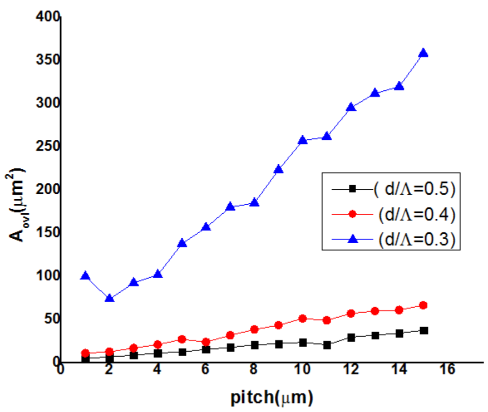

Fig. 7. Change in overlap area for different d/Λ ratio with respect to increasing pitch

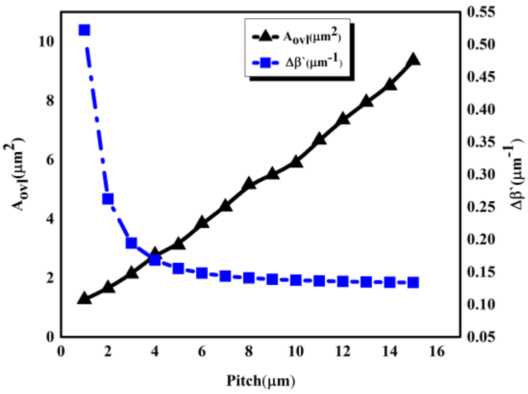

Fig. 8. Relative variation between $\Delta\beta$`(doted dash) and A_{ovl} (solid line) with respect to pitch

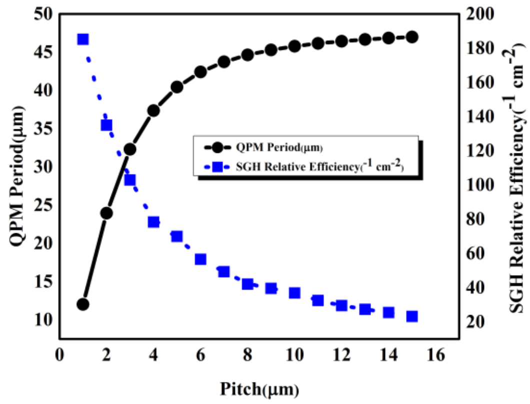

Fig. 9. SGH Relative efficiency (dotted) and QPM period with varying pitch (solid line)

As depicted in Fig. 8 the overlap area plays a vital role in the conversion efficiency as well as the phase mismatch factor. We can compute SHG in terms of relative efficiency ($(\eta_R = \eta / P_\omega l^2)$ which is defined as the ratio of absolute SHG efficiency to the product of fundamental input power and square of the length of the fiber. As, anticipated Fig. 9 clearly proves the efficiency increases as the pitch is decreased and the efficiency is maximum when it reaches 1 μm, since the phase mismatch factor is at its minimum. But poling of the fiber at 1 μm, impound condition which is impossible. In recent past thermal poling has been accomplished with core diameter of 6 μm with the electrode separation of 70 μm [34]leading to a significant increase of nonlinear optical susceptibility ($\chi^{(2)}$) [35][37] reaching the value of bulk material ~1 pm/V, d_{33} = 0.1591 pm/V. As the length of the PQF is considered to be 10 cm with d/Λ=0.5, we could achieve a relative efficiency of 56.57% W^{-1} cm^{-2}. The relative efficiency obtained with the proposed PQF is relatively higher than the previously reported values [22].

5. Conclusion

This research work have proposed a novel five -fold symmetric photonic quasi-crystal fiber mainly targeting in improving the factors responsible for efficiency of the second harmonic generation. We have optimized the design in having lowest phase mismatch vector with the overlap area between the fundamental mode and the second harmonics by the relative distance between the air holes and achieving a maximum relative efficiency of SGH to 56.57 %$W^{-1}cm^{-2}$. Form the above analysis, it is clear that overlap area plays a vital role in deciding the efficiency, but still if we need to improvise the efficiency then the research can be directed towards optimizing the d_{33} component by having multi-component glasses like germane-silicate and chalcogenide glasses with high nonlinear optical susceptibility can be accomplished conjointly with appropriate poling technique.

Acknowledgment

This research work is supported by the Department of Science and Technology, under the scheme Fund for Improvement of Science and Technology Infrastructure in Higher Educational Institutions (FIST) (SR/FST/ETI-288/2011).

