Optical solitons with Sasa-Satsuma equation by *F* -expansion scheme

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This paper applies the F - expansion scheme to derive optical soliton solution to Sasa-Satsuma equation with Kerr law nonlinearity. Bright, dark and combo optical soliton solutions are derived by the aid of this integration scheme.

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1. Introduction

Optical soliton perturbation is one of the most sought after areas of research in the field of nonlinear optics that is applicable to optical fibers, couplers, metamaterials and metasurfaces. There are sevearl types of perturbation that are studied in this context using a variety of mathematical approaches that lead to meaningful and worthy results in this field [1-25]. While sevearl models are available to address this, the current paper will consider the well-known Sasa-Satsuma equation (SSE) that is basically the perturbed nonlinear Schrödinger's equation (NLSE) that is the global model studied all over in the context of fiber optic dynamics. Here all of the perturbations are of Hamiltonian type thus rendering SSE completely integrable for Kerr law nonlinearity. This paper is thus going to take a fresh look at the model using one of the well-known mathematical techniques that is the F-expansion scheme. It will reveal several forms of soliton solutions namely bright, dark, singular as well as combo-type solitons. The results are all derived and discussed in the subsequent sections.

2. Governing model

The dynamic model for optical soliton perturbation namely the SSE, with Hamiltonian type perturbations, that stems out of NLSE is given by [2, 5, 23, 24]:

$$\begin{gathered} i\frac{\partial \psi}{\partial z}+s_1\frac{\partial^2 \psi}{\partial t^2}+i\varepsilon s_3\frac{\partial^3 \psi}{\partial t^3}+s_2\left|\psi\right|^2\psi+ \\ i\varepsilon s_4\left|\psi\right|^2\frac{\partial \psi}{\partial t}+i\varepsilon s_5\psi\frac{\partial\left|\psi\right|^2}{\partial t}=0, \end{gathered} \tag{1}$$

where $\psi(z,t)$ is a complex variable and $\varepsilon, s_1, s_2, s_3, s_4, s_5,$ are real parameters. In (1), the first term represents the evolution of optical pulses, while the second term, namely the coefficient of s_1 is the group-velocity dispersion (GVD) and the coefficient of s_3 represents self-phase modulation (SPM) with Kerr law nonlinearity. From the perturbation terms s_3 gives the coefficient of third order dispersion (3OD) and finally s_4 and s_5 are due to self-steepening and nonlinear dispersion. The perturbation parameter ε accounts for quasi-monochromaticity, a factor that accounts for smallness of the perturbative effects.

By substituting complex function $\psi(z,t)=u(z,t)+iv(z,t)$, where $u(z,t)$ and $v(z,t)$ are real functions of z and t, into equation (1), we obtained a coupled nonlinear partial differential equations. We discuss the existence of a Lagrangian and the invariant variational principle for SSE. SSE is reduced to a system of a coupled second order equation and is expressed in the following form:

$$G(u,v)=\frac{\partial u}{\partial z}+s_1\frac{\partial^2 v}{\partial t^2}+s_2\left(vu^2+v^3\right)+\varepsilon s_3\frac{\partial^3 u}{\partial t^3}+ \varepsilon s_4\left(u^2+v^2\right)\frac{\partial u}{\partial t}+2\varepsilon s_5\left(u^2\frac{\partial u}{\partial t}+uv\frac{\partial v}{\partial t}\right),$$

$$H(u,v)=-\frac{\partial v}{\partial z}+s_1\frac{\partial^2 u}{\partial t^2}+s_2\left(uv^2+u^3\right)- \varepsilon s_3\frac{\partial^3 v}{\partial t^3}-\varepsilon s_4\left(u^2+v^2\right)\frac{\partial v}{\partial t}-2\varepsilon s_5\left(v^2\frac{\partial v}{\partial t}+uv\frac{\partial u}{\partial t}\right). \tag{2}$$

