Optical solitons with resonant nonlinear Schrödinger's equation using G'/G-expansion scheme

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This paper implements G'/G-expansion scheme to retrieve soliton solution to the resonant nonlinear Schrödinger's equation. Both cubic form and power law nonlinearity are considered in this paper. The results appear with constraint conditions that guarantee the existence of these solitons. As a byproduct, singular periodic solutions are revealed.

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1. Introduction

Optical solitons is a major area of research in nonlinear fiber optics. The governing equation is the nonlinear Schrödinger's equation (NLSE) that appears with numerous forms of nonlinearity [1-15]. There are several variational forms of NLSE that are studied in this context. One of them is Schrödinger-Hirota equation that is studied to describe dispersive optical solitons. Another form is chiral NLSE that is studied in the context of quantum Hall effect. This paper is going to address resonant NLSE (RNLSE) that is studied in the context of Madelung fluids.

There are two nonlinear forms where RNLSE will be addressed in this paper. These are cubic nonlinearity, also known as Kerr law nonlinearity, and power law nonlinearity, which collapses to Kerr law when the power law nonlinearity parameter is set to unity. The cases for parabolic law and dual-power law are already addressed earlier [4]. In order to keep it on a generalized perspective, RNLSE is considered with time-dependent coefficients in this paper. The integration algorithm that will be implemented here is G'/G-expansion scheme. This will lead to dark and singular solitons solutions. As a byproduct, singular periodic solutions are revealed that are also listed in this paper.

2. Succinct overview of G'/G-expansion

The algorithm for this integration scheme is pretty much well known to the scientific community. Nevertheless it is worthwhile to describe this scheme succinctly. The main algorithmic steps are as follows [9]:

Step-1: Suppose a nonlinear partial differential equation (PDE) with time-dependent coefficients

$$P(u, u_t, u_x, u_{xx}, u_{xt}, u_{tt}...) = 0 \tag{1}$$

can be converted to an ordinary differential equation (ODE)

$$Q(U, U', U'', U'''...) = 0 \tag{2}$$

using a traveling wave variable $u(x,t) = U(\xi)$, $\xi = x - v(t)t$, where $U = U(\xi)$ is an unknown function, Q is a polynomial in the variable U and its derivatives. If all terms contain derivatives, then Eq. (2) is integrated where integration constants are considered zeros.

Step-2: Suppose that the solution of ODE (2) can be expressed by a polynomial in G'/G as follows:

$$U(\xi) = \sum_{l=0}^{N} a_l \left(\frac{G'(\xi)}{G(\xi)} \right)^l \tag{3}$$

where a_l are real constants with $a_N \neq 0$ and N is a positive integer to be determined. The function $G(\xi)$ is the solution of the auxiliary linear ordinary differential equation

$$G''(\xi) + \lambda G'(\xi) + \mu G(\xi) = 0 \tag{4}$$

where λ and μ are real constants to be determined.

Step-3: Determining N, it can be accomplished by balancing the linear term of the highest-order derivatives with the highest-order nonlinear term in Eq. (2).

Step-4: Substituting the general solution (3) together with (4) into Eq. (2) yields an algebraic equation involving powers of G'/G. Equating the coefficients of each power of G'/G to zero gives a system of algebraic equations for a_l, λ, μ and $v(t)$. Then one can solve the system with the aid of a computer algebra system, such as Mathematica, to determine these constants. Depending on the sign of the discriminant $\Delta = \lambda^2 - 4\mu$, solutions of Eq. (2) can be obtained. So, we can obtain exact solutions of the given Eq. (1).

