# Projectile proximity position measurement and target damage correlation calculation method based on light field imaging

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To improve the target damage efficiency under projectile proximity, this paper utilizes light field imaging method to establish projectile proximity position parameters and warhead fragment spatial position testing model, studies the correlation calculation method between warhead fragment and the damaged target. The calculation principle of warhead fragment spatial position is established in the intersection of two light field cameras, combines acquired warhead fragment spatial position parameters to give the distribution characteristics of the warhead fragment field, and the dynamic dispersion velocity of the warhead fragment field, obtains the calculation model of spatial warhead fragment kinetic energy, derives the target damage efficiency evaluation function of multiple warhead fragments penetrating the target by synthesizing warhead fragments vector velocity and target damage area and warhead fragment mass. Through calculation and analysis, the results show that the target damage probability increases with the increase of penetrating intersection area between warhead fragment and target. Combine with testing experiment and compare with the traditional method, we can obtain warhead fragments spatial position by light field imaging method, give calculation results of target damage probability, verifies correlation calculation method of warhead fragments and target damage fragments and target damage probability.

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#### 1. Introduction

The Damage of projectile proximity against spatial target is an important part of measurement efficiency of proximity fuse. Projectile proximity spatial target is closely related to the performance parameters of the warhead fragment warhead and the dispersion parameters of the warhead fragments. The damage efficiency of the target is evaluated, it is necessary to master the correlation calculation method between the performance parameters of the warhead fragment field and the target damage, the method is that target damage correlation satisfies a certain precondition, the warhead fragment field formed by projectile proximity can have the damage efficiency on the target [1]. At the same time, in the process of the warhead fragment dispersion, with the influence of air resistance, the velocity of the warhead fragment decay regularly, and the damage energy of the warhead fragment change with the velocity of the target, when the warhead fragment penetrate the target, causes certain impact on the damage probability of the target. In order to objectively evaluate the target damage efficiency of the warhead fragment field under projectile proximity, it is necessary to study the target damage probability under the multi-attitude intersection of the projectile and the target [2]. For the analysis and research of the target damage, reference [3] establishes the damage evaluation model of the Bayesian warship battlefield, studies mainly the network algorithm,

obtains a large number date of equipment war damage, utilizes the Bayesian network model to quantitatively express the damage relationship between the components, this method need collect a large amount of data, so it is difficult to realize in ordinary scientific research projects. Reference [4] proposes the projection sphere method to calculate the number and distribution of the warhead fragment when the warhead fragment damages the target, however, this method needs to give the mathematical description of the target's shape, and not all warhead fragments in the shadow area damage target, so it is difficult to solve the problem of accurately evaluating damage efficiency of the target. Reference [5] analyzes the coverage area of the warhead fragment, the distribution density of the warhead fragment, and the number of the warhead fragment when the warhead fragment damages target, ignores whether the velocity vector of the target and the warhead fragment are in the same plane. Reference [6] calculates the coverage area of the warhead fragment to the target by comparing the coverage boundary coordinates of the warhead fragment on the target, and the size of the target, derives the damage probability of the target, only study the case of the cylindrical target. Reference [7] combines the requirement of damage data of new armored equipment under the condition of battlefield confrontation, according to the equivalent damage experiment of the typical damage elements, utilizes the software simulation and analysis to obtain the damage data of the equipment. These references analyze the mechanism and the calculation principle of the various target damage under different attitude, however, there is relatively little research that the correlation calculation of target damage is caused by the warhead fragment under projectile proximity. Because the projectile penetration the target is mainly dependent on the kinetic energy of the warhead fragment, the spatial distribution density of the warhead fragment, and the distance between the projectile and the target, it is an important basis for precisely striking the target by establishing the model of damage correlation between the spatial warhead fragment and the target.

This paper studies the dynamic distribution density calculation method of the warhead fragment by the characteristics of projectile proximity and the warhead fragment dispersion, analyses the motion law of the warhead fragment, and gives the motion track of a warhead fragment, obtains kinetic energy calculation function of the warhead fragment, establishes the target damage probability function when the warhead fragment penetrates the target.

### 2. Projectile proximity warhead fragment spatial position measurement by light field imaging method

In the field of shooting testing, projectile proximity warhead fragment spatial position refers to the relative position of explosion, that the fuze ignites projectile detonating device when the artillery fires target at predetermined muzzle distance and predetermined height target. The target is usually used for testing, the target is placed at a certain height above the ground, the height of the target from the ground is H. When the projectile approaches a certain range of the target, the fuze explodes by using its own control priming device to ignite the gunpowder in the projectile body, utilizes the explosion of the projectile body to produce some warhead fragments, which damage the target. This paper uses two light field cameras intersection testing method to overcome the problem that the projectile burst location is obscured by the target in the traditional testing method, the testing principle is shown in Fig. 1.



