Optical solitons in dual-core fibers with inter-modal dispersion

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This paper obtains optical 1-soliton solutions in dual-core fibers with inter modal dispersion. These solitons are constructed with two types of nonlinearities namely, Kerr law and power law by the aid of ansatz approach. Additionally, the constraint conditions, for the existence of the soliton solutions are listed.

1. Introduction

Optical solitons is one of the most fascinating areas of research at the present time. These soliton molecules are basic ingredients for information transfer, through optical fibers for trans-continental and trans-oceanic distances [1-30]. Therefore, it is imperative to address the dynamics of these soliton pulses from a mathematical perspective. This will lead to a deeper understanding of the engineering aspects of these solitons that will lead to unprecedented novelty.

This paper will study the different kind of solitons in dual-core optical fibers from a purely mathematical standpoint. The focus of this paper therefore will be to extract exact 1-soliton solution for the governing model. This model is described the coupled nonlinear Schrödinger's equation (NLSE). There are several integration tools available to solve the model. A few of them are traveling waves, homotopy analysis method, variational principle, Kudryashov's method, simplest equation method, tanh-expansion scheme, extended tanh method and several others.

The rest of the article is organized as follows: In Section-2 the model has been described. The different kind of soliton solutions to the application: decoupled NLSEs for two-core fiber are constructed in next Sections 3 and 4 with Kerr and power law nonlinearities, respectively. In last Section 5, the conclusions are drawn.

2. The model

Pulse propagation in a decoupled two-core fibers has distinction from continuous wave propagation. In a conventional two core fiber, pulse propagation has been studied extensively by solving the coupled mode equations; where the light coupling between the two cores is characterized by a structure dependent parameter called the coupling coefficients. The model for decoupled NLSE read as [10, 15]:

$$
i\left(\frac{\partial \psi_1}{\partial t} + a_1 \frac{\partial \psi_2}{\partial x}\right) + b_1 \frac{\partial^2 \psi_1}{\partial x^2} \\
+ c_1 \frac{\partial^2 \psi_1}{\partial x \partial t} + d_1 F\left(|\psi_1|^2\right)\psi_1 + k_1 \psi_2 = 0
\tag{1}
$$

$$
i\left(\frac{\partial \psi_2}{\partial t} + a_2 \frac{\partial \psi_1}{\partial x}\right) + b_2 \frac{\partial^2 \psi_2}{\partial x^2} \\
+ c_2 \frac{\partial^2 \psi_2}{\partial x \partial t} + d_2 F\left(|\psi_2|^2\right)\psi_2 + k_2 \psi_1 = 0
\tag{2}
$$

where ψ_1 and ψ_2 are the field envelopes, while x is the propagation co-ordinate and $1/a_j$ are group velocity mismatch, b_j are group velocity dispersion, c_j represent spatio-temporal dispersion and k_j are linear coupling

coefficients, for $j = 1, 2$. It may also be noted that d_j are defined by $2\pi n_2 / \vartheta A_{eff}$, where n_2, ϑ and A_{eff} are nonlinear refractive index, the wavelength and effective mode area of each wavelength, respectively. For more details, see [1, 2, 26].

The functional F represents the nonlinearity type. There are two types of nonlinearities: Kerr and power laws are being studied in this article. The functional F is real-valued algebraic function where it is necessary to have smoothness of the complex function $F\left(|\psi_j|^2\right)\psi_j : C \mapsto C$ for $j = 1, 2$. Treating the complex plane C as a two-dimensional linear space R^2, the function $F\left(|\psi_j|^2\right)\psi_j$ is k times continuously differentiable, so that

$$F\left(|\psi_j|^2\right)\psi_j \in \bigcup_{m,n=1}^{\infty} C^k\left((-n,n) \times (-m,m); R^2\right) \quad (3)$$

In order to study this coupled system are being split into

$$\psi_j(x,t) = P_j(x,t)e^{i\phi} \quad (4)$$

Here, $P_j(x,t)$, for $j = 1, 2$ are the amplitude components of the wave profiles, while ϕ is the phase component of the profiles where

$$\phi = -\kappa x + \omega t + \theta \quad (5)$$

The parameters κ, ω and θ are the wave number, frequency and the phase constant, respectively. Substitute equations (4) and (5) into equations (1) and (2), and decomposed into real and imaginary parts.