3 Application to RNLSE

In this section, we shall apply G'/G-expansion scheme to solve the RNLSE with time-dependent coefficients [4, 6]. The general form of RNLSE with an arbitrary nonlinear form is [1]:

$$i\psi_t + \alpha(t)\psi_{xx} + \beta(t)F\left(|\psi|^2\right)\psi + \gamma(t)\left(\frac{|\psi|_{xx}}{|\psi|}\right)\psi = 0 \tag{5}$$

In equation (5), the first term is the linear temporal evolution, while $\alpha(t)$ is time-dependent coefficient of group-velocity dispersion (GVD) and $\beta(t)$ is time-dependent coefficient of nonlinearity. Finally, $\gamma(t)$ is quantum or Bohm potential that arises in chiral solitons with quantum Hall effect [1]. It is also seen in the context of Madelung fluid in quantum mechanics [1]. Also, the functional F meets the following technical condition:

For technical condition, F is a real-valued algebraic functional and it is necessary to have the smoothness of the complex function $F\left(|\psi|^2\right)\psi : C \mapsto C$. Considering the complex plane C as a two-dimensional linear space R^2, the function $F\left(|\psi|^2\right)\psi$ is k times continuously differentiable, so that [1]

$$F\left(|\psi|^2\right)\psi \in \bigcup_{m,n=1}^{\infty} C^k\left((-n,n)\times(-m,m); R^2\right)$$

This paper will obtain soliton solutions to (1) for Kerr law and power law nonlinearity. The cases for parabolic law and dual-power laws are reported earlier [4].

3.1 Kerr law

The Kerr law nonlinearity is the case when $F(s) = s$. This is also occasionally referred to as cubic nonlinearity. In fiber optics, this nonlinearity is also known as Kerr nonlinearity and appears when refractive index of light is intensity dependent. Additionally, this nonlinearity is studied in water waves. In dimensionless form, RNLSE for Kerr law nonlinearity is written as [7, 8]

$$i\psi_t + \alpha(t)\psi_{xx} + \beta(t)|\psi|^2\psi + \gamma(t)\left(\frac{|\psi|_{xx}}{|\psi|}\right)\psi = 0 \tag{6}$$

Under the traveling wave hypothesis

$$\psi(x,t) = U(\xi)e^{i(-\kappa x + \omega(t)t)} \tag{7-1}$$

$$\xi = x + 2\kappa\int_0^t \alpha(t)dt \tag{7-2}$$

we have

$$\left(\alpha(t)+\gamma(t)\right)U'' - \left(t\frac{d\omega(t)}{dt} + \omega(t) + \kappa^2\alpha(t)\right)U + \beta(t)U^3 = 0 \tag{8}$$

Balancing U'' with U^3 in Eq. (8), then we get $N = 1$. We then suppose that Eq. (8) has the following formal solution

$$U = a_1\left(\frac{G'}{G}\right) + a_0 \tag{9}$$

Therefore, we have

$$U'' = 2a_1\left(\frac{G'}{G}\right)^3 + 3a_1\lambda\left(\frac{G'}{G}\right)^2 + a_1(\lambda^2 + 2\mu)\left(\frac{G'}{G}\right) + a_1\lambda\mu \tag{10}$$

Using the solution procedure of the G'/G-expansion method, we obtain the system of algebraic equations as follows:

$\left(\frac{G'}{G}\right)^3$ Coeff.:

$$a_1\left(2\left(\alpha(t)+\gamma(t)\right) + \beta(t)a_1^2\right) = 0 \tag{11}$$

$\left(\frac{G'}{G}\right)^2$ Coeff.:

$$a_1\big(\lambda(\alpha(t)+\gamma(t))+\beta(t)a_0a_1\big)=0 \tag{12}$$

$\left(\frac{G'}{G}\right)^1$ Coeff.:

$$a_1\begin{pmatrix}-t\frac{d\omega(t)}{dt}-\omega(t)\\ -\kappa^2\alpha(t)+3\beta(t)a_0^2\\ +(\lambda^2+2\mu)(\alpha(t)+\gamma(t))\end{pmatrix}=0 \tag{13}$$

$\left(\frac{G'}{G}\right)^0$ Coeff.:

$$a_0\begin{pmatrix}-t\frac{d\omega(t)}{dt}-\omega(t)\\ -\kappa^2\alpha(t)+\beta(t)a_0^2\end{pmatrix} + \mu\lambda a_1\big(\alpha(t)+\gamma(t)\big)=0 \tag{14}$$

