OPTOELECTRONICS AND ADVANCED MATERIALS – RAPID COMMUNICATIONS Vol. 11, No. 7-8, July-August 2017, p. 409 - 414

PEMC backed chiral nihility Gregorian system

AHSAN ILLAHI[a], ABDUL GHAFFAR[b,*], MAJEED A. S. ALKANHAL[c], YASIN KHAN[d], ABDUL AZIZ[e], Q. A. NAQVI[e], MUHAMMAD SHARIF[f]
[a]*Research in Modeling and Simulation (RIMS) Group, Department of Physics, COMSATS Institute of Information Technology, Islamabad, Pakistan*
[b]*Department of Physics, University of Agriculture, Faisalabad, Pakistan*
[c]*Department of Electrical Engineering, King Saud University, Saudi Arabia*
[d]*Department of Physics, Allama Iqbal Open University, Islamabad, Pakistan*
[e]*Department of Electronics, Quaid-i-Azam University, Islamabad, Pakistan*
[f]*Department of Physics, Government College, University, Faisalabad, Pakistan*

Field expressions valid near the caustic of a two dimensional Gregorian system of perfect electromagnetic conductor backed by chiral nihility material are derived using Maslov's method. Numerical computations have been made to work out the field patterns around the caustic of a Gregorian system. Reliance of cross-polarized and co polarized field components for various values of admittance factor of perfect electromagnetic conductor have been studied and interesting results have been highlighted

(Received October 5, 2016; accepted August 9, 2017)

Keywords: PEMC, Chiral Nihility, Maslov's method, Gregorian system

1. Introduction

Electromagnetic waves found in the focal region of focusing system have many applications in areas, such as microwave antennas and integrated optical systems etc. Geometrical optics approximation is a usual technique for obtaining high frequency fields in homogeneous and inhomogeneous medium but it does not give results in the neighborhood of caustic area. So an alternative method is needed to overcome this discrepancy which is Maslov method. Maslov's method predicts field in the focal region [1, 2]. This method combines the simplicity of GO and generality of the Fourier transform method in his procedure. This technique made use of the fact that singularity in different domains does not coincide. Hence by representing the GO fields in terms of mixed coordinates space coordinates and wave vector coordinates can provide the remedy to find the solution at caustic. Maslov's method has been used for variety of reflector by many authors [2-20].

Chiral nihility is a material obtained from chiral material with the real part of permittivity and permeability simultaneously having zero values, in other words refractive index becomes zero at certain frequency known as chiral nihility frequency [21-27].

PEMC is combination of Perfect electric conductor (PEC) and perfect magnetic conductor (PMC) in generalized form. At PEMC interface certain linear combination of electromagnetic fields are required to vanish and has been discussed in [28-35] . This medium is also termed as the axion in the literature. This generalized media does not allow any type EM energy to enter, so it can act as boundary material. Non-reciprocity of the PEMC boundary can be demonstrated by showing that the polarization of plane wave reflected from its surface is rotated, the sense and angle of rotation depending on the admittance parameter.

In present work, we have derived high frequency field expression which is valid in the caustic region of the PEMC backed chiral nihility Gregorian system (a kind of a dual reflector antenna) by using Maslov's method. Effect of the PEMC parameter on the field amplitude in directions along the coordinate axis is noted. Interpretation of the results is given taking into account the co and cross polarization of field patterns.

2. Two dimensional Gregorian reflector

The Gregorian reflector system comprises of parabolic as main reflector (chiral nihility coated material backed by PEMC material, in present discussion) and elliptical (PEC) as sub reflector shown in Fig. 1. Gregorian system has many benefits over a parabolic reflector. The Gregorian system can be replaced with a single equivalent parabolic reflector with the same aperture as that of main reflector but enhanced focal length. The focal length of equivalent parabola is given by the following relation [36]

$$f_e = -\left(\frac{c+a}{c-a}\right)f \tag{1}$$

where $(c+a)/(c-a)$ is known as the image field magnification. Thus in the presence of secondary reflector the focal length of the main reflector is increased while providing a convenient location of feed near the vertex of the parabola [37].