The real part equations for the two components are

$$-\omega P_1 + \kappa a_1 P_2 + b_1\left(\frac{\partial^2 P_1}{\partial x^2} - \kappa^2 P_1\right)$$
$$+ c_1\left(\kappa\omega P_1 + \frac{\partial^2 P_1}{\partial x \partial t}\right) + d_1 F\left(P_1^2\right)P_1 \quad (6)$$
$$+ k_1 P_2 = 0$$

$$-\omega P_2 + \kappa a_2 P_1 + b_2\left(\frac{\partial^2 P_2}{\partial x^2} - \kappa^2 P_2\right)$$
$$+ c_2\left(\kappa\omega P_2 + \frac{\partial^2 P_2}{\partial x \partial t}\right) + d_2 F\left(P_2^2\right)P_2 \quad (7)$$
$$+ k_2 P_1 = 0$$

The imaginary part equations for the two components lead to the velocity of the solitons as

$$v = \frac{a_1 + \omega c_1 - 2\kappa b_1}{1 - c_1 \kappa} \quad (8)$$

and

$$v = \frac{a_2 + \omega c_2 - 2\kappa b_2}{1 - c_2 \kappa} \quad (9)$$

Since the wave profiles can be written as $g(x - vt)$, where v is the velocity and g is the functional form of the wave profile. Next, equating the two velocities with each other leads to a constraint relation between the soliton parameters as

$$\omega = \frac{(2\kappa b_1 - a_1)(1 - \kappa c_2) - (2\kappa b_2 - a_2)(1 - \kappa c_1)}{c_1 - c_2} \quad (10)$$

where $c_1 \neq c_2$. This relation holds for both Kerr and power laws of nonlinearity as well as for bright, dark and singular solitons for both of these laws. The real part equations given by (6) and (7) will now be analyzed separately in the next 3 and 4 sections, based on the type of nonlinearity.

3. Kerr law nonlinearity

For Kerr law nonlinearity, we have $F(\psi) = \psi$. So, equations (1) and (2) take the form [3, 14, 16-20]

$$i\left(\frac{\partial \psi_1}{\partial t} + a_1 \frac{\partial \psi_2}{\partial x}\right) + b_1 \frac{\partial^2 \psi_1}{\partial x^2} + c_1 \frac{\partial^2 \psi_1}{\partial x \partial t} + d_1 |\psi_1|^2 \psi_1 + k_1 \psi_2 = 0 \quad (11)$$

$$i\left(\frac{\partial \psi_2}{\partial t} + a_2 \frac{\partial \psi_1}{\partial x}\right) + b_2 \frac{\partial^2 \psi_2}{\partial x^2} + c_2 \frac{\partial^2 \psi_2}{\partial x \partial t} + d_2 |\psi_2|^2 \psi_2 + k_2 \psi_1 = 0 \quad (12)$$

Hence, the real part equations for the components are

$$-\omega P_1 + \kappa a_1 P_2 + b_1\left(\frac{\partial^2 P_1}{\partial x^2} - \kappa^2 P_1\right)$$

$$+ c_1\left(\kappa\omega P_1 + \frac{\partial^2 P_1}{\partial x \partial t}\right) + d_1 P_1^3 \tag{13}$$

$$+ k_1 P_2 = 0$$

$$-\omega P_2 + \kappa a_2 P_1 + b_2\left(\frac{\partial^2 P_2}{\partial x^2} - \kappa^2 P_2\right)$$

$$+ c_2\left(\kappa\omega P_2 + \frac{\partial^2 P_2}{\partial x \partial t}\right) + d_2 P_2^3 \tag{14}$$

$$+ k_2 P_1 = 0$$

The real part equations (13) and (14) are being further analyzed on the type of solitons that are considered. The study is, thus, divided into the following four subsections.

The following first subsection deals with the bright optical solitons.

3.1 Bright solitons

To construct the bright solitons, we have the ansatz hypothesis of the form [16-20]

$$P_j(x,t) = A_j \operatorname{sech}^{p_j}\xi \tag{15-1}$$

and

$$\xi = B(x - vt) \tag{15-2}$$

where A_j for $j = 1, 2$ are the amplitudes of the solitons in two components, while B and v are the inverse width and velocity of the solitons, respectively. It can, thus, be written after substituting the derivatives of equation (15) in real part equations (13) and (14) with $n = 3 - j$ for $j = 1, 2$, then we have

$$-A_j\omega \operatorname{sech}^{p_j}\xi + \kappa a_j A_n \operatorname{sech}^{p_j}\xi$$
$$+ b_j p_j^2 A_j B^2 \operatorname{sech}^{p_j}\xi - b_j p_j(p_j + 1)$$
$$\times A_j B^2 \operatorname{sech}^{p_j+2}\xi - b_j\kappa^2 A_j \operatorname{sech}^{p_j}\xi$$
$$+ c_j\kappa\omega A_j \operatorname{sech}^{p_j}\xi - c_j p_j^2 v A_j B^2 \operatorname{sech}^{p_j}\xi \tag{16}$$
$$+ c_j p_j(1 + p_j)v A_j B^2 \operatorname{sech}^{p_j+2}\xi$$
$$+ d_j A_j^3 \operatorname{sech}^{3p_j}\xi + \kappa_j A_n \operatorname{sech}^{p_n}\xi = 0$$