Solving the previous system of algebraic equations, we obtain the following result

$$a_0=\pm\frac{\lambda}{2}\sqrt{-\frac{2(\alpha(t)+\gamma(t))}{\beta(t)}} \tag{15-1}$$

$$a_1=\frac{2a_0}{\lambda} \tag{15-2}$$

$$\omega(t)=-\frac{1}{t}\int_0^t\begin{Bmatrix}\alpha(t)\kappa^2\\ +\frac{1}{2}(\alpha(t)+\gamma(t))(\lambda^2-4\mu)\end{Bmatrix}dt \tag{15-3}$$

where κ, λ and μ are arbitrary real constants. The solution of Eq. (5) corresponding to (15) has the following cases

Case-1: If $\Delta=\lambda^2-4\mu>0$, we obtain

$$\psi(\xi)=\pm\sqrt{-\frac{(\alpha(t)+\gamma(t))(\lambda^2-4\mu)}{2\beta(t)}}\times\left\{\frac{\left(C_1\sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)+C_2\cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)\right)}{\left(C_1\cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)+C_2\sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)\right)}\right\}\times e^{i\left(-\kappa x-\int_0^t\left\{\alpha(t)\kappa^2+\frac{1}{2}(\alpha(t)+\gamma(t))(\lambda^2-4\mu)\right\}dt\right)} \tag{16}$$

If we set $C_2=0$, $C_1\neq 0$

$$\psi_1(x,t)=\pm\sqrt{-\frac{(\alpha(t)+\gamma(t))(\lambda^2-4\mu)}{2\beta(t)}}\times\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(x+2\kappa\int_0^t\alpha(t)dt)\right)\times e^{i\left(-\kappa x-\int_0^t\left\{\alpha(t)\kappa^2+\frac{1}{2}(\alpha(t)+\gamma(t))(\lambda^2-4\mu)\right\}dt\right)} \tag{17}$$

If we set $C_1=0$, $C_2\neq 0$, we obtain

$$\psi_2(x,t)=\pm\sqrt{-\frac{(\alpha(t)+\gamma(t))(\lambda^2-4\mu)}{2\beta(t)}}\times\coth\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(x+2\kappa\int_0^t\alpha(t)dt)\right)\times e^{i\left(-\kappa x-\int_0^t\left\{\alpha(t)\kappa^2+\frac{1}{2}(\alpha(t)+\gamma(t))(\lambda^2-4\mu)\right\}dt\right)} \tag{18}$$

Case-2: If $\Delta=\lambda^2-4\mu<0$, we obtain

$$\psi(\xi)=\pm\sqrt{-\frac{\left(\alpha(t)+\gamma(t)\right)\left(4\mu-\lambda^2\right)}{2\beta(t)}}\times\left\{\frac{\left(\begin{array}{l}-C_1\sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)\\+C_2\cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)\end{array}\right)}{\left(\begin{array}{l}C_1\cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)\\+C_2\sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)\end{array}\right)}\right\}\times e^{i\left(-\kappa x-\int_0^t\left\{\begin{array}{l}\alpha(t)\kappa^2\\+\frac{1}{2}(\alpha(t)+\gamma(t))(\lambda^2-4\mu)\end{array}\right\}dt\right)} \quad (19)$$

If we set $C_2=0$, $C_1\neq 0$, we obtain

$$\psi_3(x,t)=\pm\sqrt{-\frac{\left(\alpha(t)+\gamma(t)\right)\left(4\mu-\lambda^2\right)}{2\beta(t)}}\times\tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(x+2\kappa\int_0^t\alpha(t)dt)\right)\times e^{i\left(-\kappa x-\int_0^t\left\{\begin{array}{l}\alpha(t)\kappa^2\\+\frac{1}{2}(\alpha(t)+\gamma(t))(\lambda^2-4\mu)\end{array}\right\}dt\right)} \quad (20)$$