The consistency conditions are expressed in [10, 18], furthermore, the system of split SSE satisfies above conditions, then we have a functional integral $J(u,v)$ in the following form as inidicated below:

$$J(u,v)=\frac{1}{4}\int_\Omega\left[\begin{array}{l}-2u\frac{\partial v}{\partial z}+2s_1u\frac{\partial^2 u}{\partial t^2}+s_2\left(u^2v^2+u^4\right)-\\ 2\varepsilon s_3u\frac{\partial^3 v}{\partial t^3}-\varepsilon s_4\left(u^3+uv^2\right)\frac{\partial v}{\partial t}-\\ 2\varepsilon s_5\left(uv^2\frac{\partial v}{\partial t}+u^2v\frac{\partial u}{\partial t}\right)+2v\frac{\partial u}{\partial z}+\\ 2s_1v\frac{\partial^2 v}{\partial t^2}+s_2\left(v^2u^2+v^4\right)+\\ 2\varepsilon s_3v\frac{\partial^3 u}{\partial t^3}+\varepsilon s_4\left(vu^2+v^3\right)\frac{\partial u}{\partial t}+\\ 2\varepsilon s_5\left(vu^2\frac{\partial u}{\partial t}+uv^2\frac{\partial v}{\partial t}\right)\end{array}\right]d\Gamma, \tag{3}$$

where $d\Gamma=dtdz$. By choosing the boundary on u_z and v_z to be such that the boundary terms vanish, we get the following Lagrangian L:

$$L(u,v)=\frac{1}{4}\left[\begin{array}{l}2\left(v\frac{\partial u}{\partial z}-u\frac{\partial v}{\partial z}\right)+2s_1\left(\left|\frac{\partial u}{\partial t}\right|^2+\left|\frac{\partial v}{\partial t}\right|^2\right)+\\ s_2\left(v^4+2u^2v^2+u^4\right)+\\ 2\varepsilon s_3\left(\frac{\partial v}{\partial t}\frac{\partial^2 u}{\partial t^2}-\frac{\partial u}{\partial t}\frac{\partial^2 v}{\partial t^2}\right)+\\ \varepsilon s_4\left(\left(vu^2+v^3\right)\frac{\partial u}{\partial t}-\left(u^3+uv^2\right)\frac{\partial v}{\partial t}\right)\end{array}\right] \tag{4}$$

It is deemed necessary to check our calculations, we use L in the Euler-Lagrange equations:

$$\frac{\partial L}{\partial u}-\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial u_t}\right)-\frac{\partial}{\partial t^2}\left(\frac{\partial L}{\partial u_{tt}}\right)-\frac{\partial}{\partial z}\left(\frac{\partial L}{\partial u_z}\right)=0,$$

$$\frac{\partial L}{\partial v}-\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial v_t}\right)-\frac{\partial}{\partial t^2}\left(\frac{\partial L}{\partial v_{tt}}\right)-\frac{\partial}{\partial z}\left(\frac{\partial L}{\partial v_z}\right)=0, \tag{5}$$

which lead to equation (1). In order to solve SSE, we assume that

$$\psi(z,t)=\phi(\xi)\exp(i(\kappa t-\Omega z)),\qquad \xi=(kt-\omega z), \tag{6}$$

where κ and Ω are real parameters, $\phi(z,t)$ is the linear phase shift function with k and ω are the normalized wave vector and frequency. By substituting from equation (6) into equation (1), we obtained real and imaginary parts of DNLS equation as follows:

$$\varphi(\xi)\left(s_3\kappa^3\varepsilon-s_1\kappa^2+\omega\right)+\varphi(\xi)^3\left(s_2-s_4\kappa\varepsilon\right)+ k^2\left(s_1-3s_3\kappa\varepsilon\right)\varphi''(\xi)=0 \tag{7}$$

$$\left(s_4+2s_5\right)k\varepsilon\varphi(\xi)^2\varphi'(\xi)-\varphi'(\xi) \left(\kappa k\left(3s_3\kappa\varepsilon-2s_1\right)+\Omega\right)+s_3k^3\varepsilon\varphi^{(3)}(\xi)=0, \tag{8}$$