From equation (16), equating the exponent pair $(p_j + 2, 3p_j)$ leads to

$$p_j = 1 \text{ for } j = 1, 2.$$

A system of equations is obtained after setting the coefficients of linearly independent function, $\operatorname{sech}^{p_j+i}\xi$ where $j = 1, 2$ and $i = 0, 2$, to zero in last equation (16). After solving the system, following results are in place.

The wave number of the solitons are given by

$$\omega = \frac{2\kappa^2 b_j A_j - 2(k_j + \kappa a_j)A_n - d_j A_j^3}{2A_j(c_j\kappa - 1)} \tag{17}$$

and

$$\omega = \frac{\kappa^2 b_j - B^2(b_j - c_j v)}{c_j\kappa - 1} \tag{18}$$

The width of the solitons is given by

$$B = A_j\sqrt{\frac{d_j}{2(b_j - c_j v)}} \tag{19}$$

This width provokes the constraint condition, for $j = 1, 2$, that is

$$d_j(b_j - c_j v) > 0 \tag{20}$$

After setting widths of the soliton equal to one another for $j = 1, 2$ in equation (19) gives condition.

$$\frac{A_1}{A_2} = \sqrt{\frac{d_2(b_1 - c_1 v)}{d_1(b_2 - c_2 v)}} \tag{21}$$

By inserting equations (19) and (21) into (17), one gets

$$\omega = \frac{\kappa^2 b_j - B^2(b_j - c_j v)}{c_j\kappa - 1}$$

$$- \frac{k_j + \kappa a_j}{c_j\kappa - 1}\left[\frac{d_j(b_n - c_n v)}{d_n(b_j - c_j v)}\right]^{\frac{1}{2}} \tag{22}$$

After setting the two values of ω equal to one another for $j = 1, 2$ in equation (18). This leads to the width of the soliton B in terms of the given parameter as

$$B =$$

$$\kappa\left[\frac{b_1(c_2\kappa - 1) - b_2(c_1\kappa - 1)}{(b_1 - c_1 v)(c_2\kappa - 1) - (b_2 - c_2 v)(c_1\kappa - 1)}\right]^{\frac{1}{2}} \tag{23}$$

The above equation gives a constraint condition, that is

$$[b_1(c_2\kappa-1)-b_2(c_1\kappa-1)]$$
$$\times[(b_1-c_1v)(c_2\kappa-1)-(b_2-c_2v)(c_1\kappa-1)]>0$$
$$(24)$$

The equations (19) and (23) leads to the following amplitudes of the solitons in terms of given parameters

$$A_j =$$

$$\kappa\left\{\frac{2(b_j-c_jv)[b_1(c_2\kappa-1)-b_2(c_1\kappa-1)]}{(b_1-c_1v)(c_2\kappa-1)-(b_2-c_2v)(c_1\kappa-1)}\right\}^{\frac{1}{2}}$$
$$(25)$$

Thus, the bright 1-soliton solution to the coupled system (11) and (12), for $j=1,2$ is given by

$$\psi_j(x,t)=A_j \operatorname{sech}[B(x-vt)]e^{i(-\kappa x+\omega t+\theta)} \quad (26)$$

3.2 Dark solitons

To construct the dark solitons, we have the ansatz hypothesis of the form

$$P_j(x,t)=A_j \tanh^{p_j}\xi \qquad (27\text{-}1)$$

and

$$\xi = B(x-vt) \qquad (27\text{-}2)$$

Here A_j for $j=1,2$ and B are free parameters of the solitons and v is the speed of dark solitons, respectively. It can, thus, be written after substituting the derivatives of the equation (27) in real part equations (13) and (14), then we have

$$-A_j(\omega+b_j\kappa^2-c_j\omega\kappa)\tanh^{p_j}\xi$$
$$+A_n(\kappa a_j+k_j)\tanh^{p_j}\xi$$
$$+d_jA_j^3\tanh^{3p_j}\xi+b_jp_jA_jB^2$$
$$\times\left\{\begin{array}{l}(p_j-1)\tanh^{p_j-2}\xi-2p_j\tanh^{p_j}\xi\\+(p_j+1)\tanh^{p_j+2}\xi\end{array}\right\} \quad (28)$$
$$-c_jp_jvA_jB^2$$
$$\times\left\{\begin{array}{l}(p_j-1)\tanh^{p_j-2}\xi-2p_j\tanh^{p_j}\xi\\+(p_j+1)\tanh^{p_j+2}\xi\end{array}\right\}=0$$

