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Optical solitons for nonlinear coupled Klein-Gordon equations

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In this paper, we obtained the 1-soliton solution of the coupled, two coupled and three coupled nonlinear Klein-Gordon equations. By using ansatz method, we found exact analytical bright and dark optical soliton solutions for the considered models. As a result, exact solutions with amplitude, inverse width, velocity and free parameters are obtained. The power of this manageable method is presented by applying it to several examples. This approach can also be applied to other nonlinear differential equations system.

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1. Introduction

In the theoretical investigation of the dynamics of nonlinear waves in physical systems several kinds of nonlinear partial differential equations (NLPDEs) has made significant progress in the past decades. There has been a growing interest in finding exact analytical solutions to nonlinear wave equations by using appropriate techniques. Particularly, the existence of soliton solutions for nonlinear partial differential equations are of great importance because of their potential application in many areas such as chaos, mathematical biology, diffusion process, quantum mechanics, optical fibers, neural physics, chemical physics, solid state physics, plasma physics and so on. It should be noted that the propagation behavior of nonlinear waves depends on the model coefficients which can be constant or variable parameters depending on the physical situation.

There are various methods of integration of which used to carry out the integration of NLPDEs. Namely, F-expansion method [1,2], homotopy perturbation method [3,4], tanh-sech method [5-7], extended tanh method [8,9], $\left(\frac{G'}{G}\right)$ -expansion method [10,11], sine-cosine method [12,13], exp-function method [14,15], homogeneous balance method [16,17], Jacobi elliptic function method [18], trial equation method [19,20], first integral method [21,22], others.

Optical solitons are wave packets or pulses which propagate in nonlinear dispersive media. Due to dynamical balance between the nonlinear and dispersive effects these waves retain a stable waveform. A soliton is a very special type of solitary wave, which also keeps its waveform after collision with other solitons [23]. Solitons in photonic crystal fibers as well as diffraction Bragg gratings have been studied. In addition, theories of dispersion managed solitons,quasi-linear pulses have also been developed. Dark solitons are also known as topological optical solitons in the context of nonlinear optics media. Dark optical solitons are more stable in presence of noise and spreads more slowly in presence of loss, in the optical communication systems, which compared with bright solitons in [24-27].

The integrability properties such as bilinear form and soliton solution of the coupled, two, three and N-coupled Klein–Gordon equations have been constructed by Alagesan in [28]. In this paper, one such modern method of integrability will be applied to carry out the integration of coupled nonlinear Klein-Gordon equations. The technique that will be adopted to integrate such equations is the solitary wave ansatz method.

The paper is organized as follows: In section 2, we derived the bright and dark optical soliton solutions of coupled nonlinear Klein-Gordon equations. In section 3, we apply the ansatz method to the two coupled nonlinear Klein-Gordon equations. In section 4, we apply the ansatz method to the three coupled nonlinear Klein-Gordon equations and establish many soliton solutions. In the last section, we briefly make a summary to the results obtained in the former sections.

2. Coupled nonlinear Klein-Gordon equations

Nonlinear Klein–Gordon equations couple with a scaler field v is of the form

$$u_{xx} - u_{tt} - u + 2u^3 + 2uv = 0, \tag{1}$$

$$v_x - v_t - 4uu_t = 0, \tag{2}$$

introduced in [29].

The solitary wave ansatz method proposed by Biswas [30] and Triki et al. [31] is particularly notable in its power and applicability in solving nonlinear problems, and it has been successfully applied to many kinds of nonlinear partial differential equations [32-35].

2.1 Bright optical soliton solution

To find exact bright optical soliton solution of equations (1)-(2), we introduce the following solitary wave ansatz

$$u(x,t) = A_1 sech^p \tau, \tag{3}$$

$$v(x,t) = A_2 sech^r \tau, \tag{4}$$

and

$$\tau = \eta\left(x - vt\right), \tag{5}$$

where A_1, A_2, η and v are constant coefficients. Here A_1 and A_2 are the soliton amplitude, η is the inverse width of the soliton and v is the soliton velocity. The exponents p and r are unknown at this point and will be determined later. From the ansatz Eqs. (3)-(5) we get,

$$u_{xx} = p^2\eta^2 A_1 sech^p\tau - p(p+1)\eta^2 A_1 sech^{p+2}\tau, \tag{6}$$

$$u_{tt} = p^2\eta^2 v^2 A_1 sech^p\tau - p(p+1)v^2\eta^2 A_1 sech^{p+2}\tau, \tag{7}$$

$$u^3 = A_1^3 sech^{3p}\tau, \tag{8}$$

$$v_x = -A_2 r\eta sech^r\tau \tanh\tau, \tag{9}$$

$$v_t = A_2 rv\eta sech^r\tau \tanh\tau, \tag{10}$$

$$uu_t = A_1^2 pv\eta sech^{2p}\tau \tanh\tau. \tag{11}$$

Substituting (6)-(11) into (1)-(2) yields

$$\begin{aligned} &p^2\eta^2 A_1 sech^p\tau - p(p+1)\eta^2 A_1 sech^{p+2}\tau \\ &-p^2\eta^2 v^2 A_1 sech^p\tau + p(p+1)v^2\eta^2 A_1 sech^{p+2}\tau \\ &-A_1 sech^p\tau + 2A_1^3 sech^{3p}\tau + 2A_1A_2 sech^{p+r}\tau = 0, \end{aligned} \tag{12}$$

and

$$\begin{aligned} &-A_2 r\eta sech^r\tau \tanh\tau - A_2 rv\eta sech^r\tau \tanh\tau \\ &-4A_1^2 pv\eta sech^{2p}\tau \tanh\tau = 0. \end{aligned} \tag{13}$$

Equating the exponents of $sech^{3p}\tau$ and $sech^{p+2}\tau$ term in equation (12), we have

$$3p = p + 2, \tag{14}$$

and consequently

$$p = 1. \tag{15}$$

Again from (13) equating the exponents of $sech^{2p}\tau \tanh\tau$ and $sech^r\tau \tanh\tau$ gives

$$2p = r, \tag{16}$$

which leads to

$$r = 2. \tag{17}$$

With (15) setting the coefficients of $sech^p\tau$ to zero in Eq. (12),

$$p^2\eta^2 A_1 - p^2\eta^2 v^2 A_1 - A_1 = 0, \tag{18}$$

using Eqs. (15) and (17), we have

$$v = \pm\frac{\sqrt{\eta^2 - 1}}{\eta} \quad ,\eta \neq 0. \tag{19}$$

Again from Eq. (13), setting the coefficients of $sech^{2p}\tau \tanh\tau$ and $sech^r\tau \tanh\tau$ terms to zero one obtains:

$$-A_1 r\eta - A_2 rv\eta - 4A_1^2 pv\eta = 0, \tag{20}$$

and using Eqs. (15), (17) and (19):

$$A_1 = \frac{\sqrt{-2A_2\sqrt{\eta^2 - 1}\left(\eta + \sqrt{\eta^2 - 1}\right)}}{2\sqrt{\eta^2 - 1}}. \tag{21}$$