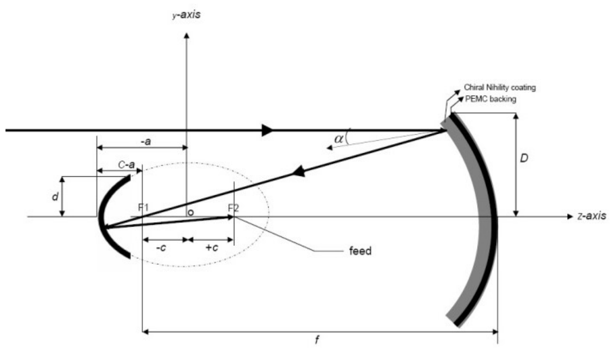

Fig. 1. Parameters in Gregorian system

In addition to the ability to place the feed at a convenient location, additional benefits of Gregorian system include, to reduce spill-over and side-lobe radiation. The equation of each reflector is given by

$$\varsigma_1 = f - \frac{\xi_1^{\,2}}{4f} - c \qquad \text{Parabola} \qquad (2)$$

$$\varsigma_2 = a\left[\frac{1-\xi_2^{\,2}}{b^2}\right]^{1/2} \qquad \text{Ellipse} \qquad (3)$$

where $c^2 = a^2 - b^2$, $aR_2 = a^2 + c\varsigma_2$, $aR_1 = a^2 - c\varsigma_2$ where (ξ_1, ς_1) and (ξ_2, ς_2) are the coordinates of the parabolic and elliptic reflectors, respectively. R_1 and R_2 are the distances from the point (ξ_2, ς_2) to the focal points $z = -c$ and $z = c$ respectively. Incident wave is given by $\mathbf{E}^{\mathrm{i}} = \mathbf{u}_{\mathrm{x}} \exp(ik_0 z)$. Thus the GO expression of the reflected wave is

$$\mathbf{E}^{\mathrm{r}} = \mathbf{E}^{\mathrm{r}}{}_0 J^{-1/2} \exp[-jk(S_0 + t_1 + t)] \qquad (4)$$

where $J = 1 - \dfrac{t}{R_2}$. It is freely observed that the GO field of the reflected wave becomes invalid at the point F_2 as is expected [5]. Using Maslov's method electromagnetic reflected field in the focal point is obtained as

$$\mathbf{E}^{\mathrm{r}} = -\sqrt{\frac{k}{j2\pi}} \int_{-\infty}^{\infty} \mathbf{E}^{\mathrm{r}}{}_0 \left[\frac{D(t)}{D(0)} \frac{\partial p_{z2}}{\partial z}\right]^{-1/2}$$
$$\times \exp[-jk(S_0 + t_1 + t - z(y, p_{z2}) p_z + z p_{z2})] dp_{z2} \qquad (5)$$

The value of $\left[J(t)\dfrac{\partial p_{z2}}{\partial z}\right]^{-1/2}$ is given by [5]

$$\left[J(t)\frac{\partial p_{z2}}{\partial z}\right]^{-1/2} = \frac{\sqrt{R_1}}{\sin(2\psi - 2\alpha)}$$

The phase function $S(p_z)$ is given by [5]

$$S(p_z) = S_0 + t_1 + S_{ex}$$

where $S_0 = -\varsigma_1 = f\left[\dfrac{2\cos 2\alpha}{1+\cos 2\alpha}\right] - c$

$$t = \sqrt{(y-\xi_2)^2 + (z-\varsigma_2)^2}$$

$$t_1 = \sqrt{(\xi_1-\xi_2)^2 + (\varsigma_1-\varsigma_2)^2}$$

$$S_{ex} = -\rho\cos(2\psi - 2\alpha + \phi) + \xi_2 \sin(2\psi - 2\alpha)$$
$$+\varsigma_2 \cos(2\psi - 2\alpha)$$

$$\xi_2 = b\sqrt{\frac{1-\cos^2\psi}{\cos^2(\psi - 2\alpha)}}$$

$$\varsigma_2 = \frac{a\cos\psi}{\cos(\psi - 2\alpha)}$$

$$\xi_1 = -2f\tan\alpha$$

$$\varsigma_1 = \frac{f\cos 2\alpha}{\cos^2\alpha} - c$$

$$\tan\psi = -\frac{a^2\xi_2}{b^2\varsigma_2}$$

$$\tan\phi = \frac{y}{x}$$

Field expression may be written as

$$\mathbf{E}^{\mathrm{r}} = -\sqrt{\frac{k}{j2\pi}}\left[\int_{A_1}^{A_2} + \int_{-A_2}^{-A_1}\right]\Big\{A_{co}\; x\exp(-jk_{0z}z)$$
$$+A_{cr}\left(\frac{jk_{0z}}{k_0}y + \frac{jk_y}{k_0}z\right)\sqrt{R_1}$$
$$\times \exp[-jk(S_0 + t_1 + S_{ex})]d(2\alpha) \qquad (6)$$

