Parametric dependence of the transmitted intensity in optical bistable fiber grating devices

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Based on the nonlinear coupled mode theory, by using the reversely recursive transmission matrix method, we analyze the dependence of the transmitted field amplitude on the optical bistable fiber grating devices parameters. The results show that, the main characteristic features of the optical hysteresis cycles will be influenced greatly by the couple coefficient, the grating length and the frequency of the incident light, respectively. The parameters-dependent bistability is due to the difference of the axial distribution of forward wave intensity in the gratings.

1. Introduction

Nonlinear Bragg gratings (NLBG) are rapidly emerging as one of the most rewarding areas in nonlinear optics [1-3]. Wave propagation in the gratings is characteristed by the presence of stop bands and pass bands. In the coupled mode theory, a wave whose frequence lies within a stop band is strongly reflected. Its amplitude decays exponentially with propagation distance into the medium. On the other hand, a wave whose frequency lies outside the stop band can pass through the structure unimpeded. In 1979, Winful et al. first studied the effect of a nonliear dielectric constant on the transmission properties of a periodic structure [4]. They found that an intense wave could alter the refractive index of the structure enough to tune itself out of the stop band, for certain intensities total transmission occurs, at these intensities a sptial resonance is excited within the distributed feedback resonator. The phenomena that occur in the nonlinear periedic structures are optical bistability, pulse compression, solition propagation, self-pulsations and chaos. A lot of work was done on the study of the NLBG optical bistable characteristics from different perspective [5-12]. Among these, analyses have principally concentrated on the case where the optical hysteresis cycles characteristic features depend on the product of the couple coefficient and the gratings length. However, either from a theoretical or from an experimental point of view, one can equally choose to vary one of the physical parameters respectively. In this connection, the analysis of the hysteresic cycles (when they exist) could be of particular interest. This is the purpose of the present work.

This paper is organized as follows. Section 2 gives the theoretical model and the reversely recursive transmission matrix method. The operation condition selection and bistability performance comparisons of NLBG are provided in Section 3. Finally, Section 4 concludes our results.

2. Theoretical model

Inside fiber gratings, the z-axial distribution of refractive index can be described by

$$n(z) = n_0 + n_1(z)\cos\left[2\beta_B(z)z + \Omega(z)\right] + n_2|E(z)|^2, \quad (1)$$

where $E(z)$ is the inner electric field of gratings, $\beta_B(z)$ is the Bragg wave vector, $\Omega(z)$ is the spatial phase shift. n_0, $n_1(z)$ and n_2 denote the effective mode refractive index, linear refractive index modulation amplitude, and nonlinear refractive index coefficient, respectively.

The inner electric field can be expressed by

$$E(z) = E_+(z)\exp\{i[\beta_B(z)z - \omega t]\} + E_-(z)\exp\{-i[\beta_B(z)z + \omega t]\}$$
$$(2)$$

where ω is the carrier angular frequency, t is the time, E_+ and E_- represent the slowly varying amplitude of

forward and backward wave, respectively. In this paper, we assume that the incident wave is continuous wave or quasi-continuous wave, or the pulse width of the incident wave is much longer than the transmission time of the incident wave inside the gratings. Substituting Eqs. (1) and (2) into the wave equations, and neglecting the loss and material dispersion (the nonlinear medium of NLBG is assumed to be Erbium-doped fiber without pump, even though its loss and material dispersion coefficients near $1.55\mu m$ are large, the total loss and material dispersion are negligible due to very short length selected in calculations), the response time of material is very fast enough, as well as the carrier wavelength is close to Bragg wavelength, one can obtain the following nonlinear coupled mode equations [11]

$$\frac{\partial E_+}{\partial z} + \frac{1}{v_g}\frac{\partial E_+}{\partial t} = i[\delta E_+ + \gamma(|E_+|^2 + 2|E_-|^2)E_+ + kE_-], \quad (3a)$$

$$-\frac{\partial E_-}{\partial z} + \frac{1}{v_g}\frac{\partial E_-}{\partial t} = i[\delta E_- + \gamma(|E_-|^2 + 2|E_+|^2)E_+ + kE_+], \quad (3b)$$

where v_g is the light group velocity in the grating medium, δ, Γ and k account for the detuning from the Bragg vector, nonlinear coefficient, and coupling coefficient, respectively, which can be expressed by

$$\delta = \beta - \beta_B(z) = n_0\frac{\omega}{c} - \beta_B(z),$$

$$\gamma = \frac{\pi n_2}{\lambda_B},$$

$$k = \frac{\pi n_1(z)}{\lambda_B}, \quad (4)$$

where c is the light velocity in vacuum, $\lambda_B = 2n_0\Lambda$ is the Bragg wavelength, Λ is the grating period,

The boundary conditions are given by

$$z = 0: \quad E_+(0,t) = E_i(0,t) \quad E_r(0,t) = E_-(0,t), \quad (5a)$$

$$z = L: \quad E_-(L,t) = 0, \quad E_t(L,t) = E_+(L,t), \quad (5b)$$

where E_i, E_r and E_t are the slowly varying amplitudes of the incident, reflected and transmitted wave, respectively.