If we set $C_1=0$, $C_2\neq 0$, we obtain

$$\psi_4(x,t)=\pm\sqrt{-\frac{\left(\alpha(t)+\gamma(t)\right)\left(4\mu-\lambda^2\right)}{2\beta(t)}}\times\cot\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(x+2\kappa\int_0^t\alpha(t)dt)\right)\times e^{i\left(-\kappa x-\int_0^t\left\{\begin{array}{l}\alpha(t)\kappa^2\\+\frac{1}{2}(\alpha(t)+\gamma(t))(\lambda^2-4\mu)\end{array}\right\}dt\right)} \quad (21)$$

The solutions given by (17) and (18) are dark and singular solitons respectively that exist provided

$$\beta(t)\left(\alpha(t)+\gamma(t)\right)>0 \quad (22)$$

while the solutions (20) and (21) are singular periodic structures that exist if

$$\beta(t)\left(\alpha(t)+\gamma(t)\right)<0 \quad (23)$$

3.2 Power law

The power law nonlinearity arises when $F(s)=s^n$, where the parameter n accounts for nonlinearity parameter. This kind of law appears in the context of plasma physics, turbulence theory and occasionally in nonlinear fiber optics. It needs to be however noted that one must have

$$0<n<2 \quad (24)$$

in order to avoid self-focusing singularity and soliton collapse in optics [7, 8]. For power law nonlinearity, the R-NLSE takes the form [7, 8]

$$i\psi_t+\alpha(t)\psi_{xx}+\beta(t)|\psi|^{2n}\psi+\gamma(t)\left(\frac{|\psi|_{xx}}{|\psi|}\right)\psi=0 \quad (25)$$

For searching the one-soliton solution for the above model, we use the same wave transformation

$$\psi(x,t)=U(\xi)e^{i(-\kappa x+\omega(t)t)} \quad (26\text{-}1)$$

$$\xi=x+2\kappa\int_0^t\alpha(t)dt \quad (26\text{-}2)$$

Substituting Eq. (26) into Eq. (25), we obtain ordinary differential equation:

$$\left(\alpha(t)+\gamma(t)\right)U''-\left(t\frac{d\omega(t)}{dt}+\omega(t)+\kappa^2\alpha(t)\right)U+\beta(t)U^{2n+1}=0 \quad (27)$$

To obtain an analytic solution, we use the transformation $U=V^{\frac{1}{2n}}$ in Eq. (27) to find

$$\left(\alpha(t)+\gamma(t)\right)\left((1-2n)(V')^2+2nVV''\right)-4\left(t\frac{d\omega(t)}{dt}+\omega(t)+\kappa^2\alpha(t)\right)n^2V^2+4\beta(t)n^2V^3=0 \quad (28)$$

Balancing VV'' with V^3 in Eq. (28), then we get $N=2$. We then assume that Eq. (28) has the following formal solution

$$V=a_2\left(\frac{G'}{G}\right)^2+a_1\left(\frac{G'}{G}\right)+a_0 \tag{29}$$

Therefore, we have

$$\begin{aligned}V' &= -2a_1\left(\frac{G'}{G}\right)^3-(2a_2\lambda+a_1)\left(\frac{G'}{G}\right)^2\\ &\quad -(a_1\lambda+2a_2\mu)\left(\frac{G'}{G}\right)-a_1\mu\end{aligned} \tag{30}$$

$$\begin{aligned}V'' &= 6a_2\left(\frac{G'}{G}\right)^4+2(5a_2\lambda+a_1)\left(\frac{G'}{G}\right)^3\\ &\quad +(4a_2(\lambda^2+2\mu)+3a_1\lambda)\left(\frac{G'}{G}\right)^2\\ &\quad +(a_1(\lambda^2+2\mu)+6a_2\lambda\mu)\left(\frac{G'}{G}\right)\\ &\quad +\mu(a_1\lambda+2a_2\mu)\end{aligned} \tag{31}$$

Using the solution procedure of the G'/G-expansion method, we obtain the system of algebraic equations as follows:

$\left(\frac{G'}{G}\right)^6$ Coeff.:

$$4a_2^2\big(a_2\beta(t)n^2+(n+1)\big(\alpha(t)+\gamma(t)\big)\big)=0 \tag{32}$$

$\left(\frac{G'}{G}\right)^5$ Coeff.:

$$4a_2^2\begin{pmatrix}a_1(3a_2\beta(t)n^2\\ +(2n+1)\big(\alpha(t)+\gamma(t)\big)\\ +a_2\lambda(n+2)\big(\alpha(t)+\gamma(t)\big)\end{pmatrix}=0 \tag{33}$$

$\left(\frac{G'}{G}\right)^4$ Coeff.:

$$a_1^2\begin{pmatrix}12a_2\beta(t)n^2+(2n+1)\big(\alpha(t)+\gamma(t)\big)\\ +4a_2^2\big(\alpha(t)+\gamma(t)\big)(\lambda^2+2\mu)\\ -4n^2a_2^2\left(t\frac{d\omega(t)}{dt}+\omega(t)+\kappa^2\alpha(t)\right)\\ +12a_0a_2n\big(a_2\beta(t)n+\alpha(t)+\gamma(t)\big)\\ +2a_2a_1\lambda(5n+4)\big(\alpha(t)+\gamma(t)\big)\end{pmatrix}=0 \tag{34}$$

$\left(\frac{G'}{G}\right)^3$ Coeff.:

$$\begin{aligned}&2a_1^3\beta(t)n^2+2a_2a_1\big(\alpha(t)+\gamma(t)\big)(\lambda^2+2\mu)\\ &-4a_2a_1n^2\left(t\frac{d\omega(t)}{dt}+\omega(t)+\kappa^2\alpha(t)\right)\\ &+na_2a_1\big(\alpha(t)+\gamma(t)\big)(\lambda^2+2\mu)\\ &+2a_0n(5a_2\lambda(\alpha(t)+\gamma(t)))\\ &+a_1\big(6a_2\beta(t)n+\alpha(t)+\gamma(t)\big)\\ &-2a_2^2\lambda\mu(n-2)\big(\alpha(t)+\gamma(t)\big)\\ &+a_1^2\lambda(n+1)\big(\alpha(t)+\gamma(t)\big)=0\end{aligned} \tag{35}$$

$\left(\frac{G'}{G}\right)^2$ Coeff.:

$$\begin{aligned}&12a_2a_0^2\beta(t)n^2+a_1^2\begin{pmatrix}\big(\alpha(t)+\gamma(t)\big)(\lambda^2+2\mu)\\ -4n^2\left(t\frac{d\omega(t)}{dt}+\omega(t)+\kappa^2\alpha(t)\right)\end{pmatrix}\\ &-2a_1a_2\lambda\mu(n-4)\big(\alpha(t)+\gamma(t)\big)-4a_2^2\mu^2(n-1)\big(\alpha(t)+\gamma(t)\big)\\ &+6a_1a_0n\lambda\big(\alpha(t)+\gamma(t)\big)+12na_0a_1^2\beta(t)n\\ &+8na_0a_2\begin{pmatrix}\big(\alpha(t)+\gamma(t)\big)(\lambda^2+2\mu)\\ -n\left(t\frac{d\omega(t)}{dt}+\omega(t)+\kappa^2\alpha(t)\right)\end{pmatrix}=0\end{aligned} \tag{36}$$

$\left(\frac{G'}{G}\right)^1$ Coeff.:

$$\begin{aligned}&6a_1a_0^2\beta(t)-a_1\mu(n-1)\big(\alpha(t)+\gamma(t)\big)(a_1\lambda+2a_2\mu)\\ &+6na_0a_2\lambda\mu\big(\alpha(t)+\gamma(t)\big)\\ &+na_0a_1\begin{pmatrix}\big(\alpha(t)+\gamma(t)\big)(\lambda^2+2\mu)\\ -4n\left(t\frac{d\omega(t)}{dt}+\omega(t)+\kappa^2\alpha(t)\right)\end{pmatrix}=0\end{aligned} \tag{37}$$