where A_{co} and A_{cr} are values of the co-polarized reflected fields given as [27]

$$A_{co} = -\exp(-2jk_{0z}d_1) + \left(\frac{2j}{M\eta + j}\right)\exp(jk_{0z}d_1)$$
$$\times\left\{E^+\exp(jk_z d_1) + E^+\exp(jk_z d_1)\right\} \qquad (7a)$$

$$A_{cr} = \frac{k_z k_0}{k k_{0z}} \left(\frac{2j}{M\eta + j} \right) \exp(jk_{0z} d_1)$$
$$\times \left\{ E^+ \exp(jk_z d_1) - E^+ \exp(jk_z d_1) \right\} \qquad (7b)$$

where

$$E^- = -E^+ \left(\frac{jR_f + M\eta}{-jR_f + M\eta} \right) \exp(2jk_z d_1)$$
$$E^+ =^+ \frac{2\exp(jk_{0z} d_1 - jk_z d_1)}{P - QL}$$
$$P = 2\frac{j + M\eta R_f}{M\eta + j}$$
$$Q = \frac{jR_f + M\eta}{-jR_f + M\eta}$$
$$L = 2\frac{j - M\eta R_f}{M\eta + j}$$
$$R_f = \frac{k_0 k_z}{k k_{0z}}$$

and $k_y^2 + k_z^2 = k^2$ where $k = \omega\kappa$ at the nihility frequency and κ is chirality parameter. It has been assumed that the impedance of chiral nihility medium is closed to the surrounding medium ($\eta_0 \quad \eta$). In above equation, $k_0 = \omega\sqrt{\varepsilon_0 \mu_0}$, k_{0z} and k_y satisfy the following dispersion relation $k_y^2 + k_{0z}^2 = k_0^2$.

When $M \to \pm\infty$ (PEC), $A_{cr} = 0$, $A_{co} = -1$, therefore (6) becomes

$$\mathbf{E}^r = -\sqrt{\frac{k}{j2\pi}} \left[\int_{A_1}^{A_2} + \int_{-A_2}^{-A_1} \right] \sqrt{R_1}$$
$$\times \exp[-jk(S_0 + t_1 + S_{ex})] d(2\alpha) \qquad (8)$$

In the above equation R_1, S_0, t_1 and S_{ex} are expresses in terms of α and A_1, A_2 are the subtention angles 2α at the edges of the parabolic and elliptic cylinders. It may be noted that limits of the integrals in equation (6) are determined using the following relations [5]

$$A_1 = \phi_v = 2\arctan\left(\frac{D}{2f} \right)$$
$$A_2 = \arctan\left(\frac{d}{2c} \right)$$

where D and d are the apertures of the apertures of the main reflector and sub-reflector respectively.

3. Numerical results and discussion

In the preceding section computational results of co-polarized and cross-polarized components of the reflected field, given by Eq. (6), along the reflector axes are shown in Fig. 2-5. The aperture of the reflector is chosen as 17λ. It has been observed that admittance parameter $M\eta$ affects the amplitude of the reflected field while pattern remains the same. Plots have been obtained for different values of $M\eta$. It has been observed that for $M\eta = \pm 1$, cross component of the reflected field is maximum, while co polarized component is zero. For $M\eta$=2, the amplitude of co polarized component is greater than the cross polarized component and the behavior is reversed for $M\eta$ = 3. For $M\eta$ > 3, the amplitude of the cross component decreases and finally vanishes for large values of $M\eta$. It can also be seen that for $M\eta$ = 0 and $M\eta \to \pm\infty$ cross polarized components of the reflected field disappear which represent the chiral nihility reflector backed by PMC and PEC material respectively, which is in accordance to our analytical formulation. In Fig. 5, we compared the results of PEMC backed chiral nihility reflector with that of PEC backed chiral nihility reflector. The results are in good agreement. In Fig. 6, co and cross polarized components of reflected fields are shown against admittance parameter which again endorses our earlier results.