By integrating equation (8) and taking the constant of integration to be zero, in order to make the obtained equation and equation (7) compatible, we get the values of κ and Ω as follows:

$$\left\{\kappa\to\frac{s_1-3s_3k\varepsilon}{s_3\varepsilon},\Omega\to\frac{-s_3\omega\varepsilon+8s_3^2k^3\varepsilon^2-8s_1s_3k^2\varepsilon+2s_1^2k}{s_3\varepsilon}\right\};$$

$$\left\{\begin{array}{l}\kappa\to\frac{-2s_1s_3\varepsilon-3\sqrt{2}\sqrt{3s_3^4k^2\varepsilon^4-2s_1s_3^3k\varepsilon^3}+6s_3^2k\varepsilon^2}{4s_3^2\varepsilon^2},\\ \Omega\to\frac{1}{4}\left(\begin{array}{l}-22s_3k^3\varepsilon+\\ \frac{9\sqrt{2}k^2\sqrt{s_3^3k\varepsilon^3\left(3s_3k\varepsilon-2s_1\right)}}{s_3\varepsilon}+\\ 22s_1k^2-\frac{4s_1^2k}{s_3\varepsilon}-4\omega-\\ \frac{6\sqrt{2}s_1k\sqrt{s_3^3k\varepsilon^3\left(3s_3k\varepsilon-2s_1\right)}}{s_3^2\varepsilon^2}\end{array}\right)\end{array}\right\}$$

$$\left\{\begin{array}{l}\kappa\to\frac{-2s_1s_3\varepsilon+3\sqrt{2}\sqrt{3s_3^4k^2\varepsilon^4-2s_1s_3^3k\varepsilon^3}+6s_3^2k\varepsilon^2}{4s_3^2\varepsilon^2},\\ \Omega\to\frac{1}{4}\left(\begin{array}{l}-22s_3k^3\varepsilon-\frac{9\sqrt{2}k^2\sqrt{s_3^3k\varepsilon^3\left(3s_3k\varepsilon-2s_1\right)}}{s_3\varepsilon}+\\ 22s_1k^2-\frac{4s_1^2k}{s_3\varepsilon}-4\omega+\\ \frac{6\sqrt{2}s_1k\sqrt{s_3^3k\varepsilon^3\left(3s_3k\varepsilon-2s_1\right)}}{s_3^2\varepsilon^2}\end{array}\right)\end{array}\right\} \tag{9}$$

Then equations (8) and (9) are reduced to ordinary differential equations as follows

$$\delta_1\phi(\xi)+\delta_2\phi(\xi)^3+\delta_3\phi''(\xi)=0, \tag{10}$$

where

$$\delta_1 = s_3k^3\varepsilon - s_1k^2+\omega,$$
$$\delta_2 = \frac{(s_4+2s_5)(s_1-3s_3k\varepsilon)}{3s_3},$$
$$\delta_3 = \frac{(s_1-3s_3k\varepsilon)^3}{s_3^2\varepsilon^2};$$

$$\delta_1 = s_3k^3\varepsilon - s_1k^2+\omega,$$

$$\delta_2 = \frac{(s_4+2s_5)\begin{pmatrix}2s_3\varepsilon(3s_3k\varepsilon-s_1)\mp\\ 3\sqrt{2}\sqrt{s_3^3k\varepsilon^3(3s_3k\varepsilon-2s_1)}\end{pmatrix}}{12s_3^2\varepsilon},$$

$$\delta_3 = \frac{(s_1-3s_3k\varepsilon)\begin{pmatrix}\pm3\sqrt{2}\sqrt{s_3^3k\varepsilon^3(3s_3k\varepsilon-2s_1)}+\\ 2s_3\varepsilon(s_1-3s_3k\varepsilon)\end{pmatrix}^2}{16s_3^4\varepsilon^4}$$