From Eq. (21), it is possible to see that the solitons will exist provided

$$\eta \neq \pm 1 \text{ and } A_2\left(\eta + \sqrt{\eta^2 - 1}\right) < 0. \tag{22}$$

Setting the coefficients of $sech^{p+2}\tau$, $sech^{3p}\tau$ and $sech^{p+r}\tau$ zero in Eq. (12) gives

$$-p(p+1)\eta^2 A_1 + p(p+1)v^2\eta^2 A_1 + 2A_1^3 + 2A_1A_2 = 0, \tag{23}$$

which leads to

$$A_2 = \frac{2\sqrt{\eta^2 - 1}}{\sqrt{\eta^2 - 1} - \eta}, \tag{24}$$

by using Eqs. (15), (17), (19) and (21).

Finally, the bright optical soliton solution for the

Coupled Klein-Gordon Equation (1)-(2) is

$$u(x,t) = A_1 sech\tau, \tag{25}$$

and

$$v(x,t) = A_2 sech^2\tau, \tag{26}$$

where the velocity of the solitons v is given by (19) and the soliton amplitude A_1 and A_2 are given by (21) and (24). When we write (24) in (21), we see from (19), (21) and (24) that the velocity of the solitons v and the soliton amplitude A_1 and A_2 are depend on the inverse width of the soliton η.

2.2 Dark optical soliton solution

In this subsection, we are interested in finding the dark optical soliton solution (expressed as hyperbolic tangent function), as defined in [36,37], for the considered coupled nonlinear Klein-Gordon equations (1)-(2). In order to construct optical soliton solution, we use an ansatz solution of the form

$$u(x,t) = A_1 \tanh^p \tau, \tag{27}$$

$$v(x,t) = A_2 \tanh^r \tau, \tag{28}$$

where

$$\tau = \eta\left(x - vt\right). \tag{29}$$

Here in (31)-(34), A_1, A_2 and η are the free parameters of the solitons and v is the velocity of the soliton. The exponents p and r are unknown at this point and their values will fall out in the process of deriving the solution of this equation. From (27)-(29) we have

$$u_{tt} = pv^2 A_1\eta^2\{(p-1)\tanh^{p-2}\tau - 2p\tanh^p\tau + (p+1)\tanh^{p+2}\tau\}, \tag{30}$$

$$u_{xx} = pA_1\eta^2\left\{(p-1)\tanh^{p-2}\tau - 2p\tanh^p\tau + (p+1)\tanh^{p+2}\tau\right\}, \tag{31}$$

$$u^3 = A_1^3 \tanh^{3p}\tau, \tag{32}$$

$$v_x = rA_2\eta\left\{\tanh^{r-1}\tau - \tanh^{r+1}\tau\right\}, \tag{33}$$

$$v_t = rvA_2\eta\left\{\tanh^{r+1}\tau - \tanh^{r-1}\tau\right\}, \tag{34}$$

$$uu_t = pvA_1^2\eta\left\{\tanh^{2p+1}\tau - \tanh^{2p-1}\tau\right\}. \tag{35}$$

By substituting (30)-(35) into (1)-(2) respectively, we have

$$\begin{aligned} &pv^2 A_1\eta^2\{(p-1)\tanh^{p-2}\tau - 2p\tanh^p\tau + (p+1)\tanh^{p+2}\tau\} \\ &- pA_1\eta^2\left\{(p-1)\tanh^{p-2}\tau - 2p\tanh^p\tau + (p+1)\tanh^{p+2}\tau\right\} \\ &- A_1\tanh^p\tau + 2A_1^3\tanh^{3p}\tau + 2A_1A_2\tanh^{p+r}\tau \\ &= 0, \end{aligned} \tag{36}$$

and

$$\begin{aligned} &rA_2\eta\left\{\tanh^{r-1}\tau - \tanh^{r+1}\tau\right\} - rvA_2\eta\left\{\tanh^{r+1}\tau - \tanh^{r-1}\tau\right\} \\ &-4pvA_1^2\eta\left\{\tanh^{2p+1}\tau - \tanh^{2p-1}\tau\right\} = 0. \end{aligned} \tag{37}$$

From Eq. (36), equating the exponents of $\tanh^{3p}\tau$ and $\tanh^{p+2}\tau$ we have

$$3p = p + 2, \tag{38}$$

and consequently

$$p = 1. \tag{39}$$

Again from Eq. (37) equating the exponents of $\tanh^{2p+1}$ and $\tanh^{r+1}\tau$ gives

$$2p + 1 = r + 1, \tag{40}$$

which leads to

$$r = 2. \tag{41}$$

with Eq. (39).

From Eq. (37), vanishing the coefficients of the linearly independent functions $\tanh^{2p+j}\tau$ and $\tanh^{p+j}\tau$ (with $p=1$ and $r=2$), where $j=-1,1$ we have

$$2A_2\eta + 2vA_2\eta + 4vA_1^2\eta = 0, \tag{42}$$

which gives rise to

$$A_2 = -\frac{2A_1^2 v}{v+1}. \tag{43}$$

Again from Eq. (36), vanishing the coefficients of $\tanh^p\tau$ one obtains:

$$-2p^2v^2A_1\eta^2 + 2p^2A_1\eta^2 - A_1 = 0, \tag{44}$$

which gives rise to

$$\eta = \pm \frac{1}{\sqrt{2v^2 - 2}}, \tag{45}$$

by using Eqs. (39), (41) and (43).