References

[1] Yadav, Ksenia, C. L. Callender, C. W. Smelser, C. Ledderhof, C. Blanchetiere, S. Jacob, J. Albert. Opt. Express **19**(27), 26975 (2011).
[2] M. Houe, P. D. Townsend. Appl. Phys. **28,** 1747(1995).
[3] An, Honglin, Simon Fleming, Opt. Express **20**(7), 7436 (2012).
[4] Grechin, Sergei G, Valentin G Dmitriev. Quantum Electron. **31**, 933 (2001).
[5] H. Takebe, P. G. Kazansky, P. St. J. Russell, K. Morinaga, Opt. Lett. **21**(7), 468 (1996).
[6] Guo, Hairun, Xianglong Zeng, Binbin Zhou, Morten Bache. J. Opt. Soc. Am. B **30**(3) 494.
[7] M. M. Fejer, G. A. Magel, D. H. Jundt, R. L. Byer. IEEE J. Quantum Electron. **28**(11), 2631 (1992).
[8] Saitoh, Kunimasa, Nikolaos Florous, Masanori Koshiba. Opt. Express **13**(21), 8365 (2005).
[9] Sivacoumar Rajalingam, Z. C. Alex. Research Journal of Applied Sciences, Engineering and Technology, **9**(9), 786 (2015).
[10] K. P. Hansen, J. R. Jensen, C. Jacobsen, H. R. Simonsen, J. Broeng, P. M. W. Skovgaard, A. Petersson, A. Bjarklev, Optical Fiber Comm. Conference (OSA), **70** (2002).
[11] Xiaoshi Zhang, Amy L. Lytle, Tenio Popmintchev, Xibin Zhou, Henry C. Kapteyn, Margaret M. Murnane, Oren Cohen, Nature Physics **3**, (2007).
[12] Y. D. Hazan, J. B. Mac Chesney, T. E. Stockert, D. J. Trevor, R. S. Windeler, US Patent 6467312 B1

(2000).
[13] R. T. Bise, D. J. Trevor. OFC/NFOEC Technical Digest. Optical Fiber Communication Conference, (2005).
[14] Ebendorff - Heidepriem, Heike, Tanya M. Monro. Opt. Express **15**(23), 15086 (2007).
[15] Yajima, Tamotsu, Jun Yamamoto, Futoshi Ishii, Toshihiko Hirooka, Masato Yoshida, Masataka Nakazawa. Opt. Express, **21**(25), 30500 (2013).
[16] James W. Fleming, Appl. Opt. **23**, 4486 (1984).
[17] Levine, Dov, Paul Joseph Steinhardt. Phys. Rev. Lett. **53**(26), 2477 (1984).
[18] Soan Kim, Chul-Sik Kee, Jongmin Lee, Opt. Express **15**, 13221 (2007)
[19] I. N. M Wijeratne, N. Kejalakshmy, A. Agrawal, B. M. A. Rahman, K. T. V. Grattan, Photonics Journal IEEE, **4(**2), 357 (2012).
[20] Bhattacharjee, Ritapa, K. Senthilnathan, S. Sivabalan, P. Ramesh Babu. Optical Materials **35,** 2132 (2013).
[21] Xia, Jiangfan, Zhiyi Wei, Jie Zhang. Optics & Laser Technology **32**, 241 (2000).
[22] Bache, Morten, Hanne Nielsen, Jesper Lægsgaard, Ole Bang. Opt. Lett. Optics Letters, 1612 (2006).
[23] Li, Guangyu, Kim A. Winick, Ali A. Said, Mark Dugan, Philippe Bado. Opt. Express **17**(11), 9442 (2009).
[24] Grubsky, Victor, Arthur Savchenko Opt. Express **13**(18), 6798 (2005).
[25] A. Ozcan, M. J. F. Digonnet, G. S. Kino, F. Ay, A. Aydinli. Opt. Express **12**(20), 4698 (2004).
[26] Efimov, Anatoly, Antoinette Taylor, Fiorenzo Omenetto, Jonathan Knight, William Wadsworth, Philip Russell. Opt. Express **11**(20), 2567 (2003).
[27] Efimov, Anatoly, Antoinette Taylor, Fiorenzo Omenetto, Jonathan Knight, William Wadsworth, P. Russell. Opt. Express **11**(8), 910 (2003).
[28] A. M. Gerasimov, N. D. Kundikova, Yu. V. Miklyaev, D. G. Pikhulya, M. V. Terpugov. Nanosystems: Physics, Chemistry, Mathematics, **5**(3), 374 (2014).
[29] T. M. Monro, V. Pruneri, N. G. R. Broderick, D. Faccio, P. G. Kazansky, D.J. Richardson. IEEE Photon. Technol. Lett. **13**(9), 981 (2001).
[30] R. Eckardt, J. Reintjes. IEEE J. Quantum Electron. **20**(10), 1178 (1984).
[31] A. Bahrampour, A. Iadicicco, E. Foomezhi, S. Momeni, A. R. Bhrampour, Photonics Conference, Third Mediterranean, (2014).
[32] Nadia Mattiucci, Giuseppe D'Aguanno, Michael Scalora, Mark J. Bloemer, J. Opt. Soc. Am. B **24**, 877 (2007).
[33] C. Flueraru, C. P. Grover. IEEE Photon. Technol. Lett. **15**(5), 697 (2003).
[34] D. Faccio, A. Busacca, W. Belardi, V. Pruneri, P. G. Kazansky, T. M. Monro, D. J. Richardson, B. Grappe, M. Cooper, C. N. Pannell. Electron. Lett. **37**(2), 107 (2001).
[35] B. Dick, A. Gierulski, G. Marowsky, G. A. Reider. Appl. Phys. B **38,** 107 (1985).
[36] Sircilli, Francisco, Marcos Antonio Ruggieri Franco, Valdir Augusto Serrao. EUROCON (2007).
[37] A. V. Tarasishin, S. A. Magnitskii, A. M. Zheltikov. Laser Physics, **11**(1), 31 (2001).
[38] D. Lorenc, M. Aranyosiova, R. Buczynski, R. Stepien, I. Burgar, A. Vincze, D. Velic. Appl. Phys B **93**, 531 (2008).

*Corresponding author: rsivacoumar@vit.ac.in