3. Soliton solutions

We implement the improved auxiliary equation mapping method with to retrieve soliton solutions of SSE. The advantage of this method is that it proves us with a new and more general traveling wave solutions for many nonlinear evolution equations, it provides a variety of soliton solutions. SSE dynamical equation has general solution in series form as given by:

$$\varphi(\xi)=\sum_{i=0}^{n}a_iF^i(\xi)+\sum_{i=-1}^{-n}b_{-i}F^i(\xi)+ \sum_{i=2}^{n}c_iF^{i-2}(\xi)F'(\xi)+\sum_{i=-1}^{-n}d_{-i}F^i(\xi)F'(\xi), \tag{11}$$

where $a_0,a_1,........a_n,b_1,........b_n,c_2,........c_n,d_1,........d_n$ are arbitrary constants, the value of $F(\theta)$ and $F'(\theta)$ satisfy the following two cases

$$F'(\xi)=\sqrt{\alpha_2F^2(\xi)+\alpha_3F^3(\xi)+\alpha_4F^4(\xi)};$$
$$F'(\xi)=\sqrt{\alpha_2F^2(\xi)+\alpha_4F^4(\xi)+\alpha_6F^6(\xi)}, \tag{12}$$

where $\xi = kt-\omega z$ and $\alpha_i, i=2,3,4,6$ are arbitrary constants and k and ω are wave length and frequency respectively. The constant positive integer m is determined later. By using Eqs. (11) and (12) into Eq. (10), we obtained algebraic system of equations.

Balancing the highest order nonlinear term and the highest order linear partial derivative term in equation (10) yields the value of $m=2$. The solution of equation (10) takes the form

$$\varphi(\xi)=a_0+a_1F(\xi)+a_2F^2(\xi)+\frac{b_1}{F(\xi)}+ \frac{b_2}{F^2(\xi)}+c_2F'(\xi)+d_1\frac{F'(\xi)}{F(\xi)}+d_2\frac{F'(\xi)}{F^2(\xi)}. \tag{13}$$

By inserting equation (13) into equation (10) and collecting coefficients of $F'^j(\theta)F^i(\theta)(j=0,1;\ i=0,1,2,3,..\ .n)$, and setting each of the coefficients to zero yields an over-determined system of algebraic equations. Upon solving the system, parameters $a_0,a_1,a_2,b_1,b_2,\ c_2,d_1,d_2$ can be determined as:

Families of coefficients I

$$a_0=0,\quad a_1=\sqrt{\frac{\alpha_4\left(k^3s_3(-\varepsilon)+k^2s_1-\omega\right)}{2\delta_3}},$$
$$d_1=\sqrt{\frac{k^3s_3(-\varepsilon)+k^2s_1-\omega}{2\delta_3}}$$

$$a_2=b_1=c_2=b_2=d_2=0,$$
$$\alpha_2=\frac{2(s_4+2s_5)(s_1-3ks_3\varepsilon)}{3s_3\left(k^3s_3\varepsilon-k^2s_1+\omega\right)},\quad \delta_3=\frac{(s_1-3ks_3\varepsilon)^3}{s_3^2\varepsilon^2} \tag{14}$$

Families of coefficients II

$$a_0=\alpha_3\sqrt{\frac{(s_4+2s_5)(2k^3s_3\varepsilon-\omega)}{\delta_3\left(3\alpha_3^2k^2s_3+8\alpha_4(s_4+2s_5)\right)}},$$
$$a_1=4\alpha_4\sqrt{\frac{(s_4+2s_5)(2k^3s_3\varepsilon-\omega)}{\delta_3\left(3\alpha_3^2k^2s_3+8\alpha_4(s_4+2s_5)\right)}}$$

$$a_2=b_1=d_1=c_2=b_2=d_2=0,$$
$$\alpha_2=\frac{\alpha_3^2}{4\alpha_4},\quad \delta_3=\frac{(s_1-3ks_3\varepsilon)^3}{s_3^2\varepsilon^2}$$

$$s_1 = \frac{3s_3\left(\alpha_3^2\left(k^3 s_3\varepsilon + \omega\right) + 8\alpha_4 k\left(s_4 + 2s_5\right)\varepsilon\right)}{3\alpha_3^2 k^2 s_3 + 8\alpha_4\left(s_4 + 2s_5\right)}, \quad (15)$$

