Optimization of space structures

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Design optimization of space structures considering geometrical and material nonlinearities is achieved by continuous virtual sub population (CVSP) evolutionary algorithm. The design variables are cross-sectional areas of the elements. Design constraints include structural and stability constraints. Tension and compression stresses are limited to their critical values and nodal displacements are restricted to their allowable values. The overall loss of stability is also checked during the optimization process and some restrictions on design variable are considered. The test examples presented demonstrate that further reduction is possible in the structural weight by including the nonlinear behavior in the optimal design process.

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1. Introduction

Nowadays, space structures are widely used because of their efficiency and numerous advantages comparing with conventional form of structures. These structures are being seen in many constructions such as, exhibition halls, stadiums, bridges, pools and aircraft hangars and so on. Therefore, sufficient attention must be considered for systematic design of these structures. For this purpose, design of space structures can be conveniently achieved by employing optimization techniques. It is obvious that an optimal design has a great influence on the economy and safety of all the structures. In recent years, much progress has been made in optimum design of space structures by considering linear behavior [1-3]. It is observed that some trusses appear nonlinear behavior even in usual range of loading [4, 5]. Therefore, neglecting of nonlinear effects in design optimization of these structures may be led to uneconomic design.

In this study, design of space structures for optimal weight considering geometrical and material nonlinear behavior is presented using a continuous evolutionary algorithm. All of the optimization problems have two main phases: analysis phase and optimization phase. We employ ANSYS (nonlinear finite element program) in analysis phase. In the optimization phase, we utilize virtual sub population (VSP) algorithm [6]. The design variables are cross sectional areas of the structures. The design constraints involved here are structural, stability and geometrical constraints. Nodal displacements are restricted to its upper bounds. Tension and compression axial stresses are limited to yield and buckling stresses, respectively. Loss of overall stability of structure is also checked throughout the optimization process.

Some illustrative examples are presented to demonstrate the effectiveness of proposed method for optimum design of space structures. The numerical results reveal that taking into account nonlinear behavior of structures affects the safety and economy of the design.

2. Theoretical background of nonlinear analysis

In a linear static analysis we implicitly assume that the deflections and strains are very small and the stresses are smaller than the material yield stresses. Consequently, the stiffness can be considered to remain constant (i.e., independent of the displacements and forces) and the finite element equilibrium equations are linear.

$$\{P\} = [\boldsymbol{K}]\{\delta\} \tag{1}$$

where $\{P\}$, $[\boldsymbol{K}]$, and $\{\delta\}$ are the external load vector, stiffness matrix and nodal displacements vector, respectively.

This linearity implies that any increase or decrease in the load will produce proportional increase or decrease in displacements, strains and stresses. But we know that, in many structures, at or near failure (ultimate) loads, the deflections and the stresses do not change proportionately with the loads. Either the stresses are so high that they no longer obey Hooke's law (linear stress to strain relationship) or else there are such large deflections that the compatibility equations (strain to displacement relationship) cease to be linear. These two conditions are called material nonlinearity and geometric nonlinearity, respectively.

In this study, a finite elements model based on geometrical and material nonlinear analysis of space structures including plasticity, and large deflection capabilities is presented by ANSYS [7]. In this model a 3-D truss element called link8 is used. The 3-D truss element is a uniaxial tension-compression element with three degrees of freedom at each node. In elasto-plastic analysis the von mises yield function is used as yield criterion. In a stress state where only one direct stress, say σ_{xx}, is nonzero, the onset of plasticity is defined by the condition

$\sigma_{xx}= \sigma_y$. Flow rule in this model is associative and the hardening rule is multilinear isotropic hardening [7].

3. Steps in nonlinear analysis combining geometrical and material nonlinearities

Here, instead of the linear strain-displacement relation, the nonlinear Green's strain [8] is used:

$$\varepsilon_G = \frac{l_n^2 - l_0^2}{2l_0^2} \tag{2}$$

where ε_G is the nonlinear Green's strain, l_n and l_0 are the length of space truss element after and before the deflection, respectively.

Since Green's strains are used, the stresses in an analysis, including geometric nonlinearity, will be 2nd Piola-Kirchoff [8] stresses.

Since the strains are nonlinear functions of the displacements or when the stresses reach values exceeding the yield stresses of the material, the stress to strain relationship is nonlinear. In these cases, the stiffness is dependent on the displacements and the straines. Obviously, the solution of the displacements can not be obtained in a single step. Instead, the analysis is carried out by the incremental method [8] combined with some iterative equilibrium corrections at every step.