From equation (28), equating the exponent pair $(p_j+2, 3p_j)$ leads to

$$p_j = 1$$

We obtain a system of equations after setting the coefficients of linearly independent function, $\tanh^{p_j\pm l}\xi$ where $j=1,2$ and $l=0,2$, to zero in last equations. Solving the above system gives the wave number of the solitons:

$$\omega=\frac{b_j\kappa^2+2B^2(b_j-c_jv)}{c_j\kappa-1} \qquad (29)$$

and

$$\omega=\frac{b_jA_j\kappa^2-(k_j+\kappa a_j)-d_jA_j^3}{A_j(c_j\kappa-1)} \qquad (30)$$

while the free parameter B is given by

$$B=A_j\sqrt{-\frac{d_j}{2(b_j-c_jv)}} \qquad (31)$$

This free parameter provokes the constraint conditions for $j=1,2$, that are

$$d_j(b_j-c_jv)<0 \qquad (32)$$

After setting the two values of B equal to one another for $j=1,2$ in equation (31), one recovers another relation of the form given in equation (21). By inserting the (21) and (31) into (30), implies

$$\omega=\frac{b_j\kappa^2+2B^2(b_j-c_jv)}{c_j\kappa-1}$$
$$-\frac{k_j+\kappa a_j}{c_j\kappa-1}\left[\frac{d_j(b_n-c_nv)}{d_n(b_j-c_jv)}\right]^{\frac{1}{2}} \qquad (33)$$

After setting the two values of ω equal to one another for $j=1,2$ in equation (29) yields another relation for the free parameter B in the following form

$$B=$$

$$\frac{\kappa}{\sqrt{2}}\left[\frac{b_1(c_2\kappa-1)-b_2(c_1\kappa-1)}{(b_2-c_2v)(c_1\kappa-1)-(b_1-c_1v)(c_2\kappa-1)}\right]^{\frac{1}{2}} \quad (34)$$

The above equation gives another constraint condition, that is

$$[b_1(c_2\kappa - 1) - b_2(c_1\kappa - 1)]$$
$$\times[(b_2 - c_2 v)(c_1\kappa - 1) - (b_1 - c_1 v)(c_2\kappa - 1)] > 0 \tag{35}$$

The equations (31) and (34) leads to the free parameters A_j of dark solitons as

$$A_j^2 d_j = -2\kappa^2(b_j - c_j v)$$

$$\times\left[\frac{b_1(c_2\kappa - 1) - b_2(c_1\kappa - 1)}{(b_2 - c_2 v)(c_1\kappa - 1) - (b_1 - c_1 v)(c_2\kappa - 1)}\right]^{\frac{1}{2}} \tag{36}$$

Thus, the dark 1-soliton solution to the coupled NLSEs (11) and (12), for $j = 1, 2$ is given by

$$\psi_j(x,t) = A_j \tanh[B(x - vt)]e^{i(-\kappa x + \omega t + \theta)} \tag{37}$$

where $v = b_j/c_j$ and $\kappa = -k_j/a_j$.

In the following subsection singular solitons of form-I, are being constructed.

3.3 Singular solitons (Form-I)

To construct the singular solitons of Form-I, we have the ansatz hypothesis of the form

$$P_j(x,t) = A_j \coth^{p_j}\xi \tag{38-1}$$

and

$$\xi = B(x - vt) \tag{38-2}$$

where A_j for $j = 1, 2$, B and v are defined in previous subsections. It can, thus, be followed After substituting the derivatives of equation (38) in real part equations (13) and (14), then we have

$$-A_j(\omega + b_j\kappa^2 - c_j\omega\kappa)\coth^{p_j}\xi$$
$$+A_n(\kappa a_j + k_j)\coth^{p_j}\xi$$
$$+d_j A_j^3 \coth^{3p_j}\xi + b_j p_j A_j B^2$$
$$\times\begin{Bmatrix}(p_j - 1)\coth^{p_j-2}\xi - 2p_j\coth^{p_j}\xi \\ +(p_j + 1)\coth^{p_j+2}\xi\end{Bmatrix} \tag{39}$$
$$-c_j p_j v A_j B^2$$
$$\times\begin{Bmatrix}(p_j - 1)\coth^{p_j-2}\xi - 2\coth^{p_j}\xi \\ +(p_j + 1)\coth^{p_j+2}\xi\end{Bmatrix} = 0$$