Fig. 1. The principle diagram of double light field cameras intersection

In Fig. 1, A and B are two positions that two light field cameras is arranged during the experiment, and are placed on both sides below the target projection, if O' is the target projection center, the distance between the target projection center and the center of two light field cameras are respectively,  $K_1$  and  $K_2$ , namely,  $O'O_1 = K_1$ ,  $O'O_2 = K_2$ ;  $O_1$  and  $O_2$  are the center of lens axis of two light field cameras, f is the focal length of two light field cameras,  $\theta_1$  and  $\theta_2$  are the pitch angle of two light field cameras,  $\omega_1$  and  $\omega_2$  are the angle between the target and the projectile burst location,  $\mathcal{E}_1$  and  $\mathcal{E}_2$  are the angle between the projectile burst location and the O'YZ plane,  $P_1(x_1, y_1)$  and  $P_2(x_2, y_2)$  are the pixel coordinates of imaging of the projectile burst location in the plane of the target. The area *abcd* constitute the detection area of the system, the size of the detection area is decided by the field angle of light field camera. In the experiment, according to the test requirements of the system, we can adjust the  $\theta_1$  and  $\theta_2$ , and  $K_1$  and  $K_2$ , the purpose is to meet the scope of the test system, namely, the scope of the detection area is determined.

According to the binocular vision principle and the digital refocusing principle of the light field camera, obtain projectile proximity warhead fragment spatial position, which is  $(x_{mb}, y_{mb}, z_{mb})$ , and can be expressed by (1).

$$\begin{cases} x_{mb} = \frac{\left(K_1 + K_2\right) \cdot \tan \theta_1}{\tan \theta_1 + \tan \theta_2} - f \cdot \frac{\sin \omega_2}{\sin \left(\theta_2 + \omega_2\right)} \\ y_{mb} = \left(K_1 + K_2\right) \cdot \tan \omega_2 - f \cdot \frac{\sin \omega_2 \cdot \tan \omega_1}{\sin \left(\theta_2 + \omega_2\right)} - H \quad (1) \\ z_{mb} = f \cdot \frac{\sin \omega_2}{2\sin \left(\theta_2 + \omega_2\right)} \cdot \frac{\cot \varepsilon_2 - \cot \varepsilon_1}{\cot \varepsilon_2 \cdot \cot \varepsilon_1} \end{cases}$$

In (1),  $\theta_1$  and  $\theta_2$ ,  $K_1$  and  $K_2$  can be obtained by testing,  $\omega_1$  and  $\omega_2$  is be calculated by (2).

$$\begin{cases} \omega_{1} = \arctan\left(\sqrt{(x_{1}')^{2} + (y_{1}')^{2}} / f\right) \\ \omega_{2} = \arctan\left(\sqrt{(x_{2}')^{2} + (y_{2}')^{2}} / f\right) \end{cases}$$
(2)

In (2),  $(x_1', y_1')$  and  $(x_2', y_2')$  can be obtained by the digital refocusing of the light field camera. Because spatial target is too large during projectile proximity, lead to occlusion the projectile burst location, which affects the information of the warhead fragment be obtained. The light field camera replace the area array camera, utilizes the digital refocusing technology of light field camera, refocus on the projectile burst location or the spatial warhead fragment position, obtain more accurate pixel

location coordinate of the projectile burst location or the spatial warhead fragment position. The digital refocusing principle of light field camera is shown in Fig. 2, if  $\alpha$  is the position parameter by changing microlens and focus plane, f is the focal length of the light field camera, make the light field camera refocus by changing  $\alpha$  and f, form the pixel coordinates of the projectile burst location or the spatial warhead fragment position by refocusing, the pixel coordinates are  $(x_1', y_1')$  and  $(x_2', y_2')$ , can be shown in (3) and (4).



Fig. 2. Digital refocusing principle of the light field camera

$$\begin{cases} x_1' = \frac{x_1}{\alpha_i} + u_i \left( 1 - \frac{1}{\alpha_i} \right) \\ y_1' = \frac{y_1}{\alpha_i} + v_i \left( 1 - \frac{1}{\alpha_i} \right) \\ x_2' = \frac{x_2}{\alpha_i} + u_i \left( 1 - \frac{1}{\alpha_i} \right) \\ y_2' = \frac{y_2}{\alpha_i} + v_i \left( 1 - \frac{1}{\alpha_i} \right) \end{cases}$$
(3)

In (3),  $(u_i, v_i)$  is the position which light enters array

microlens,  $\alpha_i$  is the corresponding position parameter according to different focusing degrees. The formulas (2) and (3) are brought into formula (1), the three-dimensional coordinates of the projectile burst location or the spatial warhead fragment position can be obtained.