In this work, using the Newton-Raphson method of solution, the following steps are used:

1. Form tangent stiffness matrix $[\boldsymbol{K}_t]$ with the latest values of displacements and stresses. This involves calculating and using the elasto-plastic material stiffness matrix, $[\boldsymbol{D}_{ep}]$, for those points which are plastic. Use the linear material stiffness matrix, $[\boldsymbol{D}]$ for points remaining elastic or unloading.

$$[\boldsymbol{K}_t] = \sum_{1}^{N_e} \int_e [\boldsymbol{B}]^{\mathrm{T}} [\boldsymbol{D}_{ep}][\boldsymbol{B}] \mathrm{d}\, v \tag{3}$$

To start the process in iteration 1 of load step 1, the linear stiffness matrix is used, assuming that stresses and strains obey Hooke's law.

2. Solve the incremental displacements:

$$\{\Delta\delta\} = [\boldsymbol{K}_t]^{-1}\{\Delta P\} + \{\psi\} \tag{4}$$

where $\{\Delta P\}$ is part of the load vector to be applied at the current increment (to be used only at the first iteration of a load step) and $\{\psi\}$ is residual force vector. Use zero values for $\{\psi\}$ at the first iteration of the first load step.

3. Add the incremental/corrective displacements $\{\Delta\delta\}$ to the total displacements $\{\delta\}$:

$$\{\delta\} = \{\delta\} + \{\Delta\delta\} \tag{5}$$

4. Calculate strains using the nonlinear Green's strain based on the latest estimate of the displacements and the incremental strains $\{\mathrm{d}\varepsilon\}$.

5. Calculate total stresses, using the linear elastic stress strain relation:

$$\{\sigma\} = \{\sigma\} + [\boldsymbol{D}]\{\mathrm{d}\,\varepsilon\} \tag{6}$$

6. Check to see if the estimated stresses are within the elastic limit:
If $\{\sigma\} > \sigma_y$, current point is plastic GO TO step 7.
If $f(\sigma) < \sigma_y$, current point is elastic GO TO step 8.

7. Calculate the elastic part $\{\varepsilon_{ee}\}$ and the plastic part $\{\varepsilon_{ep}\}$ of the incremental strains. The plastic strain increment is computed using the normality flow rule [9].

8. Calculate contributions of the current Gauss point to the element internal forces $\{f\}_i$:

$$\{f\}_i = \int_e [\boldsymbol{B}]^{\mathrm{T}}\{\sigma\}\mathrm{d}\,v \tag{7}$$

Repeat steps 4 through 6 for all the Gauss points of all elements.

9. Calculate residual forces as:

$$\{\psi\} = \int_V ([\boldsymbol{B}]^{\mathrm{T}}\{\sigma\}\mathrm{d}\,v - \{F\}) = \sum_{i=1}^{N_e}\{f\}_i - \{F\} \tag{8}$$

where the vector $\{F\}$ contains the (cumulative) external forces and includes any reactive forces at supports.
Incorrect assumptions in displacements will, in general, result in the vector $\{\psi\}$ being nonzero. The residual force $\{\psi\}$ can be visualized as additional nodal forces required bringing the assumed displacements into nodal equilibrium.

10. If $\| \psi \| >$ CTOL, the current increment has not converged. Apply equilibrium correction by repeating steps 1 through 9.

The convergence tolerance CTOL is defined as the ratio between the length of the residual force vector and the total (accumulated) external force vector acting at the current step. It can be set by the user in the set up mode. Typical value of CTOL is 0.01
(1% of the external forces acting at any given step).

11. If $\| \psi \| <$CTOL, current increment has converged. Go to step 12.

12. If all the load steps are done, stop. Otherwise, set $\{\Delta P\}$ = incremental loads to be applied at the next increment and repeat steps 1 through 9.

To increase the speed and accuracy of the nonlinear analysis, first, the applied loads are segmented into some loads termed supsteps and then, in each supstep Newton-Raphson method is used.

4. Optimum design problem formulation

It is shown that consideration of nonlinear behavior in the optimum design of structures not only provides more realistic results, but also produces lighter structures [4, 5]. Nonlinear structural behavior arises from a number of causes, which can be grouped into geometrical and material nonlinearity. If a structure experiences large deformations, its changing geometric con***Fig***uration can cause the structure to respond nonlinearly. Nonlinear stress-strain relationships are a common cause of material nonlinear behavior. One of the main factors that can influence a material's stress-strain properties is load history in elasto-plastic response.