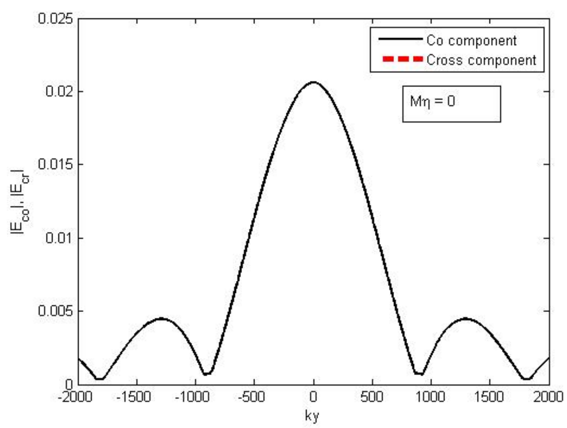

Fig. 2 (a). Cross and co polarized field reflected from system for $M\eta$ = 0, versus ky

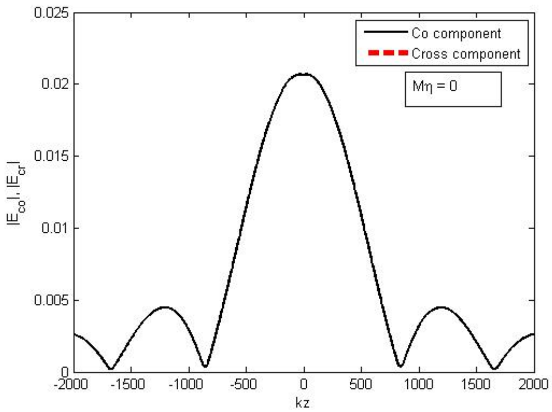

Fig. 2 (b). Cross and co polarized field reflected from system for $M\eta$ = 0, versus kz

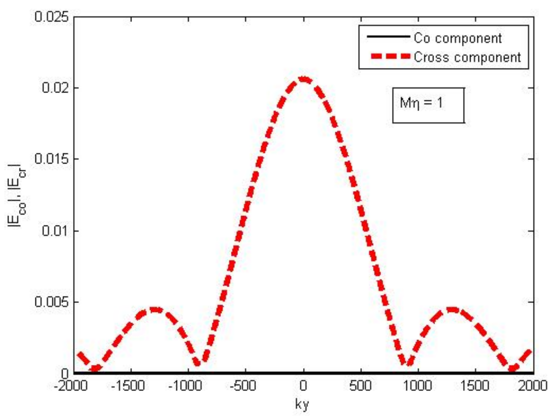

Fig. 3 (a). Cross and co polarized field reflected from system for $M\eta$ = 1, versus ky

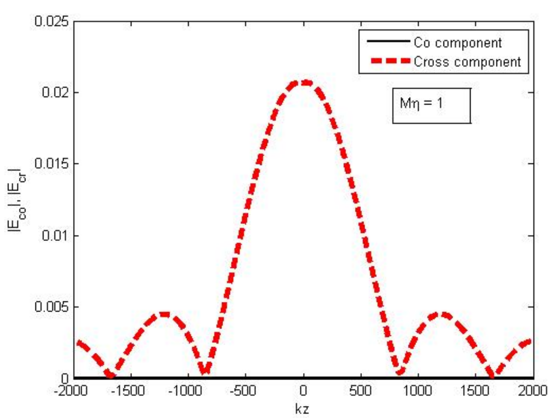

Fig. 3 (b). Cross and co polarized field reflected from system for $M\eta$ = 1, versus kz

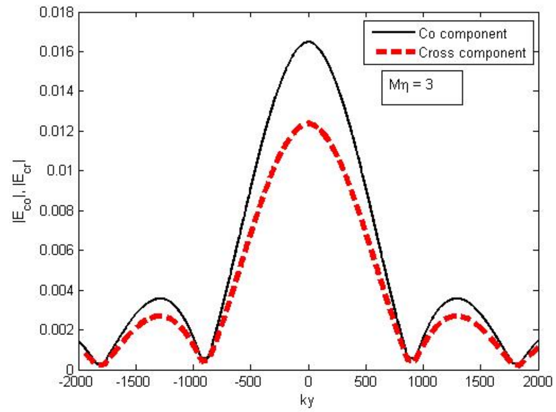

Fig. 4 (a). Cross and co polarized field reflected from system for $M\eta$ = 3, versus ky

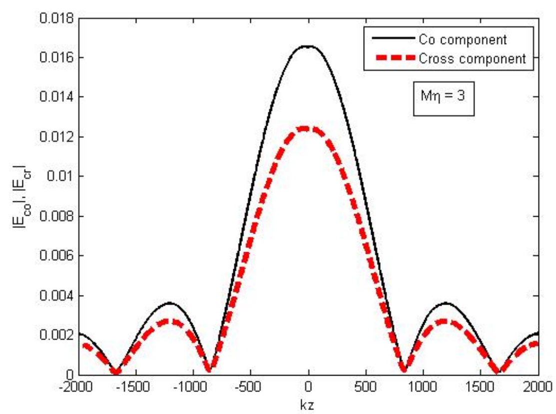

Fig. 4 (b). Cross and co polarized field reflected from system for $M\eta$ = 3, versus kz