# 3. Correlation calculation method of target damage

### **3.1.** The distribution characteristics of the warhead fragment field under projectile proximity

The shell forms a large number of warhead fragments after projectile explodes, the warhead fragment spreads in the space under the action of projectile initial velocity, form the spatial warhead fragment field, which is similar to the conical shape. The uncertainty of the warhead fragments dispersion angle can result in multiple attitude intersection between the warhead fragment field and the target [8-9]. In order to calculate target damage efficiency under multi-attitude intersection confrontation, the spatial distribution characteristics of the warhead fragment field should be calculated first, the spatial warhead fragment field is formed in the instant of projectile explosion, as shown as Fig. 3.



Fig. 3. Warhead fragment dispersion diagram in the moment of projectile explosion

In Fig. 3, if  $O - X_{mv}Y_{mv}Z_{mv}$  is the coordinate system of the projectile,  $(x_{mb}, y_{mb}, z_{mb})$  is the projectile burst location,  $\phi_1$  is the minimum dispersion angle of the warhead fragment,  $\phi_2$  is the maximum dispersion angle of the warhead fragment,  $\phi$  is the average dispersion angle of the warhead fragment,  $\phi = (\phi_1 + \phi_2)/2$ , *R* is static damage radius of the warhead fragment field,  $A_s$  is static distribution area of effective warhead fragments. This paper establishes the coordinate system of the projectile by the projectile burst location as the origin, the spatial warhead fragments are uniformly distributed in the conical ring surrounded by  $\phi_1$  and  $\phi_2$ , so  $R = \sqrt{x_{mb}^2 + y_{mb}^2 + z_{mb}^2}$ , *N* is the number of effective warhead fragments in the conical ring surrounded by  $\phi_1$  and  $\phi_2$ ,  $A_s$  in static explosion experiment of projectile is described in (5).

$$A_{s} = 2\pi R^{2} \left(\cos\varphi_{1} - \cos\varphi_{2}\right) \tag{5}$$

If  $\phi_r$  is static dispersion angle of the r-*th* warhead fragment, the distribution density of the warhead fragment in the instant of projectile explosion is as follows:

$$\rho(\varphi_r) = \frac{N}{2\pi R^2 (\cos\varphi_1 - \cos\varphi_2)} \tag{6}$$

When the projectile explodes, the final velocity of the projectile will affect the warhead fragment field, so it is necessary to superpose the velocity vector of the projectile's end point when the testing system calculate the motion parameters of the warhead fragment field. Under the dynamic condition, we denote  $V_{m0}$  as the final velocity of the projectile explosion,  $V_{f0}$  as the theoretical initial velocity of the warhead fragment,  $V_{m0}$  affects the spatial distribution of the warhead fragment and the velocity of the warhead fragment. In the process of the warhead fragment changes from  $V_{f0}$  to  $V_{f1}$ , then the dynamic dispersion velocity of the warhead fragment field can be obtained as shown by formula (7).

$$v_{f_1} = \sqrt{v_{m0}^2 + v_{f0}^2 + 2v_{m0}v_{f0}\cos\phi_r}$$
(7)

The dynamic dispersion angle of the warhead fragment changes from  $\phi_r$  to  $\phi_{vr}$ , the warhead fragment density distribution is still similar to the normal distribution [10], and then the calculation method of dynamic dispersion angle of the warhead fragment is shown in formula (8).

$$v_{f_1} = \sqrt{v_{m0}^2 + v_{f0}^2 + 2v_{m0}v_{f0}\cos\phi_r} \qquad (8)$$

Therefore, based on the above analysis, we can calculate spatial distribution density of dynamic warhead fragment under projectile proximity, it is shown in (9).

$$\rho(\varphi_{vr}) = \frac{N}{2\pi R^2 \cos \varphi_{vr}} \tag{9}$$

The warhead fragment spreads out uniformly with different dispersion angle across different spherical bands of the warhead fragment field, cause spatial distribution density of the warhead fragment change, also make the number of warhead fragments different when the warhead fragment penetrates target, cause the damage probability of the target different. At the same time, because of the influence of air resistance, the velocity of the warhead fragment change gradually during the process of the warhead fragment dispersion, resulting in the warhead fragment deviates from the original track and unable to damage the target or fall in the other position of the target, the result is that target is not precisely strike. Therefore, we analyze the effectiveness of target damage, should analyze the dynamic dispersion velocity of the warhead fragment field.

# **3.2.** Dynamic dispersion velocity of the warhead fragment field

The warhead fragment field is composed of the warhead fragments around projectile explosion position in the instant of projectile explosion. In the warhead fragment field, assuming that the size, the shape, the quality and the material of a warhead fragment are same,  $m_0$  is the mass of a warhead fragment, these warhead fragments accord to normal distribution. In order to analyze the damage efficiency of the warhead fragment motion parameter on the target, determine the position whether has slipped or detached that projectile warhead fragment collides with target surface. This paper utilizes the finite element method to divide the warhead fragment field into *n* warhead fragments micro-element beam. Assuming that a warhead fragment micro-element beam is infinitely closer to the motion track of a warhead fragment, namely, we denote the projectile burst location as starting point, the dispersion angle of the warhead fragment as the direction trajectory, the distribution of the warhead fragment micro-element beam is shown in Fig. 4. Among,  $\phi_{\rm ur}$  is the *r*-th warhead fragment dispersion angle,  $\gamma$  is the deflection angle of the warhead fragment,  $r = 1, \dots, i$ ,  $(x_{mr}, y_{mr}, z_{mr})$  is the divergent coordinates of the *r*-th warhead fragment, which is expressed by (10).