From equation (39), equating the exponent pair $(p_j + 2, 3p_j)$ leads to

$$p_j = 1$$

Setting the coefficients of linearly independent function, $\coth^{p_j \pm l}\xi$ where $j = 1, 2$ and $l = 0, 2$, to zero and by solving the obtained set of equations.

It may be noted that the same results can be obtained as in equations (29)-(36) for dark solitons.

Thus, the singular 1-soliton solution to the coupled NLSEs (11) and (12), for $j = 1, 2$ is given by

$$\psi_j(x,t) = A_j \coth[B(x - vt)]e^{i(-\kappa x + \omega t + \theta)} \tag{40}$$

where $v = b_j/c_j$ and $\kappa = -k_j/a_j$.

3.4 Singular solitons (Form-II)

To construct the singular solitons of Form-II, we have the ansatz hypothesis of the form

$$P_j(x,t) = A_j \operatorname{csch}^{p_j}\xi \tag{41-1}$$

and

$$\xi = B(x - vt) \tag{41-2}$$

where A_j, B and v are defined in previous subsections. It can, thus, be followed

$$-A_j\omega\operatorname{csch}^{p_j}\xi + \kappa a_j A_n\operatorname{csch}^{p_n}\xi$$
$$-b_j p_j^2 A_j B^2\operatorname{csch}^{p_j}\xi + b_j p_j(p_j + 1)$$
$$\times A_j B^2\operatorname{csch}^{p_j+2}\xi - b_j\kappa^2 A_j\operatorname{csch}^{p_j}\xi$$
$$+c_j\kappa\omega A_j\operatorname{csch}^{p_j}\xi - c_j p_j^2 v A_j B^2\operatorname{csch}^{p_j}\xi \tag{42}$$
$$-c_j p_j(1 + p_j)v A_j B^2\operatorname{csch}^{p_j+2}\xi$$
$$+d_j A_j^3\operatorname{csch}^{3p_j}\xi + \kappa_j A_j\operatorname{csch}^{p_j}\xi = 0$$

From equation (42), equating the exponent pair $(p_j + 2, 3p_j)$ leads to $p_j = 1$, we obtain a system of equations after setting the coefficients of linearly independent function, $\operatorname{csch}^{p_j+l}\xi$ where $j = 1, 2$ and $l = 0, 2$, to zero in last equation (42). After solving the system, the wave number of the solitons are given by

$$\omega =$$

$$\frac{2b_j A_j \kappa^2 - 2(k_j + \kappa a_j)A_n - d_j A_j^3}{A_j(c_j\kappa - 1)} \quad (43)$$

and

$$\omega = \frac{b_j\kappa^2 + B^2(b_j - c_j v)}{c_j\kappa - 1} \quad (44)$$

The free parameter B of the solitons is

$$B = A_j \sqrt{-\frac{d_j}{2(b_j - c_j v)}} \quad (45)$$

which introduces constraint condition, for $j = 1, 2$,

$$d_j(b_j - c_j v) < 0 \quad (46)$$

After setting the two B's equal to one another for $j = 1, 2$ in equation (45) leads to another relation which is similar to (21). By inserting equations (45) and (21) into (43), one obtains

$$\omega = \frac{b_j\kappa^2 + B^2(b_j - c_j v)}{c_j\kappa - 1}$$
$$- \frac{k_j + \kappa a_j}{c_j\kappa - 1}\left[\frac{d_j(b_n - c_n v)}{d_n(b_j - c_j v)}\right]^{\frac{1}{2}} \quad (47)$$

After setting the two values of ω equal to one another for $j = 1, 2$ in equation (44) leads to another relation for B:

$$B =$$

$$\kappa\left[\frac{b_2(c_1\kappa - 1) - b_1(c_2\kappa - 1)}{(b_1 - c_1 v)(c_2\kappa - 1) - (b_2 - c_2 v)(c_1\kappa - 1)}\right]^{\frac{1}{2}} \quad (48)$$