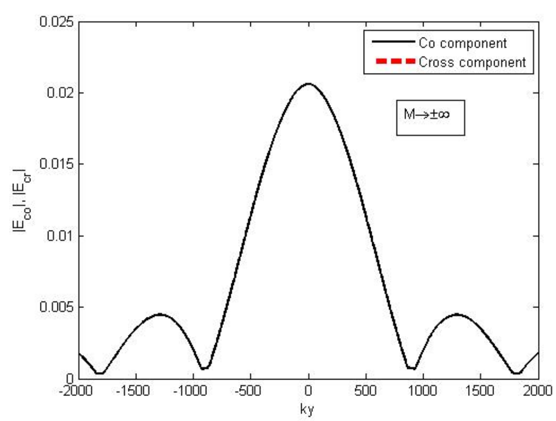

Fig. 5 (a). Cross and co polarized field reflected from system for $M\eta \to \pm\infty$, versus ky

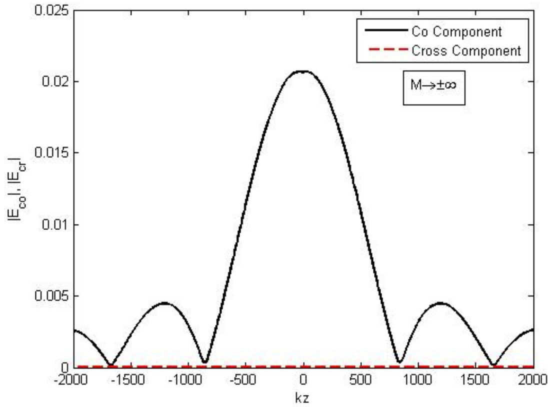

Fig. 5 (b). Cross and co polarized field reflected from system for $M\eta \to \pm\infty$, versus kz

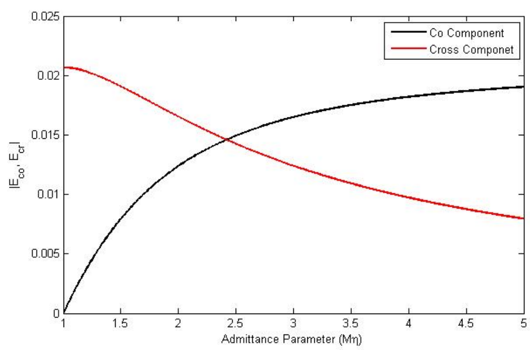

Fig. 6. Cross and co polarized field reflected from system versus $M\eta$ for fixed reflector axis i.e. kz=ky=5

4. Conclusion

The results presented demonstrate that the Maslov's method is straightforward remedy and provides an alternate tool to conventional GO method for evaluating diffraction field in the caustic region of focusing systems. In this work a dual reflector antenna called Gregorian system is studied. The important task of the work is the effects of material parameters (PEMC and chiral nihility) used in this system.

We have started from $M\eta$ = 0 (PMC) boundary and observed that field is rotated giving rise to increase and decrease in the amplitude of co and cross components for different values of $M\eta$. Finally, we reach $M\eta \to \pm\infty$ (PEC boundary). These findings may find potential use in some applications where controlled intensity of co and cross polarized field is required. Another striking feature can be seen that the factor thickness didn't appear in the expression of the reflected field. It means that the thickness of the reflector coating is irrelevant. It is perhaps due to the fact that in chiral nihility material, the two eigen-waves are circularly polarized but one of them is a backward wave.

Acknowledgment

The authors would like to extend their sincere appreciation to the Deanship of Scientific Research (DSR) at King Saud University for its funding of this research through the Research Group Project No. RG-1436-001.