Fig. 4. Warhead fragment micro-element beam schematic

$$\begin{cases} x_{mr} = x_{mb} + R\xi_1 \tan(2\pi\xi_2) \\ y_{mr} = y_{mb} + R\xi_1 \tan(2\pi\xi_2) \\ z_{mr} = z_{mb} + R\xi_1 \tan(2\pi\xi_2) \end{cases}$$
(10)

In (10),  $(\xi_1, \xi_2)$  is a group of discrete random numbers, accord to the uniform distribution of the warhead fragment. We ignore the influence of acceleration of the gravity, according to Fig. 4, calculate the linear motion equation of warhead fragment beam along three coordinate axes in the coordinate system of the projectile, as shown in formula (11).

$$\begin{cases} L_{mx} = L \cdot \cos \phi_{vr} \\ L_{mx} = L \cdot \cos \phi_{vr} \cdot \cos \gamma \\ L_{mx} = L \cdot \sin \phi_{vr} \cdot \sin \gamma \end{cases}$$
(11)

In (11), L is the distance of warhead fragment flight, which is expressed by (12).

$$L = \sqrt{\left(x_{mr} - x_{mb}\right)^2 + \left(y_{mr} - y_{mb}\right)^2 + \left(z_{mr} - z_{mb}\right)^2}$$
(12)

After the projectile explodes, the warhead fragments fly out in a certain direction. Because of the influence of air resistance, the velocity of the warhead fragment continuously attenuates, assuming that  $V_{f0}$  is the initial velocity of the warhead fragment, A is average upwind area of warhead fragment perpendicular to the flight direction at some time,  $\rho_0$  is the density of air, C is the drag coefficient of air, D is the diameter of the warhead fragment, the attenuation velocity of the warhead fragment after flying distance of L is shown in formula (13).

$$v_f(L) = v_{f0} e^{C\rho_0 A D/2m_0}$$
 (13)

If  $(x_{mt}, y_{mt}, z_{mt})$  is the spatial position coordinate of the target in the coordinate system of the projectile, when the spatial dynamic dispersion coordinate of the warhead fragment is equal to the spatial position coordinate of the target, that is,  $(x_{mr}, y_{mr}, z_{mr}) = (x_{mt}, y_{mt}, z_{mt})$ , the warhead fragment hit the target, the velocity of the warhead fragment hit the target is shown in (14).

$$v_{ft} = v_f(L)e^{\frac{C\rho_0 m_0 AD}{2}L'}$$
(14)

In (14),  $\phi$  is average dispersion angle of the warhead fragment,  $t_1$  is the time when the first warhead fragment micro-beam hits at the target,  $t_i$  is the *i*-th warhead fragment micro-beam hits the target, L' is the flight distance between the projectile burst location and the target, at this time,

$$L' = \sqrt{\left(x_{mt} - x_{mb}\right)^{2} + \left(y_{mt} - y_{mb}\right)^{2} + \left(z_{mt} - z_{mb}\right)^{2}}$$

 $t_1$  and  $t_i$  are shown in (15) and (16).

$$t_{1} = \frac{L'\sin(180^{\circ} - \varphi)}{\sin(90^{\circ} - \varphi_{vr})\left[v_{m}^{2} + v_{f0}^{2} - 2v_{m0}v_{f0}\cos(180^{\circ} - \varphi)\right]}$$
(15)
$$t_{i} = \frac{L'\sin(180^{\circ} - \varphi)}{\sin(90^{\circ} - \varphi_{vr})\left[v_{m}^{2} + v_{ft}^{2} - 2v_{m0}v_{ft}\cos(180^{\circ} - \varphi)\right]}$$

We utilize the distance and velocity between the warhead fragment and the target to calculate the time when a warhead fragment hits the target, the damage caused by the projectile to the target is the damage area caused to the target by accumulated the number of the warhead fragments during the period. The dynamic velocity of the warhead fragment field is an important parameter when we consider the damage efficiency caused by the warhead fragment field to the target under projectile proximity. At the same time, with the change of the velocity of the warhead fragment field, the kinetic energy of the warhead fragment on the target is different, the penetration thickness of the warhead fragment on the target change with change of the kinetic energy of the warhead fragment, the deeper the penetration thickness of the target, the greater the target damage probability, the target is easily damaged.