From Eq. (45), it is possible to see that the solitons exist provided

$$v \notin R - [-1,1]. \tag{46}$$

From Eq. (36), vanishing the coefficients of $\tanh^{3p}\tau$ $\tanh^{p+2}\tau$ and $\tanh^{p+r}\tau$ one obtains

$$p(p+1)v^2 A_1\eta^2 - p(p+1)A_1\eta^2 + 2A_1^3 + 2A_1A_2 = 0, \tag{47}$$

so that (with $p=1$ and $r=2$) using Eqs. (43), (45), we have

$$A_1 = \sqrt{\frac{v+1}{2-2v}}, \qquad v \neq 1. \tag{48}$$

Substituting Eq. (48) into Eq. (43) we obtain

$$A_2 = \pm \frac{v}{v-1}, \qquad v \neq 1. \tag{49}$$

Finally, the dark soliton solution for Coupled Klein-Gordon Equation (27)-(29) is given by

$$u(x,t) = A_1 \tanh\tau, \tag{50}$$

and

$$v(x,t) = A_2 \tanh^2\tau, \tag{51}$$

where the free parameters A_1, A_2 and η are given in (45), (48) and (49). We see that the free parameters A_1, A_2 and η are depend on the velocity of the soliton v.

Remark 1: The exact solutions of Eqs. (1)-(2) were found by using the general integral method, the tanh method, the infinite series method and the Reduced Differential Transform Method (RDTM) in [29,38-40] respectively. After, comparing our results with results in [39], we see that the results (25)-(26) are same. If proper η value are chosen, then it can be seen that the results are same. But we can say that exact solutions of Eqs. (1)-(2) result's (50)-(51), it can be seen that the results are new.

3. Two-coupled nonlinear Klein-Gordon equations

Two-coupled nonlinear Klein–Gordon equations are defined as

$$u_{1xx} - u_{1tt} - u_1 + 2\left(u_1^2 + u_2^2 + v\right)u_1 = 0, \tag{52}$$

$$u_{2xx} - u_{2tt} - u_2 + 2\left(u_1^2 + u_2^2 + v\right)u_2 = 0, \tag{53}$$

$$v_x - v_t - 4\left(u_1u_{1t} + u_2u_{2t}\right) = 0, \tag{54}$$

in [38].

3.1 Bright optical soliton solution

To study the bright optical soliton solutions of two-coupled nonlinear Klein-Gordon equations, we assume the solitary wave ansatz of the form

$$u_1(x,t) = A_1 sech^p\tau, \tag{55}$$

$$u_2(x,t) = A_2 sech^k\tau, \tag{56}$$

$$v(x,t) = A_3 sech^r\tau, \tag{57}$$

and

$$\tau = \eta\left(x - vt\right), \tag{58}$$

where A_1, A_2, A_3, η and v are constant coefficients. Here A_1, A_2 and A_3 are the solitons amplitude, η is the inverse width of the solitons and v is the solitons velocity. The exponents p, k and r are unknown at this point and will be determined later. From the ansatz (55)-(58) we obtains:

$$u_{1xx} = p^2\eta^2 A_1 sech^p\tau - p(p+1)\eta^2 A_1 sech^{p+2}\tau, \tag{59}$$

$$u_{1tt} = p^2\eta^2 v^2 A_1 sech^p\tau - p(p+1)v^2\eta^2 A_1 sech^{p+2}\tau, \tag{60}$$

$$u_1^3 = A_1^3 sech^{3p}\tau, \tag{61}$$

$$u_2^2 u_1 = A_1 A_2^2 sech^{2k+p}\tau, \tag{62}$$

$$vu_1 = A_1 A_3 sech^{p+r}\tau, \tag{63}$$

$$u_{2xx} = k^2\eta^2 A_2 sech^k\tau - k(k+1)\eta^2 A_2 sech^{k+2}\tau, \tag{64}$$

$$u_{2tt} = k^2\eta^2 v^2 A_2 sech^k\tau - k(k+1)v^2\eta^2 A_2 sech^{k+2}\tau, \tag{65}$$

$$u_2^3 = A_2^3 sech^{3k}\tau, \tag{66}$$

$$u_1^2 u_2 = A_1^2 A_2 sech^{2p+k}\tau, \tag{67}$$

$$vu_2 = A_2 A_3 sech^{k+r}\tau, \tag{68}$$

$$v_x = -A_3 r\eta sech^r\tau \tanh\tau, \tag{69}$$

$$v_t = A_3 rv\eta sech^r\tau \tanh\tau, \tag{70}$$

$$u_1 u_{1t} = A_1^2 pv\eta sech^{2p}\tau \tanh\tau, \tag{71}$$

$$u_2 u_{2t} = A_2^2 kv\eta sech^{2k}\tau \tanh\tau. \tag{72}$$

Substituting (59)-(72) into (52)-(54) yields

$$\begin{aligned} &p^2\eta^2 A_1 sech^p\tau - p(p+1)\eta^2 A_1 sech^{p+2}\tau \\ &-p^2\eta^2 v^2 A_1 sech^p\tau + p(p+1)v^2\eta^2 A_1 sech^{p+2}\tau \\ &-A_1 sech^p\tau + 2A_1^3 sech^{3p}\tau + 2A_1 A_2^2 sech^{2k+p}\tau \\ &+2A_1 A_3 sech^{p+r}\tau = 0, \end{aligned} \tag{73}$$

$$\begin{aligned} &k^2\eta^2 A_2 sech^k\tau - k(k+1)\eta^2 A_2 sech^{k+2}\tau \\ &-k^2\eta^2 v^2 A_2 sech^k\tau + k(k+1)v^2\eta^2 A_2 sech^{k+2}\tau \\ &-A_2 sech^k\tau + 2A_1^2 A_2 sech^{2p+k}\tau + 2A_2^3 sech^{3k}\tau \\ &+2A_2 A_3 sech^{k+r}\tau = 0, \end{aligned} \tag{74}$$

and

$$\begin{aligned} &-A_3 r\eta sech^r\tau \tanh\tau - A_3 rv\eta sech^r\tau \tanh\tau \\ &-4A_1^2 pv\eta sech^{2p}\tau \tanh\tau - 4A_2^2 kv\eta sech^{2k}\tau \tanh\tau \\ &= 0. \end{aligned} \tag{75}$$