The above equation gives a constraint condition, that is

$$[b_1(c_2\kappa - 1) - b_2(c_1\kappa - 1)]$$
$$\times [(b_1 - c_1 v)(c_2\kappa - 1) - (b_2 - c_2 v)(c_1\kappa - 1)] > 0 \quad (49)$$

The equations (19) and (23) produce another constraint condition for $j = 1, 2$ in the following form

$$A_j =$$

$$\kappa\left[\frac{2(b_j - c_j v)[b_1(c_2\kappa - 1) - b_2(c_1\kappa - 1)]}{(b_1 - c_1 v)(c_2\kappa - 1) - (b_2 - c_2 v)(c_1\kappa - 1)}\right]^{\frac{1}{2}} \quad (50)$$

Thus, the singular 1-soliton solution (Form-II) to the coupled NLSE (11) and (12), for $j = 1, 2$ is given by

$$\psi_j(x,t) = A_j \operatorname{csch}[B(x - vt)]e^{i(-\kappa x + \omega t + \theta)} \quad (51)$$

4. Power law nonlinearity

For power law nonlinearity, $F(\psi) = \psi^m$ where m is the power law nonlinearity factor with the restriction $0 < m < 2$ and in particular $m \neq 2$ to avoid the self-focusing singularity. So the equations (1) and (2) take the form [9, 16-20]

$$i\left(\frac{\partial\psi_j}{\partial t} + a_j\frac{\partial\psi_n}{\partial x}\right) + b_j\frac{\partial^2\psi_j}{\partial x^2}$$
$$+ c_j\frac{\partial^2\psi_j}{\partial x\partial t} + d_j|\psi_j|^{2m}\psi_j + k_j\psi_n = 0 \quad (52)$$

Hence, the real part equations for the components are

$$-\omega P_j + \kappa a_j P_n + b_j\left(\frac{\partial^2 P_j}{\partial x^2} - \kappa^2 P_j\right)$$
$$+ c_j\left(\kappa\omega P_j + \frac{\partial^2 P_j}{\partial x\partial t}\right) + d_j P_j^{2m+1}$$
$$+ k_j P_n = 0 \quad (53)$$

The real part equations (for $j = 1, 2$) are being analyzed on the type of solitons that are considered. The study is, thus, divided into the following subsections.

4.1 Bright solitons

This section will consider the case when both the components support bright solitons. Therefore, the starting hypothesis will be the same as given in section 3.1. Therefore following the same procedure as adopted in section 3.1, the real component equation (51) reduces to

$$-A_j \omega \operatorname{sech}^{P_j}\xi + \kappa a_j A_n \operatorname{sech}^{P_n}\xi$$
$$+b_j p_j^2 A_j B^2 \operatorname{sech}^{P_j}\xi - b_j p_j(p_j+1)$$
$$\times A_j B^2 \operatorname{sech}^{P_j+2}\xi - b_j \kappa^2 A_j \operatorname{sech}^{P_j}\xi$$
$$+c_j \kappa \omega A_j \operatorname{sech}^{P_j}\xi + c_j p_j^2 v A_j B^2 \operatorname{sech}^{P_j}\xi \tag{54}$$
$$+c_j p_j(1-p_j)v A_j B^2 \operatorname{sech}^{P_j+2}\xi$$
$$+d_j A_j^{2m+1} \operatorname{sech}^{(2m+1)p_j}\xi + \kappa_j A_n \operatorname{sech}^{P_n}\xi = 0$$

From equation (52), equating the exponent pair $(p_j+2,(2m+1)p_j)$ leads to

$$p_j = \frac{1}{m}$$

Setting the coefficients of linearly independent function, $\operatorname{sech}^{p_j+i}\xi$ where $j=1,2$ and $i=0,2$, to zero in last equation (52).

The wave number of the solitons are given by

$$\omega =$$
$$\frac{2m^2\kappa^2 b_j A_j - 2(k_j + \kappa a_j)A_n - d_j A_j^3}{2A_j m^2(c_j\kappa - 1)} \tag{55}$$

and

$$\omega = \frac{m^2\kappa^2 b_j - B^2(b_j - c_j v)}{m^2(c_j\kappa - 1)} \tag{56}$$