### 3.3. Target damage calculation of kinetic energy based on spatial warhead fragment field under projectile proximity

When the projectile explodes, it forms the warhead fragment field with destructive performance in space, the warhead fragments are evenly distributed. Different the spatial position of the warhead fragment, the distribution density of the warhead fragment, the mass of the warhead fragment, velocity and the falling angle of the warhead fragment, causes the target damage with different degrees [11]. This paper simplifies the target to a cylindrical shape, according to the principle of finite element analysis [12-13], the effective damage area of the target is divided into m square micro-element surface, the finite element model of the target damage under the intersection state of the projectile and the target is shown in Fig. 5.



Fig. 5. The finite plane model of the target damage under the intersection state of the warhead fragment and the target

In Fig. 5, the target is divided into *n* areas,  $C_1, \dots, C_n$ , respectively. When the surface of microelement gradually approaches the cross section of the warhead fragment beam, the damage area caused by the warhead fragment to the target is the sum of the damage area caused by a warhead fragment hit the target. Assuming that

(16)

 $(x_{rt}, y_{rt}, z_{rt})$  is the intersection position,  $S_i$  is the damage area caused by the warhead fragment field to the target, reference [14] utilizes shooting line method to calculate the intersection position between the warhead fragment and the target damage surface element, therefore, we utilize the finite element differential plane method [15] to calculate  $S_i$ , which is shown in (17).

$$S_i = \lim_{\Delta S \to \pi r^2} \sum_{0}^{N_s} F(x_{ri}, y_{ri}, z_{ri}) \Delta S$$
(17)

In (17),  $N_s$  is the number of effective warhead fragments that hit the target,  $N_s = N \cdot \rho(\phi_{vr})$ , according to the spatial dynamic distribution density of the warhead fragment, the mass of effective warhead fragment can be calculated by (18).

$$M = \int_{0}^{S_i} \rho(\phi_{vr}) dS = \int_{0}^{S_i} \frac{N}{4\pi R^2 \cdot \cos \phi_{vr}} dS \quad (18)$$

If  $E_r$  is the kinetic energy generated by a single warhead fragment, and  $E_r = 1/2 \cdot Mv^2$ , the initial equivalent energy of effective warhead fragment acting on a single square micro-element surface is formula (19):

$$E_{r} = 1/2 \cdot M v_{ft}^{2} = \int_{0}^{S_{t}} \frac{N v_{f0}^{2}}{4\pi R^{2} \cdot \cos \phi_{vr}} dS \qquad (19)$$

For m square micro-element surfaces, the initial kinetic energy of effective warhead fragment acting on the target surface is shown in formula (20).

$$E_{mr} = m \cdot 1/2 \cdot M v_{ft}^{2} = \int_{0}^{S_{t}} \frac{m \cdot N v_{f0}^{2}}{4\pi R^{2} \cdot \cos \phi_{vr}} dS \quad (20)$$

Therefore, during the process of projectile penetrated into the target, the kinetic energy of the target can be calculated by integrating the effective warhead fragment initial kinetic energy of the target plane in the time interval of  $t_1$  and  $t_i$ , the specific calculation formula is as follows:

$$E_{T} = \int_{t_{1}}^{t_{2}} E_{mr} dt = \int_{t_{1}}^{t_{2}} \int_{0}^{S_{i}} \frac{m \cdot N v_{f0}^{2}}{4\pi R^{2} \cdot \sin \phi_{vr}} dS \qquad (21)$$

Formula (21) gives the calculation method of kinetic energy of the target damage caused by the warhead fragment field, among, the damage area is related to the spatial position coordinates of the warhead fragment, the spatial position of the target and the damage radius of the warhead fragment field. The impact time is related to the dispersion velocity of the warhead fragment field, and the distance between the projectile and the target. Therefore, it is necessary to know the dynamic parameters of the warhead fragment distribution field for the solution of warhead fragment kinetic energy. However, for the target, the damage kinetic energy of the warhead fragment field need to reach a certain penetration condition, which can cause damage to the target.

#### 4. Calculation and analysis

# 4.1. Evaluation and Analysis of target damage efficiency by warhead fragment field

If a warhead fragment hits the target, in order to pierced the target by the warhead fragment to cause the target damage, the kinetic energy of the warhead fragment hitting the target should not be less than the power done by average dynamic deformation of the material of the target, which is described by (22).

$$E_{r} = 1/2 \cdot Mv^{2} \ge E_{0} = k_{1}b_{A1}\sigma_{A1}S_{i}$$
(22)

In (22),  $b_{Al}$  is thickness of target equivalent hard aluminum target,  $\sigma_{Al}$  is ultimate strength of hard aluminum,  $k_1$  is a coefficient,  $E_b$  is the volume specific kinetic energy of a warhead fragment hitting hard aluminum target, and  $E_b = E_r/b_{Al}S_i$ . The target specific kinetic energy of a warhead fragment is  $E_b = Mv_{f0}^2/2b_{Al}S_i$ , a warhead fragment hits the target plate that must be satisfied formula (23).