Equating the exponents of $sech^{3p}\tau$ and $sech^{p+2}\tau$ terms in equation (73), one obtains

$$3p = p + 2, \tag{76}$$

so that

$$p = 1. \tag{77}$$

From Eq. (74) equating the exponents of $sech^{3k}\tau$ and $sech^{k+2}\tau$,

$$3k = k + 2, \tag{78}$$

we get

$$k = 1. \tag{79}$$

Also, equating the exponents of $sech^r\tau \tanh\tau$, $sech^{2p}\tau \tanh\tau$ and $sech^{2k}\tau \tanh\tau$ term in Eq. (75) gives rise to

$$2p = 2k = r, \tag{80}$$

with Eqs. (77) and (79) that leads to

$$r = 2. \tag{81}$$

Setting the coefficients of $sech^p\tau$ zero in Eq. (73), we have

$$p^2\eta^2 A_1 - p^2\eta^2 v^2 A_1 - A_1 = 0, \tag{82}$$

and using Eq. (77) that gives:

$$v = \pm\frac{\sqrt{\eta^2 - 1}}{\eta}, \eta \neq 0. \tag{83}$$

Again from Eq. (74), setting the coefficients of $sech^{k+2}\tau$, $sech^{2p+k}\tau$, $sech^{3k}\tau$ and $sech^{k+r}\tau$ terms zero one obtains:

$$-k(k+1)\eta^2 A_2 + k(k+1)v^2\eta^2 A_2 + 2A_1^2 A_2 + 2A_2^3 + 2A_2 A_3 = 0, \tag{84}$$

the latter gives

$$A_1 = \pm\sqrt{1 - A_2^2 - A_3}, \tag{85}$$

by using Eqs. (86) and (90).

Finally, varying the coefficients of $sech^r\tau \tanh\tau$, $sech^{2p}\tau \tanh\tau$ and $sech^{2k}\tau \tanh\tau$ in Eq. (82) gives

$$-A_3 r\eta - A_3 rv\eta - 4A_1^2 pv\eta - 4A_2^2 kv\eta = 0, \tag{86}$$

this leads to

$$A_3 = \frac{2\sqrt{\eta^2 - 1}}{\sqrt{\eta^2 - 1} - \eta}, \tag{87}$$

by using Eqs. (77), (79), (81), (83) and (85).

The bright optical soliton solution for the two-Coupled nonlinear Klein-Gordon equations (52)-(54) is

$$u_1(x,t) = A_1 sech\tau, \tag{88}$$

$$u_2(x,t) = A_2 sech\tau, \tag{89}$$

and

$$v(x,t) = A_3 sech^2\tau, \tag{90}$$

where the velocity of the solitons v is given by (83) and the soliton amplitudes A_1 and A_3 are given by (85) and (87).

3.2 Dark optical soliton solution

In order the start off with the solution hypothesis, the following ansatz is assumed

$$u_1(x,t) = A_1 \tanh^p \tau, \tag{91}$$

$$u_2(x,t) = A_2 \tanh^k \tau, \tag{92}$$

$$v(x,t) = A_3 \tanh^r \tau, \tag{93}$$

and

$$\tau = \eta\left(x - vt\right). \tag{94}$$

Here in (91)-(94); A_1, A_2, A_3 and η are the free parameters of the solitons and v is the velocity of the soliton. The exponents p, k and r are unknown. These will be determined. From (91)-(94), we obtain

$$u_{1xx} = pA_1\eta^2\left\{(p-1)\tanh^{p-2}\tau - 2p\tanh^p\tau + (p+1)\tanh^{p+2}\tau\right\}, \tag{95}$$

$$u_{1tt} = pv^2A_1\eta^2\{(p-1)\tanh^{p-2}\tau - 2p\tanh^p\tau + (p+1)\tanh^{p+2}\tau\}, \tag{96}$$

$$u_1^3 = A_1^3\tanh^{3p}\tau, \tag{97}$$

$$u_2^2u_1 = A_1A_2^2\tanh^{2k+p}\tau, \tag{98}$$

$$vu_1 = A_1A_3\tanh^{p+r}\tau, \tag{99}$$

$$u_{2xx} = kA_2\eta^2\left\{(k-1)\tanh^{k-2}\tau - 2k\tanh^k\tau + (k+1)\tanh^{k+2}\tau\right\}, \tag{100}$$

$$u_{2tt} = kv^2A_2\eta^2\{(k-1)\tanh^{k-2}\tau - 2k\tanh^k\tau + (k+1)\tanh^{k+2}\tau\}, \tag{101}$$

$$u_2^3 = A_2^3\tanh^{3k}\tau, \tag{102}$$

$$u_1^2u_2 = A_1^2A_2\tanh^{2p+k}\tau, \tag{103}$$

$$vu_2 = A_2A_3\tanh^{k+r}\tau, \tag{104}$$

$$v_x = rA_3\eta\left\{\tanh^{r-1}\tau - \tanh^{r+1}\tau\right\}, \tag{105}$$

$$v_t = rvA_3\eta\left\{\tanh^{r+1}\tau - \tanh^{r-1}\tau\right\}, \tag{106}$$

$$u_1u_{1t} = pvA_1^2\eta\left\{\tanh^{2p+1}\tau - \tanh^{2p-1}\tau\right\}, \tag{107}$$

$$u_2u_{2t} = kvA_2^2\eta\left\{\tanh^{2k+1}\tau - \tanh^{2k-1}\tau\right\}. \tag{108}$$

Substituting (95)-(108) into (52)-(54) respectively yields

$$\begin{aligned} &pA_1\eta^2\left\{(p-1)\tanh^{p-2}\tau - 2p\tanh^p\tau + (p+1)\tanh^{p+2}\tau\right\} \\ &-pv^2A_1\eta^2\{(p-1)\tanh^{p-2}\tau - 2p\tanh^p\tau + (p+1)\tanh^{p+2}\tau\} \\ &-A_1\tanh^p\tau + 2A_1^3\tanh^{3p}\tau + 2A_1A_2^2\tanh^{2k+p}\tau + 2A_1A_3\tanh^{p+r}\tau \\ &= 0, \end{aligned} \tag{109}$$

$$\begin{aligned} &kA_2\eta^2\left\{(k-1)\tanh^{k-2}\tau - 2k\tanh^k\tau + (k+1)\tanh^{k+2}\tau\right\} \\ &-kv^2A_2\eta^2\{(k-1)\tanh^{k-2}\tau - 2k\tanh^k\tau + (k+1)\tanh^{k+2}\tau\} \\ &-A_2\tanh^k\tau + 2A_1^2A_2\tanh^{2p+k}\tau + 2A_2^3\tanh^{3k}\tau + 2A_2A_3\tanh^{k+r}\tau \\ &= 0, \end{aligned} \tag{110}$$

and

$$\begin{aligned} &rA_3\eta\left\{\tanh^{r-1}\tau - \tanh^{r+1}\tau\right\} - rvA_3\eta\left\{\tanh^{r+1}\tau - \tanh^{r-1}\tau\right\} \\ &-4pvA_1^2\eta\left\{\tanh^{2p+1}\tau - \tanh^{2p-1}\tau\right\} \\ &-4kvA_2^2\eta\left\{\tanh^{2k+1}\tau - \tanh^{2k-1}\tau\right\} = 0. \end{aligned} \tag{111}$$