$\left(\frac{G'}{G}\right)^0$ Coeff.:

$$4a_0^3\beta(t)n^2-4a_0^2n^2\left(t\frac{d\omega(t)}{dt}+\omega(t)+\kappa^2\alpha(t)\right)+2a_0\mu n\big(\alpha(t)+\gamma(t)\big)(a_1\lambda+2a_2\mu)-a_1^2\mu^2(2n-1)\big(\alpha(t)+\gamma(t)\big)=0 \quad (38)$$

Solving the above system of algebraic equations, we obtain the following results:

$$a_0=-\frac{(n+1)(\alpha(t)+\gamma(t))\mu}{n^2\beta(t)} \quad (39\text{-}1)$$

$$a_1=\frac{\lambda a_0}{\mu} \quad (39\text{-}2)$$

$$a_2=\frac{a_0}{\mu} \quad (39\text{-}3)$$

$$\omega(t)=-\frac{1}{t}\int_0^t\left\{\begin{array}{l}\alpha(t)\kappa^2\\-\frac{\lambda^2-4\mu}{4n^2}\big(\alpha(t)+\gamma(t)\big)\end{array}\right\}dt \quad (39\text{-}4)$$

where κ, λ and μ are arbitrary real constants. The solution of Eq. (25) corresponding to (36) has the following cases

Case-1: If $\lambda^2-4\mu>0$, there is

$$\psi(\xi)=\left(\frac{(\lambda^2-4\mu)(\alpha(t)+\gamma(t))(1+n)}{4\beta(t)n^2}\right)^{\frac{1}{2n}}\times\left\{1-\left(\frac{C_1\sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)+C_2\cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)+C_2\sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)}\right)^2\right\}^{\frac{1}{2n}}\times e^{i\left(-\kappa x-\int_0^t\left\{\alpha(t)\kappa^2-\frac{a_2}{4n^2}(\alpha(t)+\gamma(t))\right\}dt\right)} \quad (40)$$

If we set $c_2=0$, $c_1\neq 0$, we obtain

$$\psi_1(x,t)=\left[\left(\frac{(\lambda^2-4\mu)(\alpha(t)+\gamma(t))(1+n)}{4\beta(t)n^2}\right)\times\tanh^2\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\times(x+2\kappa\int_0^t\alpha(t)dt-\xi_0)\right)\right]^{\frac{1}{2n}}\times e^{i\left(-\kappa x-\int_0^t\left\{\alpha(t)\kappa^2-\frac{a_2}{4n^2}(\alpha(t)+\gamma(t))\right\}dt\right)} \quad (41)$$

If we set $c_1=0$, $c_2\neq 0$, one recovers

$$\psi_2(x,t)=\left[-\frac{(\lambda^2-4\mu)(\alpha(t)+\gamma(t))(1+n)}{4\beta(t)n^2}\times\text{csch}^2\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(x+2\kappa\int_0^t\alpha(t)dt-\xi_0)\right)\right]^{\frac{1}{2n}}\times e^{i\left(-\kappa x-\int_0^t\left\{\alpha(t)\kappa^2-\frac{a_2}{4n^2}(\alpha(t)+\gamma(t))\right\}dt\right)} \quad (42)$$

and these are dark and singular soliton solutions respectively. Their existence is guaranteed vy virtue of (22). On the other hand,

Case-2: If $\lambda^2-4\mu<0$, we have

$$\psi(\xi)=\left(\frac{(\lambda^2-4\mu)(\alpha(t)+\gamma(t))(1+n)}{4\beta(t)n^2}\right)^{\frac{1}{2n}}\times\left\{1-\left(\frac{-C_1\sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)+C_2\cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)}{C_1\cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)+C_2\sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)}\right)^2\right\}^{\frac{1}{2n}}\times e^{i\left(-\kappa x-\int_0^t\left\{\alpha(t)\kappa^2-\frac{a_2}{4n^2}(\alpha(t)+\gamma(t))\right\}dt\right)} \quad (43)$$