The width of the solitons is given by

$$B = mA_j \sqrt{\frac{d_j}{2(b_j - c_j v)}} \tag{57}$$

The width of the solitons B provokes the constraint condition, for $j=1,2$, that is

$$d_j(b_j - c_j v) > 0 \tag{58}$$

After setting the two B's equal to one another for $j=1,2$ in equation (57). This leads to the condition (21). By inserting equations (57) and (21) into (55), one recovers

$$\omega = \frac{m^2\kappa^2 b_j - B^2(b_j - c_j v)}{m^2(c_j\kappa - 1)}$$
$$-\frac{k_j + \kappa a_j}{c_j\kappa - 1}\left[\frac{d_j(b_n - c_n v)}{d_n(b_j - c_j v)}\right]^{\frac{1}{2}} \tag{59}$$

After setting the two values of ω equal to one another for $j=1,2$ in equation (56). This leads to another relation for B

$$B =$$
$$m\kappa\left[\frac{b_1(c_2\kappa - 1) - b_2(c_1\kappa - 1)}{(b_1 - c_1 v)(c_2\kappa - 1) - (b_2 - c_2 v)(c_1\kappa - 1)}\right]^{\frac{1}{2}} \tag{60}$$

The above equation gives a constraint condition

$$[b_1(c_2\kappa - 1) - b_2(c_1\kappa - 1)]$$
$$\times[(b_1 - c_1 v)(c_2\kappa - 1) - (b_2 - c_2 v)(c_1\kappa - 1)] > 0 \tag{61}$$

Thus, the bright 1-soliton solution to the coupled NLSEs (53), for $j=1,2$ is given by

$$\psi_j(x,t) = A_j \operatorname{sech}^{\frac{1}{m}}[B(x - vt)]e^{i(-\kappa x + \omega t + \theta)} \tag{62}$$

4.2 Dark solitons

This section will consider the case when both the components support bright solitons. Therefore, the starting hypothesis will be the same as given in section 3.2. Thus, following the same procedure as adopted in section 3.2, the real component equation (53) reduce to

$$-A_j(\omega + b_j\kappa^2 + c_j\omega\kappa)\tanh^{P_j}\xi$$
$$+A_n(\kappa a_j + k_j)\tanh^{P_j}\xi$$
$$+d_j A_j^{2m+1}\tanh^{(2m+1)p_j}\xi + b_j p_j A_j B^2$$
$$\times \left\{ \begin{array}{l}(p_j-1)\tanh^{P_j-2}\xi - 2p_j\tanh^{P_j}\xi \\ +(p_j+1)\tanh^{P_j+2}\xi\end{array}\right\} \tag{63}$$
$$-c_j p_j v A_j B^2$$
$$\times \left\{ \begin{array}{l}(p_j-1)\tanh^{P_j-2}\xi - 2p_j\tanh^{P_j}\xi \\ +(p_j+1)\tanh^{P_j+2}\xi\end{array}\right\} = 0$$

From equation (63), equating the exponent pair $(p_j + 2, (2m+1)p_j)$ leads to

$$p_j = \frac{1}{m}$$

Next from stand-alone linearly independent functions in (63)

$$p_j = 1$$

From the above two expressions for p_j, one arrives at $m = 1$. This shows that for dark soliton solution, power law nonlinearity reduces to Kerr law nonlinear medium. Therefore all results from (29) through (37) remain valid for this section, as well.

4.3 Singular solitons (Form-I)

This section will consider the case when both the components support singular solitons of Form-I. Therefore, the starting hypothesis will be the same as given in subsections 3.3 for singular solitons of Form-I. Thus following the same procedure as adopted in subsection 3.3, the real component equation (53) reduce to

$$
\begin{aligned}
&- A_j(\omega + b_j \kappa^2 + c_j \omega \kappa) \coth^{p_j} \xi \\
&+ A_n(\kappa a_j + k_j) \coth^{p_n} \xi \\
&+ d_j A_j^{2m+1} \coth^{(2m+1)p_j} \xi + b_j p_j A_j B^2 \\
&\times \left\{ \begin{array}{l} (p_j - 1) \coth^{p_j - 2} \xi - 2 p_j \coth^{p_j} \xi \\ + (p_j + 1) \coth^{p_j + 2} \xi \end{array} \right\} \\
&- c_j p_j v A_j B^2 \\
&\times \left\{ \begin{array}{l} (p_j - 1) \coth^{p_j - 2} \xi - 2 \coth^{p_j} \xi \\ + (p_j + 1) \coth^{p_j + 2} \xi \end{array} \right\} = 0
\end{aligned} \tag{64}
$$

From equation (64), equating the exponent pair $(p_j + 2, (2m+1)p_j)$ leads to

$$p_j = \frac{1}{m}$$

Again from linearly independent functions in (64), one recovers:

$$p_j = 1$$

Thus similarly, as in dark solitons, singular soliton (Form-I) solutions, for power law nonlinearity, reduce to the solutions of Kerr law nonlinearity. Therefore all

discussion from Section 3.3 after equation (39) are valid, for power law nonlinearity as well.