From Eq. (109), equating the exponents of $\tanh^{3p}\tau$ and $\tanh^{p+2}\tau$ we have

$$3p = p + 2, \tag{112}$$

so that

$$p = 1. \tag{113}$$

From Eq. (110), equating the exponents of $\tanh^{3k}\tau$ and $\tanh^{k+2}\tau$ we get

$$3k = k + 2, \tag{114}$$

we get

$$k = 1. \tag{115}$$

Also from Eq. (111), equating the exponents of

$\tanh^{2k+1}, \tanh^{2p+1}$ and $\tanh^{r+1}\tau$ we have

$$2p+1=2k+1=r+1, \quad (116)$$

which leads to

$$r=2, \quad (117)$$

by using Eqs. (113) and (115).

Now from Eq. (109), setting the coefficients of $\tanh^{p}\tau$ terms zero, one obtains

$$-2p^2A_1\eta^2+2p^2v^2A_1\eta^2-A_1=0, \quad (118)$$

that gives

$$\eta=\pm\frac{1}{\sqrt{2v^2-2}}, \quad (119)$$

by using Eq. (113).

From Eq. (119), it is possible to see that the solitons exist provided

$$v\notin R-[-1,1]. \quad (120)$$

Then, from Eq. (110), setting the coefficients of $\tanh^{3k}\tau$, $\tanh^{k+2}\tau$, $\tanh^{2p+k}\tau$ and $\tanh^{k+r}\tau$ terms zero, we have

$$k(k+1)A_2\eta^2-k(k+1)v^2A_2\eta^2+2A_1^2A_2+2A_2^3+2A_2A_3=0, \quad (121)$$

so that (with $k=1$)

$$A_3=\frac{1}{2}-A_1^2-A_2^2, \quad (122)$$

by using Eq. (119).

From Eq. (111), variying the coefficients of the linearly independent functions $\tanh^{2p+j}\tau$ and $\tanh^{r+j}\tau$ or $\tanh^{2k+j}\tau$ and $\tanh^{r+j}\tau$ (with $p=1$, $k=1$ and $r=2$), where $j=-1,1$ we have

$$-2A_3\eta-2vA_3\eta-4vA_1^2\eta-4vA_2^2\eta=0, \quad (123)$$

$$A_1=\sqrt{\frac{v+1+2A_2^2(v-1)}{2-2v}}, \quad (124)$$

and so the solitons exist provided $(1-v)(v+1+2A_2^2(v-1))>0$.

Substituting Eq. (124) into Eq. (122) we obtain

$$A_3=\frac{1}{2}-\frac{v+1+2A_2^2(v-1)}{2v-2}-A_2^2......,v\neq 1. \quad (125)$$

Finally, the dark optical soliton solution for two-coupled nonlinear Klein-Gordon equations (55)-(58) is

$$u_1(x,t)=A_1\tanh\tau, \quad (126)$$

$$u_2(x,t)=A_2\tanh\tau, \quad (127)$$

and

$$v(x,t)=A_3\tanh^2\tau, \quad (128)$$

where the free parameters A_1, A_3 and η are given in Eqs. (124), (122) or (125) and (119). We see that the free parameters A_1, A_3 and η are depend on the velocity of the soliton v and the other free parameter A_2.

Remark 2: Comparing our results with results in [29,38], we see that the results are new.

4. Three-coupled nonlinear Klein-Gordon equations

The three-coupled Klein–Gordon equations are in the following general form:

$$u_{1xx}-u_{1tt}-u_1+2\left(u_1^2+u_2^2+u_3^2+v\right)u_1=0, \quad (129)$$

$$u_{2xx}-u_{2tt}-u_2+2\left(u_1^2+u_2^2+u_3^2+v\right)u_2=0, \quad (130)$$

$$u_{3xx}-u_{3tt}-u_3+2\left(u_1^2+u_2^2+u_3^2+v\right)u_3=0, \quad (131)$$

$$v_x-v_t-4\left(u_1u_{1t}+u_2u_{2t}+u_3u_{3t}\right)=0. \quad (132)$$

4.1 Bright optical soliton solution

Let us begin the analysis by assuming an ansatz solution of the form

$$u_1(x,t)=A_1 sech^{p}\tau, \quad (133)$$

$$u_2(x,t)=A_2 sech^{k}\tau, \quad (134)$$

$$u_3(x,t) = A_3 sech^r \tau, \tag{135}$$

$$v(x,t) = A_4 sech^s \tau, \tag{136}$$

and

$$\tau = \eta\left(x - vt\right), \tag{137}$$

where A_1, A_2, A_3, A_4, η and v are constant coefficients. Here A_1, A_2, A_3 and A_4 are the soliton amplitudes, η is the inverse width of the solitons and v is the solitons velocity The exponents p, k, r and s are unknown at this point and will be determined later. From the ansatz (133)-(137) we obtain

$$u_{1xx} = p^2\eta^2 A_1 sech^p\tau - p(p+1)\eta^2 A_1 sech^{p+2}\tau, \tag{138}$$

$$u_{1tt} = p^2\eta^2 v^2 A_1 sech^p\tau - p(p+1)v^2\eta^2 A_1 sech^{p+2}\tau, \tag{139}$$

$$u_1^3 = A_1^3 sech^{3p}\tau, \tag{140}$$

$$u_2^2 u_1 = A_1 A_2^2 sech^{2k+p}\tau, \tag{141}$$

$$u_3^2 u_1 = A_1 A_3^2 sech^{2r+p}\tau, \tag{142}$$

$$vu_1 = A_1 A_4 sech^{p+s}\tau, \tag{143}$$

$$u_{2xx} = k^2\eta^2 A_2 sech^k\tau - k(k+1)\eta^2 A_2 sech^{k+2}\tau, \tag{144}$$

$$u_{2tt} = k^2\eta^2 v^2 A_2 sech^k\tau - k(k+1)v^2\eta^2 A_2 sech^{k+2}\tau, \tag{145}$$

