Optoelectronic system performance prediction using the modulation transfer function of the components

C. TODIRICĂ*, C. PLEȘA, D. ȚURCANU, I. NICOLA

Military Equipment and Technologies Research Agency, Bucharest, Romania

In this paper the authors present a prediction model for the performance of an optoelectronic system by calculating the modulation transfer function (MTF) of the system using the MTF of the components. The paper addresses the imaging process through the optoelectronic equipment, the MTF of the main components and their influence on the total MTF of the optoelectronic system in order to use them in the performance prediction process of an optoelectronic system at the design stage. Also, a series of MTF measurement tests performed by the authors are presented in order to compare the measured MTF values with computed ones.

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1. Introduction

Most of the optoelectronic systems have sampling optoelectronic devices [1-14]. These devices take the image from an optical device (lens) and divide it into several areas corresponding to the number of elements it consists of, the so-called pixels.

In the case of optoelectronic systems that include such devices, special problems arise when it is desired to define and measure the MTF and use it to characterize the performance of these systems. The main problem with these systems is that they do not meet the stationary condition that requires the point or line spread function not to change with the position change in the image. Several authors [15-18] have shown that the effective MTF of the sampling devices depends on the exact position of the object relative to the sampling area.

Examples of sampling systems would be: all types of systems using matrix sensors, LCD screens, scanning sensor systems (stationary condition is met only in the scan direction), displays whose image is generated by scanning (e.g. CRT displays, stationary condition is met only in the scan direction) [16-18, 21].

2. Theoretical approach

The impulse response, h(x,y), represents the smallest detail of an object that can be solved by the optoelectronic system [16, 17]. The diameter of the image formed by the optoelectronic system in response to an impulse depends both on the diffraction effects and the aberration of the optical system [19-21]. The impulse response, h(x,y), can

be interpreted as the picture irradiation distribution (W/cm^2) depending on the position.

The impulse response of an optoelectronic system is composed, as known, from the impulse response of its components. Each component of the system contributes to the degradation of the scene image. In other words, each component has an impulse response that can be applied in the same manner as the impulse response of the whole system.

The impulse response of a component may be influenced by certain physical effects. For example, the impulse response of the lens is influenced by the combination of the diffraction and aberrations. Detector response is influenced by physical form and the integration time. If the optoelectronic system does not contain sampling devices or if the sampling points are so close that the system has a continuity of sampling positions, then the impulse response of the system is given by the conventional imaging theory. It has been shown that the point spread function (PSF) of the system is the convolution of the PSF of its components ([17]):

$$h_{system}(x, y) = h_{lens}(x, y) \otimes h_{det}(x, y) \otimes h_{el}(x, y) \otimes h_{disp}(x, y)$$
(1)

where

 \otimes express the convulsion,

 $h_{\text{system}}(x, y)$ is the system impulse response,

 $h_{lens}(x, y)$ is the lens impulse response,

 $h_{det}(x, y)$ is the detector impulse response,

 $h_{el}(x, y)$ is the electronics impulse response,

 $h_{display}(x, y)$ is the display impulse response.



Fig. 1. Stages of image formation through an optoelectronic system

After applying the Fourier transform, we obtain the equation of the optical transfer function of the system as the product of the transfer functions corresponding to its components:

$$OTF_{system}(f_x, f_y) = OTF_{lens}(f_x, f_y) \cdot OTF_{det}(f_x, f_y) \\ \cdot OTF_{el}(f_x, f_y) \cdot OTF_{disp}(f_x, f_y)$$
⁽²⁾

Considering that

$$OTF(s) = MTF(s)\exp[i \cdot PTF(s)]$$
(3)

it can be written that the OTF module is given by the relation:

$$MTF_{system}(f_x, f_y) = MTF_{lens}(f_x, f_y) \cdot MTF_{det}(f_x, f_y)$$

$$\cdot MTF_{el}(f_x, f_y) \cdot MTF_{disp}(f_x, f_y)$$
(4)

2.1. Lens transfer function

To calculate the optical transfer function of the lens, it is necessary to consider two optical effects: diffraction and aberrations [1, 16, 20, 22]. The impulse response of the optics will be given by the equation:

$$h_{lens}(x, y) = h_{dif}(x, y) \otimes h_{aber}(x, y)$$
(5)

applying the Fourier transform next relationship is obtained

$$MTF_{lens}(f_x, f_y) = MTF_{dif}(f_x, f_y) \cdot MTF_{aber}(f_x, f_y)$$
(6)

The impulse response of a diffraction limited optical system is:

$$h_{dif}(x, y) = 4somb^2(r) \tag{7}$$

where

 $r = \sqrt{x^2 + y^2}$ expressed in units of $1/\lambda f_{\#}$

$$somb(r) = \frac{J_1(\pi r)}{\pi r}$$
(8)