4.4 Singular solitons (Form-II)

This section will consider the case when both the components support singular solitons of Form-II. Therefore, the starting hypothesis will be the same as given in subsections 3.4 for singular solitons of Form-II. Thus, following the same procedure as adopted in subsection 3.4, the real component equation (53) reduce to

$$
\begin{aligned}
&- A_j \omega \operatorname{csch}^{p_j} \xi + \kappa a_j A_n \operatorname{csch}^{p_m} \xi \\
&- b_j p_j^2 A_j B^2 \operatorname{csch}^{p_j} \xi - b_j p_j (p_j + 1) \\
&\times A_j B^2 \operatorname{csch}^{p_j + 2} \xi - b_j \kappa^2 A_j \operatorname{csch}^{p_j} \xi \\
&+ c_j \kappa \omega A_j \operatorname{csch}^{p_j} \xi - c_j p_j^2 v A_j B^2 \operatorname{csch}^{p_j} \xi \\
&- c_j p_j (1 + p_j) v A_j B^2 \operatorname{csch}^{p_j + 2} \xi \\
&+ d_j A_j^{2m+1} \operatorname{csch}^{(2m+1)p_j} \xi + \kappa_j A_n \operatorname{csch}^{p_n} \xi = 0
\end{aligned} \tag{65}
$$

From equation (65), equating the exponent pair $(p_j + 2, (2m+1)p_j)$ leads to

$$p_j = \frac{1}{m}$$

The system of equations after setting the coefficients of linearly independent function, $\operatorname{csch}^{p_j + l} \xi$ where $j = 1, 2$ and $l = 0, 2$, to zero in equation (65) leads to the wave number of the solitons being

$$
\omega = \frac{2m^2 b_j A_j \kappa^2 - 2(k_j + \kappa a_j) A_n - d_j A_j^3}{m^2 A_j (c_j \kappa - 1)} \tag{66}
$$

and

$$
\omega = \frac{m^2 b_j \kappa^2 + B^2 (b_j - c_j v)}{m^2 (c_j \kappa - 1)} \tag{67}
$$

The width of the solitons is given by

$$
B = m A_j \sqrt{-\frac{d_j}{2(b_j - c_j v)}} \tag{68}
$$

The width of the solitons B provokes the constraint condition, for $j = 1, 2$, as given by (32). After setting the two B's equal to one another for $j = 1, 2$ in equation (68). This leads to another relation which is similar to (21). By inserting equations (68) and (21) into (66), one obtains

$$\omega = \frac{m^2 b_j \kappa^2 + B^2(b_j - c_j v)}{m^2(c_j \kappa - 1)}$$

$$- \frac{k_j + \kappa a_j}{c_j \kappa - 1}\left[\frac{d_j(b_n - c_n v)}{d_n(b_j - c_j v)}\right]^{\frac{1}{2}} \qquad (69)$$

After setting the two values of ω equal to one another for $j = 1, 2$ in equation (67).

This leads to another relation for B

$$B =$$

$$m\kappa\left[\frac{b_2(c_1\kappa - 1) - b_1(c_2\kappa - 1)}{(b_1 - c_1 v)(c_2\kappa - 1) - (b_2 - c_2 v)(c_1\kappa - 1)}\right]^{\frac{1}{2}} \quad (70)$$

The above equation gives a constraint condition given by (49). Thus, the singular 1-soliton solution of Form-II to the coupled system (53), for $j = 1, 2$ is

$$\psi_j(x,t) = A_j \operatorname{csch}^{\frac{1}{m}}[B(x - vt)]$$
$$\times e^{i(-\kappa x + \omega t + \theta)} \qquad (71)$$

5. Conclusion

This paper obtained bright dark and singular 1-soliton solutions to the model for dual-core optical fibers. The ansatz method is applied to carry out the integration with Kerr and power laws of nonlinearities. Bright, dark and singular soliton solutions are recovered. It is observed that dark soliton and singular solitons (Form-I), for power law nonlinearity reduce to the results for Kerr law nonlinearity. The results of this paper stands on a strong footing. In future, additional integration schemes will be applied to obtain soliton as well as other solutions to dual-core fibers. Some of these additional solutions are cnoidal waves, snoidal waves, singular periodic functions and others. These results will be reported soon.

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