The optimum design problem of nonlinear space structures can be expressed as follows:

$$\begin{aligned}&\text{Minimize}: w(X)\\&\text{Subjectto}: g_q(X) \le 0 \quad q = 1,...,m\end{aligned} \tag{9}$$

where X is the vector of design variables with some unknowns, g_q are the m inequality constrains including the side constraints. Also, $w(X)$ represents the objective function that should be minimized.

4.1. Optimization Algorithm

In structural optimization problems, where the objective function and the constraints are highly non-linear functions of the design variables, the computational effort spent in gradient calculations required by the mathematical programming algorithms is usually large. In recent years, it was found that probabilistic search algorithms are computationally efficient even if greater number of optimization cycles is needed to reach the optimum. Furthermore, probabilistic methodologies were found to be more robust in finding the global optima, due to their random search, whereas mathematical programming algorithms may be trapped into local optima. Many successful applications of evolutionary algorithms are reported in the related literatures [10-15].

In the present study, to obtain the optimum design of space structures, continuous genetic algorithm based on VSP method is used. Continuous optimization method, CVSP, require less computer effort comparing with the discrete, VSP, method.

4.2. Design variables

In this study, the design variables are cross sectional areas of the space structures as:

$$X = \{x_1,...,x_n,...,x_{ng}\} \tag{10}$$

where x_n is design area of members belonging to group n and ng is the total number of groups in the structures.

4.3. Objective function

The objective function is weight of structure which can be expressed as:

$$w(X) = \sum_{n=1}^{ng} x_n \sum_{i=1}^{nm} \gamma_i l_i \tag{11}$$

where γ_i , l_i are the weight density and length of member i; nm is the number of members in group n.

4.4. Design constraints

Design constraints are divided into some groups including the structural, stability and geometrical constraints. The structural constraints are defined as:

$$g_{dj}(X) = \delta_j - \delta_{ju} \le 0 \qquad j = 1,..., p \tag{12}$$

$$g_{sk}(X) = \sigma_k - \sigma_{kcr} \le 0 \quad k = 1,..., ne \tag{13}$$

δ_j is the displacement of joint j and δ_{ju} is its upper bound; p is the number of restricted displacement. σ_k is the stress of member k and σ_{kcr} is its critical value; ne is the total number of members. The critical stress for a tension member is simply taken as the yield stress, σ_y, of steel. The critical stress of compression member is obtained according to buckling stress as:

$$\sigma_{cr} = \pi^2 E/\lambda^2 \qquad \lambda > C_c \tag{14}$$

$$\sigma_{cr} = \sigma_y (1\text{-}\lambda^2/2C_c^2) \quad \lambda < C_c \tag{15}$$

where λ is the slenderness ratio of the member and $C_c = \sqrt{2\pi^2 E/\sigma_y}$.

The constraint ensuring the stability of the space structure during the optimization process is as:

$$g_{ls}(X) = f_a - f_u \le 0 \tag{16}$$

where f_a is applied load factor and f_u is ultimate load factor determined using nonlinear analysis.

The geometric constrain is defined to limit design variable as:

$$g_{gn}^l(X) = x_{nl} - x_n \le 0 \quad , \quad n = 1,..., ng \tag{17}$$

$$g_{gn}^u(X) = x_n - x_{nu} \le 0 \quad , \quad n = 1,..., ng \tag{18}$$

where x_{nl} and x_{nu} are lower and upper bound on design variable x_n.

5. Numerical examples

The design algorithm presented is used to optimize two space structures where geometrical and material nonlinearity take into account. Optimum solutions obtained by ordinary GA and CVSP methods are compared with those of other researchers that considered linear and nonlinear behavior in optimum design of space structures [4, 5].

5.1. 25-bar space truss

The design of the space truss depicted in Fig. 1 is considered as the first example. As shown in Fig.1 the cross sectional areas of members are collected into eight groups. The loading are given in Table 3.

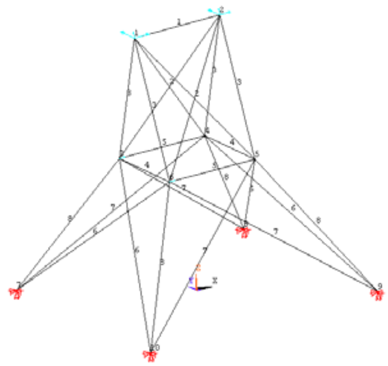

Fig. 1. 25 bar space truss.