where the sufficient conditions of stability of solutions as follows:

$$\frac{\alpha_4\left(k^3 s_3(-\varepsilon) + k^2 s_1 - \omega\right)}{2\delta_3} > 0, \qquad \delta_3 \neq 0,$$

$$\alpha_4 \neq 0, \quad s_3\left(k^3 s_3\varepsilon - k^2 s_1 + \omega\right) \neq 0,$$

$$\frac{\left(s_4 + 2s_5\right)\left(2k^3 s_3\varepsilon - \omega\right)}{\delta_3\left(3\alpha_3^2 k^2 s_3 + 8\alpha_4\left(s_4 + 2s_5\right)\right)} > 0,$$

$$s_3^2\varepsilon^2 \neq 0, \quad 3\alpha_3^2 k^2 s_3 + 8\alpha_4\left(s_4 + 2s_5\right) \neq 0,$$

Families of solutions I:

By substituting from equation (14) into equation (13), we obtain soliton solutions of equation (1):

Case I

$$\psi_{11}(z,t) = \left(\frac{\sqrt{\alpha_2}\,\mu\sqrt{k^3 s_3(-\varepsilon) + k^2 s_1 - \omega}\,\mathrm{sech}^2\left(\frac{1}{2}\sqrt{\alpha_2}(kz - t\omega) + \xi_0\right)}{2\sqrt{2\beta_3}\left(\mu\tanh\left(\frac{1}{2}\sqrt{\alpha_2}(kz - t\omega) + \xi_0\right) + 1\right)} - \frac{\alpha_2\sqrt{\alpha_4\left(k^3 s_3(-\varepsilon) + k^2 s_1 - \omega\right)}\left(\mu\tanh\left(\frac{1}{2}\sqrt{\alpha_2}(kz - t\omega) + \xi_0\right) + 1\right)}{\alpha_3\sqrt{2\delta_3}} \right) \exp(i(\kappa t - \Omega z)) \quad (16)$$

where $\alpha_3^2 = 4\alpha_2\alpha_4$, $\alpha_2 > 0$ and the value of $\mu = \pm 1$.

$$\psi_{12}(z,t) = \frac{1}{2}\sqrt{\frac{\alpha_2\left(k^3 s_3(-\varepsilon) + k^2 s_1 - \omega\right)}{2\delta_3}} \left(\frac{\mu\sinh\left(\sqrt{\alpha_2}(kz - t\omega) + \xi_0\right)}{\cosh\left(\sqrt{\alpha_2}(kz - t\omega) + \xi_0\right) + \lambda} + 1 \right)$$

$$\frac{\mu\sqrt{\alpha_2\left(k^3 s_3(-\varepsilon) + k^2 s_1 - \omega\right)}\left(\lambda\cosh\left(\sqrt{\alpha_2}(kz - t\omega) + \xi_0\right) + 1\right)}{\sqrt{2\beta_3}\left(\cosh\left(\sqrt{\alpha_2}(kz - t\omega) + \xi_0\right) + \lambda\right)}$$

$$\left(\frac{1}{\mu\sinh\left(\sqrt{\alpha_2}(kz - t\omega) + \xi_0\right) + \cosh\left(\sqrt{\alpha_2}(kz - t\omega) + \xi_0\right) + \lambda} \right) \exp(i(\kappa t - \Omega z)) \quad (17)$$

where $\alpha_3 = -\sqrt{4\alpha_2\alpha_4}$; $\alpha_2 > 0$; $\alpha_4 > 0$ and the value of $(\mu, \lambda) = (1,1),\ (1,-1),\ (-1,-1),\ (-1,1)$.