References

[1] V. P. Maslov, V. E. Nazaikinski "Asymptotics of operator and Pseudo-Differential Equations," Consultants Bureau N.Y., 1988.
[2] V. P. Maslov, "Perturbation theory and asymptotic method," Moskov., Gos. Univ., Moscow, 1976.
[3] V. P. Ziolkowski, G. A. Deschamps, Radio Sci. **19**(4), 1001 (1984).
[4] A. Aziz, Q. A. Naqvi, K. Hongo, Progress in Electromagnetics Research, PIER **71**, 227 (2007).
[5] A. Aziz, A. Ghaffar, Q. A. Naqvi, K. Hongo, J. of Electromagnetic Waves and Appl. **22**(1), 85 (2008).
[6] A. Ghaffar, Q. A. Naqvi, K. Hongo, Progress in Electromagnetics Research, PIER **72**, 215 (2007).
[7] A. Ghaffar, A. Hussain, Q. A. Naqvi, K. Hongo, J. of Electromagnetic Waves and Appl. **22**(2-3), 301 (2008).
[8] A. Ghaffar, Q. A. Naqvi, K. Hongo, J. of Electromagnetic Waves and Appl. **22**(5-6), 665 (2008).
[9] A. Hussain, Q. A. Naqvi, K. Hongo, Progress in Electromagnetics Research **73**, 107 (2007).
[10] M. Faryad, Q. A. Naqvi, J. of Electromagnetic Waves and Appl. **22**(4), 563 (2008).
[11] M. Faryad, Q. A. Naqvi, Progress in Electromagnetics Research **76**, 153 (2007).
[12] A. Ghaffar, Q. A. Naqvi, K. Hongo, Optics Communications **281**, 1343 (2008).
[13] M. R. Ashraf, A. Ghaffar, Q. A. Naqvi, J. of Electromagnetic Waves and Appl. **22**(5-6), 815 (2008).
[14] M. Faryad, Q. A. Naqvi, J. of Electromagnetic Waves and Appl. **22**(4), 965 (2008).
[15] M. A. Fiaz, A. Ghaffar, Q. A. Naqvi, J. of Electromagnetic Waves and Appl. **22**(2-3), 358 (2008).
[16] M. A. Fiaz, A. Aziz, A. Ghaffar, Q. A. Naqvi, Progress in Electromagnetics Research **81**, 135 (2008).
[17] A. Ghaffar, M. A. S. Alkanhal, IEEE Transactions on Plasma Science **43**(1), 3801 (2015).
[18] A. Ghaffar, M. A. S. Alkanhal, Chinese Optics Letters **13**(9), 090801 (2015).
[19] M. Naz, A. Ghaffar, I. Shakir, Q. A. Naqvi, J. Optoelectron. Adv. M. **17**(1-2), 27 (2015).
[20] M. Naz, A. Ghaffar, I. Shakir, Q. A. Naqvi, Optoelectron. Adv. Mat. **9**(1-2), 208 (2015).
[21] S. Tretyakov, I. Nefedov, A. Sihvola, S. Maslovski, C. Simovski, J. of Electromagnetic Waves and Appl. **17**(5), 695 (2003).
[22] Q. Cheng, T. Jun, C. Zhang, Optics Communications

276, 317 (2007).
[23] Q. A. Naqvi, Progress in Electromagnetics Research, PIER **85**, 381 (2008).
[24] Q. A. Naqvi, Opt. Commun. **282**, 2016 (2009).
[25] Q. A. Naqvi, J. of Electromagnetic Waves and Appl. **23**(5-6), 773 (2009).
[26] S. Ahmed, Q. A. Naqvi, J. of Electromagnetic Waves and Appl. **23**(5-6), 761 (2009).
[27] A. Illahi, J. of Electromagnetic Waves and Appl. **23**(7), 863 (2009).
[28] I. V. Lindell, A. H. Sihvola, J. of Electromagnetic Waves Appl. **19**(7), 861 (2005).
[29] I. V. Lindell, A. H. Sihvola, IEEE Trans. Antennas Propag. **53**(9), 3012 (2005).
[30] A. Hussain, Q. A. Naqvi, Progress in Electromagnetics Research **73**, 61 (2007).
[31] A. Hussain, Q. A. Naqvi, M. Abbas, Progress in Electromagnetics Research **71**, 85 (2007).
[32] S. Ahmed, Q. A. Naqvi, Progress in Electromagnetics Research **78**, 25 (2008).
[33] S. Ahmed, Q. A. Naqvi, J. of Electromagnetic Waves and Appl. **22**(7), 987 (2008).
[34] S. Ahmed, Q. A. Naqvi, Opt. Commun. **281**, 4211 (2008).
[35] S. Ahmed, Q. A. Naqvi, Progress in Electromagnetics Research B **10**, 75 (2008).
[36] C. Scott, Modern Methods of Reflector Antenna Analysis and Design. Boston, London: Artech House, 1990.
[37] S. J. Orfanidis, Electromagnetic Waves and Antennas, Exercise Book (2002).

*Corresponding author: aghaffar16@uaf.edu.pk