The displacements of joint 1 and 2 in the x and y directions are restricted to 10 mm. The yield stress and modulus of elasticity are 240 N/mm^2 and 207 kN/mm^2, respectively. Minimum and maximum size constraints are chosen to be 200 mm^2 and 2000 mm^2, respectively. Optimum solutions obtained by this work and those of obtained by Saka [4] are compared in Table 4.

Table 3. 25-bar space truss loading.

Joint No.	*Loading (kN)*		
	x	y	z
1	80	120	30
2	60	100	30
3	30	0	0
6	30	0	0

Table 4. Comparison of optimum solutions obtained by various methods.

Variable No.	*Optimum Design (mm)*			
	Linear behavior	Nonlinear behavior		
	Saka and Ulker [4] & this work	Saka and Ulker [4]	GA	CVSP
1	200	200	369.74	398
2	1640	750	1151.67	678
3	1568	1312	1003.09	1461
4	368	200	423.90	205
5	399	427	285.72	507
6	1492	380	319.46	349
7	1496	422	720.06	309
8	1495	1715	1643.99	1785
Weight (N)	9035.01	4973.7	5453.20	4964.7

As shown, nonlinear behavior consideration can significantly reduce the total weight of the structure. It is also observed that solution found by CVSP method is more economical than that attained by GA.

5.2. 200-bar double layer grid

The space structure, shown in Fig. 2, has 200 bars. Design optimization of this structure was carried out in ref [1] involving linear behavior. In the present work, the structure is optimized by including nonlinear behavior. The cross sectional areas of members are collected into three groups. One of the groups contains the bottom layer members. Diagonal are grouped together as another one, and top layer members are collected in the third group. Simple supports are considered on joints 1, 6, 31 and 36. The top layer joints are subjected to vertical loading of 13.5 kN and the vertical displacements of these joints are restricted to 20 mm. The yield stress and modulus of elasticity are 240 N/mm^2 and 210kN/mm^2, respectively. Minimum and maximum size constraints are chosen to be 200 mm^2 and 2000 mm^2, respectively.

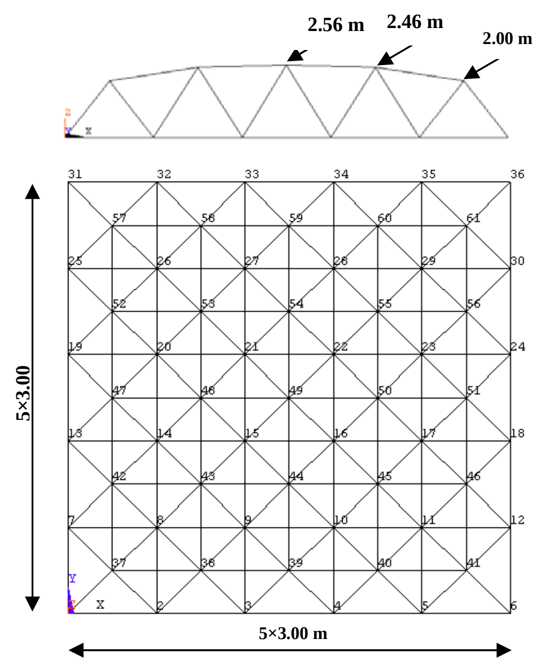

Fig. 2. 200-bar double layer grid.

Optimum solutions obtained by various methods are compared in Table 5.

Table 5. Comparison of optimum solutions obtained by various methods.

Variable No.	*Optimum Design (mm)*		
	Linear behavior	Nonlinear behavior	
	Togan and Daloglu [1] & this work	GA	CVSP
1	819.0	320.50	320.50
2	1552.0	632.33	632.33
3	1552.0	837.59	557.70
Weight (kN)	62.869	30.421	23.690

It is observed that the solutions found by considering nonlinear behavior are more economical than solution is attained by linear behavior and CVSP method can reach to better optimum design than ordinary genetic algorithm.

6. Conclusions

Optimal design of space structures involving geometrical and material nonlinearity is presented. Optimization algorithm is continuous VSP (CVSP) evolutionary algorithm. In the process of CVSP real values of design variables are incorporated in the optimization instead of discrete values. Total weight of structures is taken as objective function and the design variables are cross sectional areas of the structures. The design constraints include structural, stability and geometrical constraints. Tension and compression axial stresses are limited to their critical values and nodal displacements are restricted to its upper bounds. Stability constraint is defined to prevent from loss of overall stability of structure during the optimization process. Some illustrative examples are presented to demonstrate the effectiveness of proposed method for optimum design of space structures. The numerical results reveal that taking into account the nonlinear behavior can significantly reduce the optimum weight of structures compared with those of obtained using linear behavior.

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