Optical solitons with generalized quadratic–cubic nonlinearity

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This work obtains soliton solutions to the governing nonlinear Schrödinger's equation by traveling wave hypothesis. The model is considered with the generalized quadratic–cubic nonlinearity that is also a special case of Kudryashov's form of nonlinear refractive index setting the coefficients of nonlinear terms with negative exponents, in Kudryashov's nonlinearity, to zero. Based on the sign of the discriminant, plane waves, bright or singular solitons emerge. Notably, a major shortcoming of this approach is that traveling waves fail to recover dark optical solitons to the model. Thus, traveling wave hypothesis has its own limitations just as various other integrability approaches which has their own shortcomings–a strong message as this paper conveys. The parameter constraints for the existence of these solitons and plane waves are also presented.

1. Introduction

In recent times, there has been a surge of interest in exploring the characteristics of localized optical pulses or optical solitons as they are also called especially in the context of transportation of data through optical fibers. Several works related to pulse propagation in connection with the telecommunication have been reported in the recent past which is an indication of the importance given by industry as well as the research community towards data transportation. Among several forms of nonlinearity quadratic–cubic (QC) type nonlinearity has received a lot of attention in the recent past. There have been reports regarding various conservation laws, exact analytical and numerical soliton solutions by means of traveling wave ansatz, *F*–expansion method, variational principle and others [1–9]. In this paper, the extension of QC nonlinearity to generalized QC nonlinear form is carried out.

It must be noted that the generalized QC nonlinear form is also a special case of Kudryashov's form of nonlinear refractive index [4]. This is obtained upon setting the coefficients of nonlinear terms with negative exponents, in Kudryashov's nonlinearity, to zero. Thus,

the governing nonlinear Schrödinger's equation (NLSE) with this generalized QC nonlinearity is an intermediate form of refractive index structure that is straddled between QC nonlinearity and Kudryashov's law of refractive index. Therefore, staying intermediate, the current paper explores the model with traveling wave hypothesis to reveal bright and singular solitons as well as plane waves as the discriminant dictates. The results are exhaustively displayed in the rest of the paper. The parameter constraints are also presented that indicate the existence criteria of these solitons and plane waves.

1.1. Governing model

We consider the dynamics of the optical pulses in a nonlinear medium which exhibits two power–law nonlinear effects. The following generalized version of QC–NLSE describes pulse propagation with distributed coefficients [4, 5]:

$$i\psi_t + \beta\psi_{xx} + (\gamma_1|\psi|^p + \gamma_2|\psi|^{2p})\psi = 0. \qquad (1)$$

Here, the coefficients β and γ_j for $j = 1,2$ are $\in \mathbb{R}$ while $0 < p < 4$ as mentioned earlier [4]. The parameter β is from chromatic dispersion while γ_j represents nonlinear refractive index change.

2. Traveling wave solutions

We assume a traveling wave solution of the form

$$\psi(t,x) = U(\xi)e^{i(\kappa x + \omega t + \theta)}, \xi = x - ct. \tag{2}$$

Substituting (2) into (1) we obtain the following equations for the imaginary and real parts respectively:

$$(\beta\kappa - c)U' = 0, \tag{3}$$

$$\beta U'' - (\omega + \beta\kappa^2)U + \gamma_1 U^{p+1} + \gamma_2 U^{2p+1} = 0. \tag{4}$$

Eq. (3) may be satisfied by choosing $c = \kappa\beta$ so that the traveling wave coordinate $\xi = x - \kappa\beta t$. Furthermore on multiplying (4) by U' and integrating we have

$$\beta U'^2 - (\omega + \beta\kappa^2)U^2 + \frac{2\gamma_1}{p+2}U^{p+2}$$

$$+ \frac{\gamma_2}{(p+1)}U^{2(p+1)} = C_1. \tag{5}$$

Here C_1 is an arbitrary constant of integration which we will set to zero in the sequel. On introducing the transformation $U = z^{1/p}$ we may re-write (5) as

$$z'^2 + \delta z^4 + \epsilon z^3 - \nu z^2 = 0. \tag{6}$$

Here the constants δ, ϵ and ν are defined by

$$\delta = \frac{p^2\gamma_2}{\beta(p+1)}, \epsilon = \frac{2p^2\gamma_1}{\beta(p+2)}, \nu = \frac{p^2}{\beta}(\omega + \beta\kappa^2). \tag{7}$$

In order to find the solutions of (6) we consider the following cases:

2.1. Case–I: (Plane waves)

If $\omega + \beta\kappa^2 = 0$, the coefficient $\nu = 0$ Eq. (6) may be easily integrated to give

$$z = -\frac{2\beta(p+1)(p+2)\gamma_1}{\gamma_2\beta(p+2)^2 + \gamma_1^2 p^2(p+1)(\pm\xi+\eta)^2}, \tag{8}$$

where η is a constant of integration and we have replaced the values of δ, ϵ and ν. The amplitude is then given by

$$\psi(x,t) = \left[\frac{-2\beta(p+1)(p+2)\gamma_1}{\gamma_2\beta(p+2)^2 + 2\gamma_1^2 p^2(p+1)(\pm\xi+\eta)^2}\right]^{\frac{1}{p}}$$

$$\times e^{i(\kappa x + \omega t + \theta)}, \tag{9}$$

with $\xi = x - \beta\kappa t$.