$$u_2^3 = A_2^3 sech^{3k}\tau, \tag{146}$$

$$u_1^2 u_2 = A_1^2 A_2 sech^{2p+k}\tau, \tag{147}$$

$$u_3^2 u_2 = A_3^2 A_2 sech^{2r+k}\tau, \tag{148}$$

$$vu_2 = A_2 A_4 sech^{k+s}\tau, \tag{149}$$

$$u_{3xx} = r^2\eta^2 A_2 sech^r\tau - r(r+1)\eta^2 A_2 sech^{r+2}\tau, \tag{150}$$

$$u_{3tt} = r^2\eta^2 v^2 A_2 sech^r\tau - r(r+1)v^2\eta^2 A_2 sech^{r+2}\tau, \tag{151}$$

$$u_3^3 = A_3^3 sech^{3r}\tau, \tag{152}$$

$$u_1^2 u_3 = A_1^2 A_3 sech^{2p+r}\tau, \tag{153}$$

$$u_2^2 u_3 = A_2^2 A_3 sech^{2k+r}\tau, \tag{154}$$

$$vu_3 = A_3 A_4 sech^{r+s}\tau, \tag{155}$$

$$v_x = -A_4 s\eta sech^s\tau \tanh\tau, \tag{156}$$

$$v_t = A_4 sv\eta sech^s\tau \tanh\tau, \tag{157}$$

$$u_1 u_{1t} = A_1^2 pv\eta sech^{2p}\tau \tanh\tau, \tag{158}$$

$$u_2 u_{2t} = A_2^2 kv\eta sech^{2k}\tau \tanh\tau, \tag{159}$$

$$u_3 u_{3t} = A_3^2 rv\eta sech^{2r}\tau \tanh\tau. \tag{160}$$

Substituting (138)-(160) into (129)-(132), respectively yields

$$\begin{aligned} &p^2\eta^2 A_1 sech^p\tau - p(p+1)\eta^2 A_1 sech^{p+2}\tau \\ &-p^2\eta^2 v^2 A_1 sech^p\tau + p(p+1)v^2\eta^2 A_1 sech^{p+2}\tau \\ &-A_1 sech^p\tau + 2A_1^3 sech^{3p}\tau + 2A_1 A_2^2 sech^{2k+p}\tau \\ &+2A_3^2 A_1 sech^{2r+k}\tau + 2A_1 A_4 sech^{p+s}\tau \ = 0, \end{aligned} \tag{161}$$

$$\begin{aligned} &k^2\eta^2 A_2 sech^k\tau - k(k+1)\eta^2 A_2 sech^{k+2}\tau \\ &-k^2\eta^2 v^2 A_2 sech^k\tau + k(k+1)v^2\eta^2 A_2 sech^{k+2}\tau \\ &-A_2 sech^k\tau + 2A_1^2 A_2 sech^{2p+k}\tau + 2A_2^3 sech^{3k}\tau \\ &+2A_3^2 A_2 sech^{2r+k}\tau + 2A_2 A_4 sech^{k+s}\tau \ = 0, \end{aligned} \tag{162}$$

$$\begin{aligned} &r^2\eta^2 A_2 sech^r\tau - r(r+1)\eta^2 A_2 sech^{r+2}\tau \\ &-r^2\eta^2 v^2 A_2 sech^r\tau + r(r+1)v^2\eta^2 A_2 sech^{r+2}\tau \\ &-A_3 sech^r\tau + 2A_1^2 A_3 sech^{2p+r}\tau + 2A_2^2 A_3 sech^{2k+r}\tau \\ &+2A_3^3 sech^{3r}\tau + 2A_3 A_4 sech^{r+s}\tau \ = 0, \end{aligned} \tag{163}$$

and

$$\begin{aligned} &-A_4 s\eta sech^s\tau \tanh\tau - A_4 sv\eta sech^s\tau \tanh\tau \\ &-4A_1^2 pv\eta sech^{2p}\tau \tanh\tau - 4A_2^2 kv\eta sech^{2k}\tau \tanh\tau \\ &-4A_3^2 rv\eta sech^{2r}\tau \tanh\tau \quad = 0. \end{aligned} \tag{164}$$

Similarly, equating the exponents of $sech^{3p}\tau$ and $sech^{p+2}\tau$ terms in Eq. (161), one obtains

$$3p = p + 2, \tag{165}$$

so that

$$p = 1. \tag{166}$$

From Eq. (162) equating the exponents of $sech^{3k}\tau$ and $sech^{k+2}\tau$

$$3k = k + 2, \tag{167}$$

we get

$$k = 1. \tag{168}$$

Also, from Eq. (163), equating the exponents of $sech^{3r}\tau$ and $sech^{r+2}\tau$

$$3r = r + 2, \tag{169}$$

we get

$$r = 1. \tag{170}$$

Then equating the exponents of $sech^{s}\tau \tanh\tau$, $sech^{2p}\tau \tanh\tau$, $sech^{2k}\tau \tanh\tau$ and $sech^{2r}\tau \tanh\tau$ term in Eq. (164) gives

$$2p = 2k = 2r = s, \tag{171}$$

and by using Eqs. (166) and (168) it leads to

$$s = 2. \tag{172}$$

Setting the coefficients of $sech^{p}\tau$ zero in Eq. (161), we have

$$p^2\eta^2 A_1 - p^2\eta^2 v^2 A_1 - A_1 = 0, \tag{173}$$

and by using Eq. (166) that gives

$$v = \pm\frac{\sqrt{\eta^2 - 1}}{\eta} \quad , \eta \neq 0. \tag{174}$$

On the otherhand, from Eq. (164) setting the coefficients of $sech^{s}\tau \tanh\tau$, $sech^{2p}\tau \tanh\tau$, $sech^{2k}\tau \tanh\tau$ and $sech^{2r}\tau \tanh\tau$ terms zero we obtain

$$-A_4 s\eta - A_4 s v\eta - 4A_1^2 p v\eta - 4A_2^2 k v\eta - 4A_3^2 r v\eta = 0, \tag{175}$$

the latter gives by using Eq. (174):

$$A_4 = -\frac{2\sqrt{\eta^2 - 1}\left(A_1^2 + A_2^2 + A_3^2\right)}{\eta + \sqrt{\eta^2 - 1}}. \tag{176}$$

Setting the coefficients of $sech^{3p}\tau$, $sech^{p+r}\tau$, $sech^{2k+p}\tau$, $sech^{p+2}\tau$ and $sech^{2r+k}\tau$ in Eq. (161) zero gives rise to

$$-p(p+1)\eta^2 A_1 + p(p+1)v^2\eta^2 A_1 + 2A_1^3 + 2A_1A_2^2 + 2A_3^2A_1 + 2A_1A_4 = 0, \tag{177}$$

and this leads to

$$A_2 = \frac{\sqrt{\left(\eta - \sqrt{\eta^2 - 1}\right)\left(\eta + \sqrt{\eta^2 - 1} - A_1^2\eta + A_1^2\sqrt{\eta^2 - 1} - A_3^2\eta - A_3^2\sqrt{\eta^2 - 1}\right)}}{\sqrt{\eta^2 - 1} - \eta}, \tag{178}$$

by using Eqs. (166), (168), (170), (172), (174) and (176).