 J_1 is the Bessel Function of the first kind of order 1

The optical transfer function for a diffraction limited optical system with a circular aperture is [3, 22]:

$$OTF_{dif}(f_x, f_y) = MTF_{dif}(f_x, f_y)$$

= $\frac{2}{\pi} \cdot [\arccos(\rho) - \rho \sin(\arccos(\rho))]$ (9)

and for a rectangular aperture system:

$$OTF_{dif}(f_x, f_y) = MTF_{dif}(f_x, f_y) = 1 - \rho$$
(10)

where
$$\rho = \sqrt{f_x^2 + f_y^2}$$





Fig. 2. PSF and OTF for a diffraction limited optical system

The transfer function corresponding to the aberrations effects of the optical system can be expressed as a Gaussian function:

$$MTF_{aber}\left(f_{x},f_{y}\right) = e^{-\pi\rho^{2}\sigma_{aber}^{2}}$$
(11)

In Fig. 2 is presented an example for lens PSF and MTF representations using equations (6), (9) and (11).

A complex calculation of the transfer function corresponding to the aberrations of the optical system will be the subject of a future work.

2.2. Detector transfer function

The detector transfer function is the measure of the detector's contribution to image formation through the optoelectronic system.

In the case of a detector the object is a real image projected on it, and the image produced is in the form of an electrical signal. Thus, we can define the MTF as the ratio of the output signal to the input radiant flux modulation, which is a one-dimensional sinusoidal wave passing through the detector [15, 23-25]. MTF is represented by space frequency (usually measured in lp/mm).

The transfer function of the detector is influenced, among other things, by the detector spatial integration. Other effects, such as responsivity, are usually considered irrelevant. However, there are situations when these need to be considered [15, 23] (e.g. when the responsivity varies greatly on the surface of the detector).

As most detectors have rectangular shape, a rectangular function was used to spatially model the detector (Fig. 3):

$$h_{det}(x, y) = \frac{1}{IFOV_x IFOV_y} rect\left(\frac{x}{IFOV_x}, \frac{y}{IFOV_y}\right)$$

$$= \frac{1}{IFOV_x} rect\left(\frac{x}{IFOV_x}\right) \frac{1}{IFOV_y} rect\left(\frac{y}{IFOV_y}\right)$$
(12)

where

 $IFOV_x$, $IFOV_y$ is angular subtense, of the detector in the direction being considered, in *mrad*, and is calculated by dividing the detector size by the focal length;

$$rect(x) = \begin{cases} 1, |x| \le \frac{1}{2} \\ 0, |x| > \frac{1}{2} \end{cases}$$
(13)



Fig. 3. PSF and OTF for a detector

By Fourier transform of equation (12) the detector transfer function is obtained:

$$MTF_{det}(f_x, f_y) = sinc(IFOV_x f_x, IFOV_y f_y)$$

= sinc(IFOV_x f_x)sinc(IFOV_y f_y) (14)

where sinc(x) function is defined as:

$$sinc(x) = \frac{\sin(\pi x)}{\pi x}$$
(15)

If the value of the detector size in mm is used instead of IFOV then the MTF depending on the spatial frequency expressed in lp/mm is obtained.

2.3. Electronics transfer function

Electronic circuits are different from the other components of an optoelectronic system, being causal (an

output signal cannot precede the input signal that caused it).

The electronics transfer function is one of the most difficult to characterize and one of the most ambiguous to apply. First, it involves the transformation of temporal frequencies into spatial frequencies. This involves the use of a scanning or reading device. Second, the impulse response of an electronic circuit is unidirectional. A circuit changes the image of a point in one direction, while an optical system scatters the point image in all directions. There is often a bidirectional form of impulse response, which overruns the rule of causality. Usually, this approximation does not have a major impact on system performance estimates, as long as the electronic part is not the component limiting the system. Holst [17] and Vollmerhausen and Driggers [21] presented approximations of the transfer function of analog and digital electronic circuits that can be used in the estimation of the optoelectronic system's transfer function.

$$MTF_{electric}(f) = \frac{\sin(\pi \cdot \alpha_i \cdot f)}{\pi \cdot \alpha_i \cdot f}$$
(16)

where

 $\alpha_i = \frac{\alpha \cdot t_i}{\tau_d},$

 α is the angular dimension of the detector

 t_i integration time

 τ_{d} is the delay time corresponding to the scan of the angular dimension α .

2.4. Display transfer function

The finite size and shape of the display elements determines the display transfer function. Typically, the active element of a display is Gaussian (in the case of cathode ray tube, CRT) or rectangular (LED display). The PSF of a display has the size and shape of the active element. The only difference is that its shape and dimensions must be transformed from physical dimensions into the angular space of the detector [15, 17]. For the Gaussian element, its size in mm should be converted into an angular space equivalent to the sensor field of view.

$$\sigma_{mrad} = \sigma_{cm} \frac{FOV_{v}}{L_{v}}$$
(17)

where L_{ν} is the height of the screen in *cm* and FOV_{ν} is the sensor field of view in *mrad*. In the case of rectangular elements, their dimensions must also be transformed into the angular space of the sensor. The vertical dimension of the rectangular element is obtained with the equation (13) and the horizontal dimension is determined with a similar relation. Once these angular dimensions have been obtained, the PSF of the display element can be formulated as follows:

- for CRT display:

$$h_{display}(x, y) = \frac{1}{\sigma_{mrad}^2} \exp\left(-\pi \frac{r^2}{\sigma_{mrad}^2}\right)$$
(18)

- for LED display:

$$h_{display}(x, y) = \frac{1}{V_{mrad_h}H_{mrad_v}} \operatorname{rect}\left(\frac{x}{V_{mrad_h}}, \frac{y}{H_{mrad_v}}\right) (19)$$

where the dimensions of the element are given in the *mrad*. The transfer functions associated with these elements of the screen are determined by applying the Fourier transform to the equations above and obtaining:

- for CRT display (Fig. 4 up):

$$MTF_{display}(f_x, f_y) = \exp(-\pi \rho^2 \sigma_{nrad}^2)$$
(20)

- for LED display (Fig. 4 down):

$$MTF_{display}\left(f_{x}, f_{y}\right) = sinc\left(V_{mrad_h}f_{x}, H_{mrad_v}f_{y}\right) \quad (21)$$



Fig. 4. MTF of a CRT display (up) and a LED display (down)

3. Results and discussion

The discussions that will be presented below in this paper will refer to the MTFs of optoelectronic systems and their components. Simultaneously, for exemplification and for a better understanding of the phenomena, we chose the thermal imaging camera with a 100mm focal lens. This equipment has been chosen as an example because it has been used in most of the tests presented in this paper. Following the presented MTF measurement tests, a comparison will be made between the calculated MTF value and the measured MTF.

As shown above, the PSF convolution and the product of the MTF of the system components were performed to calculate the whole system transfer function. To evaluate the actual performance of the system, the MTF of the eye is usually not taken into account. The system can also be described as "limited" by a specific feature, meaning the resolution limit of a component is smaller than the resolution limit of the other components. For example, an optoelectronic system limited by the detector is a system whose detector degrades the image the most, and has a lower resolution frequency compared to other system components.

This approach for determining the MTF model for an optoelectronic system provides small errors (in the order of a few percent) which makes it quite useful, especially when developing an optoelectronic system to anticipate its performance.



Fig. 5. MTF chart calculated using equation (4)

Using the equations (4), (14) and (16) MTF was calculated in the case of a thermal imaging camera equipped with a 100mm lens. The device does not have its own display, so the $MTF_{display}$ values have not been included in the calculation. The values for the MTF_{lens} are those provided by the lens manufacturer.

The obtained values are graphically represented in Fig. 5.

Next, the MTF of a thermal imaging system equipped with a 100mm focal lens was measured and the results compared with those obtained by calculation for the same system.

In order to carry out the tests in this paper, optoelectronic test equipment from the Optoelectronics and Lasers Laboratory of Military Equipment and Technologies Research Agency (Fig. 6 a)) was used. In Fig. 6 b) is presented the thermal imager measurement equipment layout scheme. This includes a 30" collimator, a wheel with 12 targets, a rotary support for the equipment under test, a blackbody with high resolution temperature controller and a computer with specialized software.





Fig. 6. a) Thermal imager measurement equipment b) Thermal imager measurement equipment layout scheme

Analysing the graphical representation of the two functions (Fig. 7), it can easily be observed that the calculated values are similar to the results obtained from the measurements, the differences between the two functions not exceeding 8.6%. This value allows us to assert that the MTF of a system can be anticipated with a fairly high accuracy at the design stage. Thus, the designer will have the necessary information for the final product to meet the requirements for which it was designed.



Fig. 7. Comparison of the computed MTF with the measured MTF for a thermal imager with 100mm lens

4. Conclusions

Our experiments highlight that the impulse response of a component may be influenced by certain physical effects. The impulse response of the optical part is influenced by the combination of the diffraction effect and the aberration effect. Detector response is influenced by physical form and integration time. The system can also be described as "limited" by a certain feature of the system.

It can also be concluded that in the process of the optoelectronic system performance prediction using components MTF, the human eye is not part of the performance evaluation of an optoelectronic system and the eye optical transfer function is not included in the optical transfer function of the optoelectronic system, the PSF and the MTF of the eye being used to evaluate system limitations. Considering that the eye is the ultimate receiver of optoelectronic systems, knowing its features is important for optimal choice of the parameters of these systems.

In order to exemplify and to better understand the phenomena, a thermal imaging camera equipped with a 100 mm lens was used. First, considering the presented relationships and manufacturer data, system MTF was predicted using calculated MTFs of its components. Next, the MTF of thermal imaging system was measured and the results compared with those obtained by calculation. Analysing the results, it can easily be observed that the computed values are very close to the results obtained from the measurements, the differences between the two functions not exceeding 8.6%.

In keeping with the theoretical predictions, this approach to determine the MTF model for an optoelectronic system provides small errors (in the order of a few percent) which makes it quite useful, especially when developing an optoelectronic system to anticipate its performance.

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^{*}Corresponding author: corneltod@yahoo.com