2.2. Case–II (Solitons)

When $\omega + \beta\kappa^2 \neq 0$, we may write (6) as

$$z'^2 = \nu z^2\left[1 - \frac{\epsilon}{\nu}z - \frac{\delta}{\nu}z^2\right], \tag{10}$$

which upon making the transformation $z = \frac{1}{w}$, may be expressed as

$$w'^2 = \nu\left[\left(w - \frac{\epsilon}{2\nu}\right)^2 - \frac{\epsilon^2 + 4\nu\delta}{4\nu^2}\right]. \tag{11}$$

We can have two subcases:

2.2.1. Bright solitons

Consider the situation when $\epsilon^2 + 4\nu\delta > 0$. Setting $v = w - \frac{\epsilon}{2\nu}$ and assuming $\kappa_+^2 = \frac{\epsilon^2 + 4\nu\delta}{4\nu^2}$ we have the following equation, namely

$$v'^2 = \nu[v^2 - \kappa_+^2]. \tag{12}$$

Its solution is given by

$$v = \kappa_+\cosh(\sqrt{\nu}(\pm\xi + \eta)). \tag{13}$$

As $z = \frac{1}{w}$ with $w = v + \frac{\epsilon}{2\nu}$ we have finally for the amplitude

$$U(\xi) = \left[\frac{1/\kappa_+}{\frac{\epsilon}{2\nu\kappa_+} + \cosh(\sqrt{\nu}(\pm\xi+\eta))}\right]^{\frac{1}{p}}. \tag{14}$$

Thus, bright 1–soliton solution has the form:

$$\psi(x,t) = \frac{A}{D + \cosh[B(x-vt)]^{\frac{1}{p}}}e^{i(\kappa x + \omega t + \theta)}, \tag{15}$$

where A is the amplitude of the soliton and B is its inverse width while D is a parameter of the soliton and v represents soliton speed.
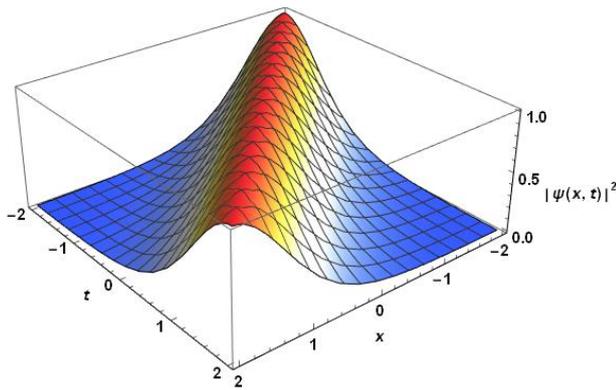
Fig. 1. 3D graphic for (15) setting all arbitrary
parameters to unity (color online)

2.2.2. Singular solitons

Here, $\epsilon^2 + 4\nu\delta < 0$. In this case, writing $\kappa_-^2 = \frac{4\nu\delta + \epsilon^2}{4\nu^2}$ and proceeding in the same manner as above we obtain

$$U = \left[\frac{\frac{1}{\kappa_-}}{\frac{\epsilon}{2\nu\kappa_-} + \sinh(\sqrt{\nu}(\pm\xi + \eta))} \right]^{1/p}. \qquad (16)$$

Therefore, the singular 1–soliton solution turns out to be:

$$\psi(x,t) = \frac{A}{D + \sinh[B(x - vt)]^{\frac{1}{p}}} e^{i(\kappa x + \omega t + \theta)}, \qquad (17)$$
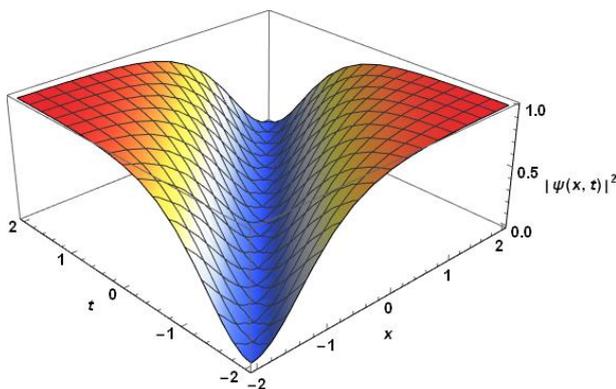
where A, B and D are free parameters.

Fig. 2. 3D graphic for (17) setting all arbitrary
parameters to unity (color online)

3. Conclusions

The current paper implemented traveling wave hypothesis to recover bright and singular solitons by traveling wave hypothesis. The sign of the discriminant yielded the type of solitons as listed. In the third situation with the discriminant, plane wave solutions have emerged. Notably, a major shortcoming of this approach is that traveling waves fail to recover dark optical solitons to the model. These were however successfully recovered and reported earlier when the more generalized model, with Kudryashov's law of refractive index, was studied with the usage of undetermined coefficients [4]. Thus, traveling wave hypothesis has its own limitations just as various other integrability approaches which has their own shortcomings– a strong message as this paper conveys.

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