Finally, the bright optical soliton solution for the three-coupled nonlinear Klein-Gordon equations (133)-(137) is as follows;

$$u_1(x,t) = A_1 sech\tau, \tag{179}$$

$$u_2(x,t) = A_2 sech\tau, \tag{180}$$

$$u_3(x,t) = A_3 sech\tau, \tag{181}$$

and

$$v(x,t) = A_4 sech^2\tau, \tag{182}$$

where the velocity of the solitons v is given by (174) and the soliton amplitudes A_4 and A_2 are given by (176) and (178).

4.2 Dark optical soliton solution

In this section, we concern with 1-soliton to the three-coupled nonlinear Klein-Gordon equations by the wave ansatz method. The starting hypothesis or ansatz is as follows;

$$u_1(x,t) = A_1 \tanh^p \tau, \tag{183}$$

$$u_2(x,t) = A_2 \tanh^k \tau, \tag{184}$$

$$u_3(x,t) = A_3 \tanh^r \tau, \tag{185}$$

$$v(x,t) = A_4 \tanh^s \tau, \tag{186}$$

and

$$\tau = \eta\left(x - vt\right), \tag{187}$$

where A_1, A_2, A_3, A_4 and η are the free parameters of the solitons and v is the velocity of the soliton. The exponents p, k, r and s are unknown. These will be determined later. From (183) - (187) we obtain

$$u_{1xx} = pA_1\eta^2\left\{(p-1)\tanh^{p-2}\tau - 2p\tanh^p\tau + (p+1)\tanh^{p+2}\tau\right\}, \tag{188}$$

$$u_{1tt} = pv^2A_1\eta^2\{(p-1)\tanh^{p-2}\tau - 2p\tanh^p\tau + (p+1)\tanh^{p+2}\tau\}, \tag{189}$$

$$u_1^3 = A_1^3\tanh^{3p}\tau, \tag{190}$$

$$u_2^2u_1 = A_1A_2^2\tanh^{2k+p}\tau, \tag{191}$$

$$u_3^2u_1 = A_1A_3^2\tanh^{2r+p}\tau, \tag{192}$$

$$vu_1 = A_1A_4\tanh^{p+s}\tau, \tag{193}$$

$$u_{2xx} = kA_2\eta^2\left\{(k-1)\tanh^{k-2}\tau - 2k\tanh^k\tau + (k+1)\tanh^{k+2}\tau\right\}, \tag{194}$$

$$u_{2tt} = kv^2A_2\eta^2\{(k-1)\tanh^{k-2}\tau - 2k\tanh^k\tau + (k+1)\tanh^{k+2}\tau\}, \tag{195}$$

$$u_2^3 = A_2^3\tanh^{3k}\tau, \tag{196}$$

$$u_1^2u_2 = A_1^2A_2\tanh^{2p+k}\tau, \tag{197}$$

$$u_3^2u_2 = A_3^2A_2\tanh^{2r+k}\tau, \tag{198}$$

$$vu_2 = A_2A_4\tanh^{k+s}\tau, \tag{199}$$

$$u_{3xx} = rA_3\eta^2\left\{(r-1)\tanh^{r-2}\tau - 2r\tanh^r\tau + (r+1)\tanh^{r+2}\tau\right\}, \tag{200}$$

$$u_{3tt} = rv^2A_3\eta^2\{(r-1)\tanh^{r-2}\tau - 2r\tanh^r\tau + (r+1)\tanh^{r+2}\tau\}, \tag{201}$$

$$u_3^3 = A_3^3\tanh^{3r}\tau, \tag{202}$$

$$u_1^2u_3 = A_1^2A_3\tanh^{2p+r}\tau, \tag{203}$$

$$u_2^2u_3 = A_2^2A_3\tanh^{2k+r}\tau, \tag{204}$$

$$vu_3 = A_3A_4\tanh^{r+s}\tau, \tag{205}$$

$$v_x = sA_3\eta\left\{\tanh^{s-1}\tau - \tanh^{s+1}\tau\right\}, \tag{206}$$

$$v_t = svA_3\eta\left\{\tanh^{s+1}\tau - \tanh^{s-1}\tau\right\}, \tag{207}$$

$$u_1u_{1t} = pvA_1^2\eta\left\{\tanh^{2p+1}\tau - \tanh^{2p-1}\tau\right\}, \tag{208}$$

$$u_2u_{2t} = kvA_2^2\eta\left\{\tanh^{2k+1}\tau - \tanh^{2k-1}\tau\right\}, \tag{209}$$

$$u_3u_{3t} = rvA_3^2\eta\left\{\tanh^{2r+1}\tau - \tanh^{2r-1}\tau\right\}. \tag{210}$$