In the steady-state, the nonlinear coupled-mode equations can be solved numerically by using the transmission matrix method (TMM), based on this method, the fiber gratings can be divided into a set of approximately periodic, uniform segments and the forward and backward fields at the j-th segments can be described as $E_{+j}(E_{-j})$ and $E_{+(j+1)}(E_{-(j+1)})$, which is defined by

$$\begin{bmatrix} E_{+(j+1)} \\ E_{-(j+1)} \end{bmatrix} = E_j^p\begin{bmatrix} E_{+j} \\ E_{-j} \end{bmatrix} \quad (6a)$$

where E_j^p is the transfer matrix for the j-th section, which is expressed by

$$E_j^p = d_j \times \begin{bmatrix} \cosh(\mu_j l) - \frac{\Delta_j}{\mu_j}\sinh(\mu_j l) & -i\frac{k_j}{\mu_j}\sinh(\mu_j l) \\ i\frac{k_j}{\mu_j}\sinh(\mu_j l) & \cosh(\mu_j l) + \frac{\Delta_j}{\mu_j}\sinh(\mu_j l) \end{bmatrix}$$

$$(6b)$$

with

$$d_j = \exp\left[\frac{i\gamma}{2}\int(|E_-|^2 - |E_+|^2)dz\right],$$

$$\mu_j^2 = \Delta_j^2 + k_j^2,$$

$$\Delta_j = \frac{i}{2l}\int[2\delta + 3\gamma(|E_+|^2 + |E_-|^2)]dz,$$

where l is the length of the j-th section. From the boundary condition (5b), caculating iteratively the eqs. (6a) and (6b), together with the boundary condition (5a), one can obtain input-output characteristics of the gratings.

3. Results and discussions

The used data in calculations are $\lambda_B = 1.55\mu m$, $n_0 = 1.46$, $n_2 = 6.9\times10^{-15}m^2/w$. To facilitate description, the input light intensity I_i, and the output I_t intensity are normalized as I_i/I_c and I_t/I_c

respectively in following discussions, where $I_c = 4\lambda_B / (3\pi n_2 L)$ is the critical input intensity.
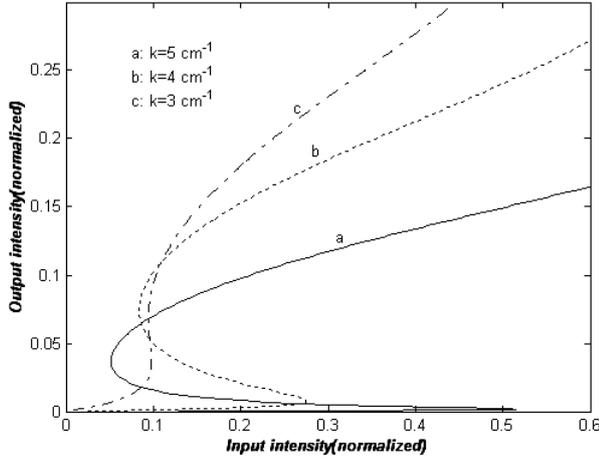
Fig. 1.(a) stable input-output characteristics of NLBG for various couple coefficient.
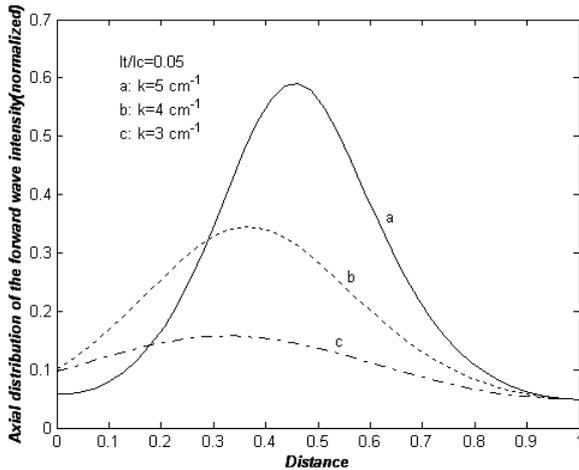
Fig. 1. (b) axial distribution of the forward wave intensity for various couple coefficient.