$$\psi_{13}(x,y,t) = \frac{\mu\sqrt{\alpha_2\left(k^3 s_3(-\varepsilon) + k^2 s_1 - \omega\right)}}{\sqrt{2\delta_3}\left(\sinh\left(\sqrt{\alpha_2}(kz - t\omega) + \xi_0\right) + p\right)}$$

$$\frac{\lambda\left(-\sqrt{p^2+1}\right)\cosh\left(\sqrt{\alpha_2}(kz - t\omega) + \xi_0\right) + p\sinh\left(\sqrt{\alpha_2}(kz - t\omega) + \xi_0\right) - 1}{\mu\cosh\left(\sqrt{\alpha_2}(kz - t\omega) + \xi_0\right) + \sinh\left(\sqrt{\alpha_2}(kz - t\omega) + \xi_0\right) + \lambda\mu\sqrt{p^2+1} + p}$$

$$-\frac{\alpha_2\sqrt{\alpha_4\left(k^3 s_3(-\varepsilon) + k^2 s_1 - \omega\right)}}{\alpha_3\sqrt{2\beta_3}} \left(\frac{\mu\left(\cosh\left(\sqrt{\alpha_2}(kz - t\omega) + \xi_0\right) + \lambda\sqrt{p^2+1}\right)}{\sinh\left(\sqrt{\alpha_2}(kz - t\omega) + \xi_0\right) + p} + 1 \right)$$

$$\cdot\exp(i(\kappa t - \Omega z)) \quad (18)$$

where μ and λ are any choices of 1 or -1, and p is an arbitrary constant, $\alpha_2 > 0$.

Case II

$$\psi_{21}(z,t) = \frac{1}{\alpha_3}\sqrt{\frac{\left(s_4 + 2s_5\right)\left(2k^3 s_3\varepsilon - \omega\right)}{\delta_3\left(3\alpha_3^2 k^2 s_3 + 8\alpha_4\left(s_4 + 2s_5\right)\right)}}$$

$$\left(\alpha_3^2 - 4\alpha_2\alpha_4\left(\mu\tanh\left(\frac{1}{2}\sqrt{\alpha_2}(kz - t\omega) + \xi_0\right) + 1\right)\right) \exp(i(\kappa t - \Omega z)) \quad (19)$$

$$\psi_{22}(z,t) = \sqrt{\frac{\left(s_4 + 2s_5\right)\left(2k^3 s_3\varepsilon - \omega\right)}{\delta_3\left(3\alpha_3^2 k^2 s_3 + 8\alpha_4\left(s_4 + 2s_5\right)\right)}}$$

$$\left(\alpha_3+2\sqrt{\alpha_2\alpha_4}\left(\frac{\mu\sinh\left(\sqrt{\alpha_2}(kz-t\omega)+\xi_0\right)}{\cosh\left(\sqrt{\alpha_2}(kz-t\omega)+\xi_0\right)+\lambda}+1\right)\right) \quad (20)$$
$$\exp(i(\kappa t-\Omega z))$$

$$\psi_{23}(z,t)=\frac{1}{\alpha_3}\sqrt{\frac{(s_4+2s_5)(2k^3s_3\varepsilon-\omega)}{\delta_3\left(3\alpha_3^2k^2s_3+8\alpha_4(s_4+2s_5)\right)}}$$

$$\left(\alpha_3^2-4\alpha_2\alpha_4\left(\frac{\mu\left(\cosh\left(\sqrt{\alpha_2}(kz-t\omega)+\xi_0\right)+\lambda\sqrt{p^2+1}\right)}{\sinh\left(\sqrt{\alpha_2}(kz-t\omega)+\xi_0\right)+p}+1\right)\right)$$
$$\exp(i(\kappa t-\Omega z)) \quad (21)$$

Families of solutions II:

By inserting from equation (15) into equation (13), we obtain the following soliton solutions of SSE:

Case I

$$\psi_{31}(z,t)=\sqrt{\frac{(s_4+2s_5)(2k^3s_3\varepsilon-\omega)}{\delta_3\left(3\alpha_4^2k^2s_3+2\alpha_6(s_4+2s_5)\right)}}$$

$$\left(\frac{8\sqrt{\frac{\alpha_2^2}{\alpha_4^2-4\alpha_2\alpha_6}}\alpha_6+\alpha_4\left(\cosh\left(2\left(\sqrt{\alpha_2}\theta+\xi_0\right)\right)-\sqrt{\frac{\alpha_4^2}{\alpha_4^2-4\alpha_2\alpha_6}}\right)}{\cosh\left(2\left(\sqrt{\alpha_2}\theta+\xi_0\right)\right)-\sqrt{\frac{\alpha_4^2}{\alpha_4^2-4\alpha_2\alpha_6}}}\right)$$
$$\exp(i(\kappa t-\Omega z)) \quad (22)$$

such that $\alpha_4^2-4\alpha_2\alpha_6$ and $\alpha_2>0$.