$$E_b \Longrightarrow k_1 \sigma_{A1} \tag{23}$$

Thus, under the condition of  $E_b \ge k_1 \sigma_{Al}$ , this paper analyzes target's damage efficiency by damage area of the vulnerable area of the target, assuming that  $S_2$  is surface area of the vulnerable area of the target,  $L_2$  is length of the vulnerable area of the target, d is the radius of the target,  $S_2 = \pi dL_2$ . The target is destroyed when  $S_i = S_2$ , the target is damaged in a certain extent for  $S_i < S_2$ . The target damage sensitivity is characterized by the ratio of damage area of the target, which is written by (24).

$$P_{2}' = \frac{S_{i}}{S_{2}} = \frac{\lim_{\Delta S \to \pi r^{2}} \sum_{0}^{N_{s}} F(x_{n}, y_{n}, z_{n}) \Delta S}{\pi dL_{2}}$$
(24)

In order to accurately analyze the damage probability of the target, we divide the damaged target into four areas, these areas are  $C_1$ ,  $C_2$ ,  $C_3$  and  $C_4$  respectively, the spatial warhead fragment hitting the surface of the target as shown in Fig. 6.



Fig. 6. Spatial warhead fragment hitting the surface of the target

In Fig. 6, the warhead fragments are distributed in different areas of the target, then cause a certain damage of the target. We denote  $C_2$  as vulnerable area of the whole target, so damage caused by the warhead fragment to vulnerable area is more easily to cause damage of the whole target, the damage probability of the whole target is as follows:

$$P = k_{c1}P_1' + k_{c2}P_2' + k_{c3}P_3' + k_{c4}P_4'$$
(25)

In (25),  $k_{c1}$ ,  $k_{c2}$ ,  $k_{c3}$ ,  $k_{c4}$  is weight of the damage probability of the whole target, which synthetically considers the ratio of area and the relation of vulnerable area. Where,  $k_{c1} + k_{c2} + k_{c3} + k_{c4} = 1$ ,  $P_1'$ ,  $P_2'$ ,  $P_3'$  and  $P_4'$  are target damage sensitivity of corresponding target area.

When the dispersion angle of warhead fragment is about  $30^{\circ}$  and  $60^{\circ}$ , according to the formula (25), the influence of the velocity of the warhead fragment on the damage probability of the target can be calculated, the simulation results is shown in Fig. 7.



### Fig. 7. Damage probability of the target for the warhead fragment divergence angle of 30° and 60°

It can be seen from Fig. 7 that the larger the flight velocity of the warhead fragment, the target is easier damaged. Because the initial velocity of the warhead fragment is higher, the kinetic energy of target damage is greater, the damage probability is higher that the warhead fragment breaks through rectangular surface elements of vulnerable section, causes the target damage serious. It can also be seen that the small the dispersion angle of the warhead fragment is easier to hit the target, the velocity of the warhead fragment is lower for achieved same damage probability, the damage probability is between 0.1 and 0.9, the initial velocity of the warhead fragment is 660m/s to 1100m/s for the divergence angle of  $30^{\circ}$ , the initial velocity of the warhead fragment is 680m/s to 1150m/s for the dispersion angle of  $60^{\circ}$ .

After the projectile explodes, the dispersion angle of warhead fragment dynamically changes, which leads to the damage radius of the warhead fragment field formed after warhead fragment dispersion, simultaneously, the flight velocity of the warhead fragment itself also changes. Therefore, the warhead fragment distribution density is affected by the dynamic dispersion angle of the warhead fragment, the flight velocity of the warhead fragment, and the damage radius of the warhead fragment field, which leads to the different warhead fragment distribution density in each period of the warhead fragment field, as the result, the warhead fragment distribution density is dynamic.

According to formulas (9) to (17), we can find that the distribution density of the warhead fragment is inversely proportional to the damage radius of the warhead fragment field, when the dispersion angle of the warhead fragment is relatively constant, if the projectile burst location is the farther, the damage radius of warhead fragment field is larger; The damage radius of warhead fragment field is the larger, the distribution density of the warhead fragment is the smaller; Therefore, the damage radius of the warhead fragment field is affected by the dispersion angle of the warhead fragment and the projectile burst location. For the design of the dispersion angle of the warhead fragment, if the dispersion angle of the warhead fragment is larger, considering the intersection attitude of projectile and target, the worst damage efficiency is that many warhead fragments can't penetrate the target in the warhead fragment field; if the dispersion angle of the warhead fragment is small, unless the projectile intersects in the head or the afterbody of the target, almost all warhead fragments can't penetrate the target in other intersection attitude, resulting in the small target damage probability and the poor target damage efficiency. To obtain the best target damage effect, according to the laws of the actual projectile burst location, the size and the mass and the flight velocity of the warhead fragment, the size of the target, and the vulnerability of the target in different regions, the dispersion angle of the warhead fragment is designed.