multistability phenomena. These features may be understood as follows: When the input intensity is low, the nonlinear effect is weak, and the transmittance is small, then the output intensity lies in the lower branch of hysteresis. Under the case, its behaviors are similar. When the input intensity exceeds the switching-on threshold to form bistablity, the nonlinear effect is strong. Meantime, for the larger couple coefficient, the inner energy of grating is more convergent (see Fig. 1(b)). As a result, a larger switching-on threshold is required to excite bistability.
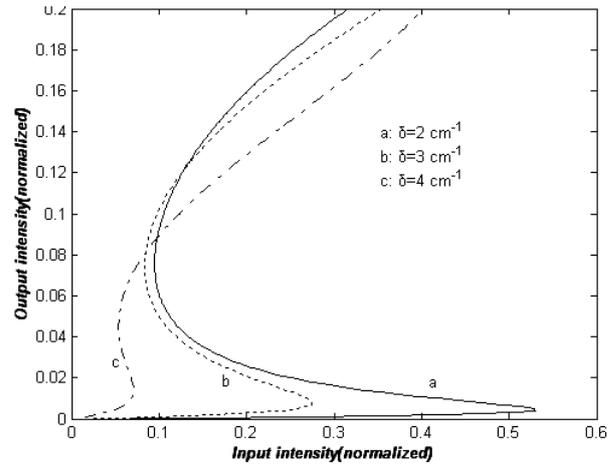
Fig. 2(a) stable input-output characteristics of NLBG for various initial detuning.
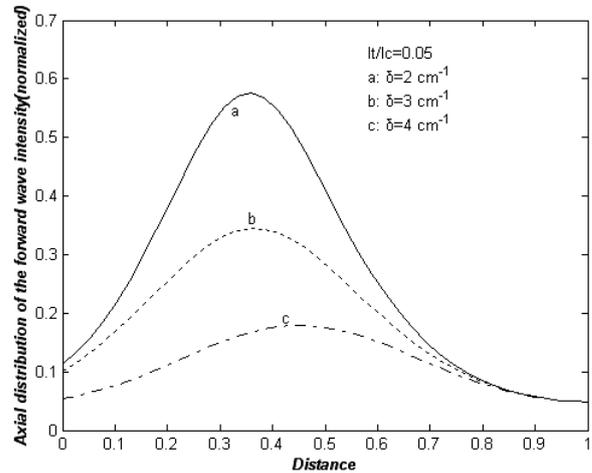
Fig. 2. (b) axial distribution of the forward wave intensity for various initial detuning.

Fig. 1(a) shows the steady-state input-output characteristics of NLBG for three different couple coefficient. To be understood easily, the axial distribution of forward wave intensity are also indicated in Fig. 1 (b), where $L = 1cm$, $\delta = 3cm^{-1}$, $I_t / I_c = 0.05$. From Fig. 1, it can be seen that, for low values of k, the feedback is insufficient to create bistability, the optical transmission mode of operation occurs at $k = 3.0cm^{-1}$. With increasement of couple coefficient, the grating internal feedback enhanced, a hysteresis loop is traced out at $k = 4cm^{-1}$, and larger values of k lead to

Fig. 2(a) shows the steady-state input-output characteristics of NLBG for three different initial detuning, and their axial distribution of forward wave intensity are also presented respectively in Fig. 2(b), where $K = 4cm^{-1}$, $L = 1cm$, $I_t / I_c = 0.05$. From the figure, it can be seen that , the detuning(the incident wavelength)

have obvious influence on the bistable characteristics: When the detuning decreases, and the incident wavelength shifts to higher values of the Bragg wavelength, grating feedback enhanced gradually, the inner energy of grating is more convergent, then the lasing threshold increases significantly, even the bistable phenomena vanishes with increasement of the detuning, in other words, the light is in a transmissive state, see Fig. 2(b). These features may be understood as follows: In the case of smaller detuning, the frequency of the incident light lies in the centre of reflection spectrum, and the transmitted light is "stopped" when the input intensity is lower, therefore, the required switching-on threshold to excite bistability is larger.
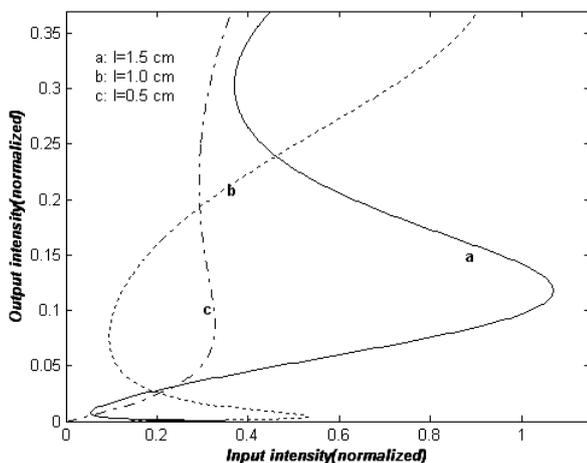
Fig. 3. Stable input-output characteristics of NLBG for various grating length.

Fig. 3 shows the steady-state input-output characteristics of NLBG for three different grating length, where $\delta = 2.0 cm^{-1}$, $k = 4.0 cm^{-1}$. From the figure, it can be seen that, the grating length have obvious influence on the bistable characteristics, such as the switching-on threshold, the on-off ratio, the width of the hysteresis and the transmittance of the upper branch. When length is smaller (< 0.5 cm), no bistable phenomena occurs. With the gradual increasement of length, the bistable effect begin to occurs, moreover the width of the hysteresis increase rapidly, for the bigger length, it exists two hysteresis. As a result, the bistable performance can be further optimized by reasonably selecting the grating length.

4. Conclusions

In summary, based on the nonlinear coupled mode theory, this paper has demonstrated the dependence of the transmitted field amplitude on the optical bistable fiber grating devices parameters in term of the reversely recursive transmission matrix method. The numerical simulations show that, the bistability performance of the nonlinear Bragg gratings has significant dependence on the the couple coefficient, the grating length and the the frequency of the incident light, respectively. The results may provide an instructive insight from a practical viewpoint.

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