If we set $c_2 = 0$, $c_1 \neq 0$, we obtain

$$\psi_3(x,t) = \left[\frac{(\lambda^2 - 4\mu)(\alpha(t)+\gamma(t))(1+n)}{4\beta(t)n^2} \times \sec^2\left(\frac{\sqrt{4\mu-\lambda^2}}{2} \times (x + 2\kappa\int_0^t \alpha(t)dt - \xi_0) \right) \right]^{\frac{1}{2n}} \times e^{i\left(-\kappa x - \int_0^t \left\{ \alpha(t)\kappa^2 - \frac{a_2}{4n^2}(\alpha(t)+\gamma(t)) \right\} dt\right)} \quad (44)$$

If we set $c_1 = 0$, $c_2 \neq 0$, we obtain

$$\psi_4(x,t) = \left[\frac{a_2(\alpha(t)+\gamma(t))(1+n)}{4\beta(t)n^2} \times \csc^2\left(\frac{\sqrt{4\mu-\lambda^2}}{2} \times (x + 2\kappa\int_0^t \alpha(t)dt - \xi_0) \right) \right]^{\frac{1}{2n}} \times e^{i\left(-\kappa x - \int_0^t \left\{ \alpha(t)\kappa^2 - \frac{a_2}{4n^2}(\alpha(t)+\gamma(t)) \right\} dt\right)} \quad (45)$$

which are singular periodic solutions that exist provided (23) holds.

4. Conclusion

This paper obtained soliton solutions to NLSE with cubic and power law nonlinearities. The integration scheme adopted is G'/G-expansion. This lead to the retrieval of dark and singular soliton solutions to the model. The results for power law nonlinearity collapses to Kerr law upon setting the power law nonlinearity parameter $n = 1$. It must be noted that it is only the dark and singular soliton solutions that are recoverable by this integration scheme. Thus bright solitons cannot be obtained using this scheme. This is a limitation of this integration scheme. However, there are other tools to secure these bright solitons as reported earlier [1, 2]. Further schemes will be explored later and the results will be reported in future.

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References

[1] A. Biswas, Quantum Physics Letters **1**, 79 (2012).
[2] M. Eslami, M. Mirzazadeh, A. Biswas, Journal of Modern Optics **60**, 1627 (2013).
[3] M. Eslami, M. Mirzazadeh, B. F. Vajargah, A. Biswas, Optik **125**, 3107 (2014).
[4] M. Mirzazadeh, M. Eslami, D. Milovic, A. Biswas, Optik **125**, 5480 (2014).
[5] M. Mirzazadeh, M. Eslami, B. F. Vajargah, A. Biswas, Optik **125**, 4246 (2014).
[6] M. Mirzazadeh, A. H. Arnous, M. F. Mahmood, E. Zerrad, A. Biswas, Submitted.
[7] H. Triki, T. Hayat, O. M. Aldossary, A. Biswas, Optics and Laser Technology **44**, 2223 (2012).
[8] H. Triki, A. Yildirim, T. Hayat, O. M. Aldossary, A. Biswas, Advanced Science Letters **16**, 309 (2012).
[9] M. L. Wang, X. Z. Li, J. L. Zhang, Physics Letters A **372**, 417 (2008).
[10] E. Zayed, K. A. Gepreel, Applied Mathematics and Computation **212**, 1 (2009).
[11] S. Zhang, J. L. Tong, W. Wang, Physics Letters A **372**, 2254 (2008).
[12] H. Zhang, Communications in Nonlinear Science and Numerical Simulation **14**, 3220 (2009)
[13] Q. Zhou, D. Yao, F. Chen, Journal of Modern Optics **60**, 1652 (2013).
[14] Q. Zhou, Q. Zhu, A. Biswas, Optica Applicata **41**, 399 (2014).
[15] Q. Zhou, Q. Zhu, Y. Liu, H. Yu, C. Wei, P. Yao, A. H. Bhrawy, A. Biswas, Laser Physics **25**, 025402 (2015).

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