# 4.2. Dynamic process simulation analysis of warhead fragment penetrating target

In order to analyze the damage efficiency of the warhead fragment to the target in the warhead fragment field, the simulation calculation is carried out from the velocity and kinetic energy of the warhead fragment in the warhead fragment field, the velocity change curve of a warhead fragment is obtained, as shown in Fig. 8; the kinetic energy change curve of a warhead fragment is given, as shown in Fig. 9. "A" in the figures represents a warhead fragment in the warhead fragment in the warhead fragment field.

Fig. 8 gives the velocity curve of a warhead fragment in the warhead fragment field, Fig. 9 shows the curve of kinetic energy of a warhead fragment in the warhead fragment field.



Fig. 8. The velocity curve of a warhead fragment in the warhead fragment field



Fig. 9. The curve of kinetic energy of a warhead fragment in the warhead fragment field

From Fig. 8, we can find that the velocity of the warhead fragment decreases gradually during the whole penetration process, because the mass attenuation of the warhead fragment is ignored during penetration, and the mass of the warhead fragment is very small. We ignore the influence of gravity, the energy of the warhead fragment is mainly converted into kinetic energy of penetration target, which is attenuated in the penetration process because of the influence of the target resistance. Fig. 9 verifies the results of Fig. 8. At the same time, as the velocity of the

warhead fragment increases, the kinetic energy of the warhead fragment increases gradually, the greater the internal capacity of the surface of the target resulting in the more serious damage to the plane of the target.

#### 5. Experiment analysis

In order to verify feasibility of testing method and correlation between warhead fragment spatial position and target damage, the testing method of two light field cameras intersection in this paper is used to measure warhead fragment spatial position in experiment, the two light field cameras are arranged on both sides of trajectory, and the target is located at the intersection point of optical axis of light field camera. We denote  $\theta_1$  as the pitch angle of the first light field camera,  $\theta_1 = 44.5^\circ$ .  $\theta_2$  as the pitch angle of the second light field camera,  $\theta_2 = 45.7^\circ$ , the focus section of two light field cameras is 9.5 to 77.8 mm, the resolution is  $480 \times 800$ , the distance between the two light field cameras is 4m, the length of damage target is 1.8 m, the diameter of damage target is 0.4m, the damage target is suspended in the air, the damage target is divided into four regions, the area of C1 accounts for 15% of total surface area of the target, the area of  $C_2$  accounts for 40% of total surface area of the target, the area of C3 accounts for 35% of total surface area of the target, the area of C<sub>4</sub> accounts for 10% of total surface area of the target. In an experiment, the projectile launching direction is perpendicular to the side of the target, the spatial position of the warhead fragment relative to the target are captured by two light field cameras, the measurement data is shown in Table 1, the result of testing is that the target is destroyed, the unit of data is in meters.

Table 1. Measurement data in an experiment

	Warhead fragment		Warhead fragment
No.	position	No.	position
	(x,y,z)		(x,y,z)
1	( 0.014, -0.052,0.172 )	16	(0.722, 0.054, -0.076)
2	(0.091, 0.136, 0.178)	17	(0.619, 0.098, -0.104)
3	(0.162, -0.092,-0.136)	18	( 0.947, -0.161,0.187 )
4	(0.113, 0.122, -0.031)	19	( 0.939, 0.149,-0.153 )
5	( 0.493, -0.150,0.065 )	20	(0.859, 0.164, 0.157)
6	(0.510, -0.189,0.143)	21	(0.351,-0.039,0.176)
7	(0.513, 0.075, -0.083)	22	(0.781,0.095,0.075)
8	( 0.476, 0.046, 0.160 )	23	( 0.488, -0.084,-0.119 )
9	(0.517, 0.155, -0.073)	24	(0.827, -0.185, -0.189)
10	( 0.357, 0.084, 0.174 )	25	(0.634, 0.180, -0.128)
11	(0.561, -0.092, -0.078)	26	(0.329, -0.127,0.021)
12	(0.507, 0.152, -0.119)	27	(0.835, 0.034, -0.059)
13	(0.707, -0.065, 0.037)	28	(1.025, -0.189,0.234)
14	( 0.947, -0.035,-0.029 )	29	(1.262, 0.161, 0.687)
15	( 0.880, 0.077, 0.046 )		

We analyze the damage probability of the target based on position of the warhead fragment field from this experiment. Spatial position of a warhead fragment according to Table 1 can be calculated that the number of the warhead fragments in the  $C_1$  area is 4, the number of the warhead fragments in the  $C_2$  area is 23, the number of the warhead fragments in the C<sub>3</sub> area is 2, the number of the warhead fragments in the C4 area is 0, if each warhead fragment has the same volume, so the damage area of in the C<sub>1</sub> area is about 0.16m<sup>2</sup>, the damage area of in the C<sub>2</sub> area is  $0.92m^2$ , the damage area of in the C<sub>3</sub> area is  $0.08m^2$ , the damage area of in the  $C_4$  area is 0. The results show that the damage area of in the C<sub>2</sub> area is a little bigger than the vulnerable area of the target, according to theoretical calculation, the target is destroyed, and namely, the damage probability of the whole target is 100%. So the theoretical conclusion is consistent with the testing result.