Substituting (188)-(210) into (129)-(132) respectively yields

$$\begin{aligned}&pA_1\eta^2\left\{(p-1)\tanh^{p-2}\tau - 2p\tanh^p\tau + (p+1)\tanh^{p+2}\tau\right\}\\ &-pv^2A_1\eta^2\{(p-1)\tanh^{p-2}\tau - 2p\tanh^p\tau + (p+1)\tanh^{p+2}\tau\}\\ &-A_1\tanh^p\tau + 2A_1^3\tanh^{3p}\tau + 2A_1A_2^2\tanh^{2k+p}\tau + 2A_1A_3^2\tanh^{2r+p}\tau + 2A_1A_4\tanh^{p+s}\tau\\ &= 0,\end{aligned} \tag{211}$$

$$\begin{aligned}&kA_2\eta^2\left\{(k-1)\tanh^{k-2}\tau - 2k\tanh^k\tau + (k+1)\tanh^{k+2}\tau\right\}\\ &-kv^2A_2\eta^2\{(k-1)\tanh^{k-2}\tau - 2k\tanh^k\tau + (k+1)\tanh^{k+2}\tau\}\\ &-A_2\tanh^k\tau + 2A_1^2A_2\tanh^{2p+k}\tau + 2A_2^3\tanh^{3k}\tau + 2A_3^2A_2\tanh^{2r+k}\tau + 2A_2A_4\tanh^{k+s}\tau\\ &= 0,\end{aligned} \tag{212}$$

$$\begin{aligned}&rA_3\eta^2\left\{(r-1)\tanh^{r-2}\tau - 2r\tanh^r\tau + (r+1)\tanh^{r+2}\tau\right\}\\ &-rv^2A_3\eta^2\{(r-1)\tanh^{r-2}\tau - 2r\tanh^r\tau + (r+1)\tanh^{r+2}\tau\}\\ &-A_3\tanh^r\tau + 2A_1^2A_3\tanh^{2p+r}\tau + 2A_3^3\tanh^{3r}\tau + 2A_2^2A_3\tanh^{2k+r}\tau + 2A_3A_4\tanh^{r+s}\tau\\ &= 0,\end{aligned} \tag{213}$$

$$\begin{aligned}&sA_3\eta\left\{\tanh^{s-1}\tau - \tanh^{s+1}\tau\right\} - svA_3\eta\left\{\tanh^{s+1}\tau - \tanh^{s-1}\tau\right\}\\ &-4pvA_1^2\eta\left\{\tanh^{2p+1}\tau - \tanh^{2p-1}\tau\right\}\\ &-4kvA_2^2\eta\left\{\tanh^{2k+1}\tau - \tanh^{2k-1}\tau\right\}\\ &-4rvA_3^2\eta\left\{\tanh^{2r+1}\tau - \tanh^{2r-1}\tau\right\} = 0.\end{aligned} \tag{214}$$

From Eq. (211) equating the exponents of $\tanh^{3p}\tau$ and $\tanh^{p+2}\tau$ we have

$$3p = p + 2, \tag{215}$$

so that

$$p = 1. \tag{216}$$

Also, from Eq. (212) equating the exponents of $\tanh^{3k}\tau$ and $\tanh^{k+2}\tau$ we have

$$3k = k + 2, \tag{217}$$

we get

$$k = 1. \tag{218}$$

On the otherhand, from Eq. (213), equating the exponents of $\tanh^{3r}\tau$ and $\tanh^{r+2}\tau$ we have

$$3r = r + 2, \qquad (219)$$

we get

$$r = 1. \qquad (220)$$

Then from Eq. (214) equating the exponents of $\tanh^{2k+1}$, $\tanh^{2p+1}$, $\tanh^{2r+1}$ and $\tanh^{s+1}\tau$ gives rise to

$$2p+1 = 2k+1 = 2r+1 = s+1, \qquad (221)$$

and by using Eqs. (216), (218) and (220) we get

$$s = 2. \qquad (222)$$

Now from Eq. (213), setting the coefficients of $\tanh^{p}\tau$ terms zero one obtains:

$$-2p^2A_3\eta^2 + 2p^2v^2A_3\eta^2 - A_3 = 0, \qquad (223)$$

and by using Eq. (216), we get

$$v = \pm\frac{\sqrt{4\eta^2+2}}{2\eta} \quad ,\eta \neq 0. \qquad (224)$$

After that, in Eq. (211), setting the coefficients of $\tanh^{3p}\tau$, $\tanh^{p+2}\tau$, $\tanh^{2k+p}\tau$, $\tanh^{2r+p}\tau$ and $\tanh^{p+r}\tau$ terms zero one obtains

$$p(p+1)A_1\eta^2 - p(p+1)v^2A_1\eta^2 + 2A_1^3 + 2A_1A_2^2 + 2A_1A_3^2 + 2A_1A_4 = 0, \qquad (225)$$

so that (with $p = 1$) and using Eq. (224)

$$A_1 = \pm\frac{\sqrt{2-4A_2^2-4A_3^2-4A_4}}{2}. \qquad (226)$$

Again from Eq. (214), setting the coefficients of the linearly independent functions $\tanh^{2p+j}\tau$ and $\tanh^{s+j}\tau$ or $\tanh^{2k+j}\tau$ and $\tanh^{s+j}\tau$ $\tanh^{2r+j}\tau$ and $\tanh^{s+j}\tau$ (with $p=1$, $k=1$, $r=1$ and $s=2$) zero, $j=-1,1$ we have

$$-2A_3\eta - 2vA_3\eta - 4vA_1^2\eta - 4vA_2^2\eta - 4vA_3^2\eta = 0, \qquad (227)$$

and

$$A_4 = \frac{\sqrt{4\eta^2+2}}{-2\eta+\sqrt{4\eta^2+2}}. \qquad (228)$$

Finally, the dark optical soliton solution for three-coupled nonlinear Klein-Gordon equations (183)-(187) is of the form

$$u_1(x,t) = A_1\tanh\tau, \qquad (229)$$

$$u_2(x,t) = A_2\tanh\tau, \qquad (230)$$

$$u_3(x,t) = A_3\tanh\tau, \qquad (231)$$

and

$$v(x,t) = A_4\tanh^2\tau, \qquad (232)$$

where the free parameters A_1 and A_4 are given by (226) and (228). The velocity of the soliton v is given by (224).

Remark 3: Comparing our results with results in [29,38], we see that our results are new.

Remark 4: It has been shown that the one- and two-coupled nonlinear Klein–Gordon equations are completely integrable and their integrability properties can also be constructed by using P-analysis [41].

5. Conclusion

In this work, we have investigated the bright and dark optical soliton solutions of three variants of the coupled Klein-Gordon equations by using the solitary wave ansatz method. We showed that all the physical parameters of the obtained solutions are depend on the others. We also proved that the exponents in the bright solitary wave solution are similar to those given in the dark solitary wave solution. Some of the results are in agreement with the results reported by others in the literature, and new results are formally developed in this work. In addition to, with the aid of Maple, it is confirmed that the solutions are correct since these solutions satisfy the original equation.

The method that is used is far less involved than the standard techniques that are used to study these kind of problems. Additionally, we see that the used method is an efficient method of integrability for constructing exact soliton solutions for such versions of the coupled nonlinear KG-type equations.

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