$$\psi_{32}(z,t)=\sqrt{\frac{(s_4+2s_5)(2k^3s_3\varepsilon-\omega)}{\delta_3\left(3\alpha_4^2k^2s_3+2\alpha_6(s_4+2s_5)\right)}}$$

$$\left(\frac{-\alpha_2\sqrt{\frac{\alpha_4^2-4\alpha_2\alpha_6}{\alpha_2^2}}\alpha_4\sin\left(2\sqrt{-\alpha_2}\theta+\xi_0\right)+\alpha_4^2-8\alpha_2\alpha_6}{\alpha_4-\alpha_2\sqrt{\frac{\alpha_4^2-4\alpha_2\alpha_6}{\alpha_2^2}}\sin\left(2\sqrt{-\alpha_2}\theta+\xi_0\right)}\right) \quad (23)$$
$$\exp(i(\kappa t-\Omega z)),$$

and $\alpha_4^2-4\alpha_2\alpha_6$ and $\alpha_2<0$.

Case II

$$\psi_{41}(z,t)=\frac{\sqrt{2}\left(\sqrt{\alpha_2}\sinh\left(2\left(\sqrt{\alpha_2}\theta+\xi_0\right)\right)+2\sqrt{\alpha_6}\sqrt{\frac{\alpha_2^2}{\alpha_4^2-4\alpha_2\alpha_6}}\right)}{\sqrt{\delta_3}\left(\cosh\left(2\left(\sqrt{\alpha_2}\theta+\xi_0\right)\right)-\sqrt{\frac{\alpha_4^2}{\alpha_4^2-4\alpha_2\alpha_6}}\right)}$$

$$\sqrt{k^3s_3(-\varepsilon)+k^2s_1-\omega}\exp(i(\kappa t-\Omega z)), \quad (24)$$

such that $k^3s_3(-\varepsilon)+k^2s_1-\omega>0$ $\alpha_4^2-4\alpha_2\alpha_6$ and $\alpha_2>0$.

$$\psi_{42}(z,t)=\frac{\sqrt{2}\alpha_2\left(\sqrt{-\alpha_2}\sqrt{\frac{\alpha_4^2-4\alpha_2\alpha_6}{\alpha_2^2}}\cos\left(2\sqrt{-\alpha_2}\theta+\xi_0\right)-2\sqrt{\alpha_6}\right)}{\sqrt{\delta_3}\left(\alpha_4-\sqrt{\alpha_4^2-4\alpha_2\alpha_6}\sin\left(2\sqrt{-\alpha_2}\theta+\xi_0\right)\right)}$$

$$\sqrt{k^3s_3(-\varepsilon)+k^2s_1-\omega}\exp(i(\kappa t-\Omega z)), \quad (25)$$

where $\alpha_4^2-4\alpha_2\alpha_6$ and $\alpha_2<0$.

The solutions list (16)-(25) represents bright, dark, singular as well as different forms of combo optical solitons as well as singular periodic solutions. Thus, a novel variety of soliton solutions are reported here, that are derived using F-expansion scheme for SSE, for the first time.

4. Conclusions

This paper secured a variety of optical soliton solutions for the SSE by the aid of F-expansion scheme. The respective sufficient conditions of integrability are also listed for these solitons to exist. The results of this paper are indeed very encouraging that serves as a way to further future research in this field. The type of nonlinearity will be extended and generalized to higher order and additional perturbation terms will be incorporated. These will lead to additional soliton solutions provided the extended and or generalized model passes the Painleve test of integrability. Such results are awaited at this time and will be reported in future.

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