In order to further verify the feasibility of target damage model, we carry out another experiment and analysis, the projectile launching direction is still perpendicular to the side of the target. The measurement data of warhead fragments position is shown in Table 2, the result of testing is that target is not completely destroyed.

Table 2. Measurement data in another experiment

No.	Warhead fragment position (x,y,z)	No.	Warhead fragment position (x,y,z)
1	(0.102, -0.143,0.156)	16	(0.965, 0.067, 0.128)
2	( 0.016, -0.010,0.068 )	17	(0.857, -0.124, -0.188)
3	( 0.091, -0.120,-0.107 )	18	(0.831, 0.129, 0.118)
4	(0.123, 0.169, -0.044)	19	( 0.923, 0.015, 0.160 )
5	(0.131, 0.140, -0.106)	20	(0.793, 0.132, -0.155)
6	( 0.036, 0.132, 0.124 )	21	(0.527,-0.021,-0.133)
7	( 0.350, -0.025,-0.079 )	22	(1.309, 0.079, -0.034)
8	( 0.546, -0.140,0.113 )	23	(1.389, -0.166,0.040)
9	(0.600, 0.144, 0.157)	24	(1.404, -0.125,0.066)
10	(0.571,-0.121,-0.136)	25	(1.272, 0.097, 0.080)
11	(0.522, -0.159,0.010)	26	(1.596, -0.170,-0.128)
12	(0.471,0.050,0.152)	27	(1.712, 0.103, 0.025)
13	( 0.703, 0.026, 0.104 )	28	(1.644, -0.058,-0.132)
14	(0.822, 0.119, -0.071)	29	(1.703, 0.108, -0.089)
15	(0.697,-0.034,0.131)	30	(1.691, -0.125,0.096)

The analysis principle is the same as Table 1, we assume that each warhead fragment has the same volume, according to Table 2, we can figure out that the damage area of in the  $C_1$  area is  $0.24m^2$ , the damage area of in the  $C_2$  area is  $0.6m^2$ , the damage area of in the  $C_3$  area is  $0.2m^2$ , the damage area of in the  $C_4$  area is  $0.16m^2$ . The results indicate that the damage area of each area is smaller than the area itself, combining with formula (25), we can determine that the damage probability of the whole target is 63.8%. The experiment combines with the spatial position of the warhead fragment, the dispersion rule of the target, calculates quantitatively the damage probability of the target is consistent with the target damage results in this

experiment. Therefore, this paper establishes the damage probability of static target, which can indicate the correctness of correlation calculation function between the warhead fragment and the damage efficiency of the target under projectile proximity. When the destroyed target is in flight state, causes the uncertainty of intersection attitude between the target and the projectile, the damage probability model of the target needs to be verified and corrected through a large number of testing in the battlefield.

#### 6. Conclusions

According to the distribution characteristics of formed the warhead fragment field in the space after projectile proximity, this paper establishes calculation model of dynamic spatial distribution density of the warhead fragment field, discusses the influence trend of different warhead fragment dispersion angle and the warhead fragment dispersion radius on the spatial distribution density, studies the motion trajectory of a warhead fragment, utilizes to finite element method to obtain the velocity, the time and the distance of warhead fragments, and gives the attenuation law of velocity of the warhead fragment after projectile proximity, the results show that velocity of the warhead fragment affect target damage. At the same time, give the kinetic energy calculation method of the warhead fragment field for penetrating target according to flight rule of the warhead fragment. Through calculation and analysis, gives the influence trend of dynamic parameters and damage coefficient on the damage probability of the target, reflects the damage precondition that the target is destroyed by formed the warhead fragment of projectile proximity. When the kinetic energy of the warhead fragment is larger than the carrying kinetic energy of material on the surface of the target, the warhead fragment can break through the surface of the target and calculate the damage probability of the target. This paper carried out experiments to analyze the damage efficiency of projectile burst location, the warhead fragments distribution and the target damage vulnerability distribution on target, verifies the feasibility of the damage probability model of the target in this paper. In the future battlefield, the uncertainty of projectile burst location, the diversity of intersection between the target and the projectile, these factors led to target damage result that need to be determined by a large number of testing. Therefore, the study of the damage probability of static target in this paper can provide the basis for research of dynamic target damage, the research of this paper has very high research significance and scientific value.

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