Optical soliton solutions for generalized NLSE using Jacobi elliptic functions

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In this paper, the generalized NLSE is studied with four forms of nonlinearity. The Jacobi elliptic functions are used to obtain exact solutions of this equation. Bright and dark optical soliton solutions are also retrieved for the generalized NLSE.

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1. Introduction

The study of optical solitons has made an overwhelming progress for the past few decades. Today there are increasing number of papers that are being published in this area of research [1-12]. The governing equation for studying the dynamics of optical soliton is the Nonlinear Schrödinger equation (NLSE). It governs the propagation of solitons through optical fibers. The NLSE has crucial mathematical features that interest the Nonlinear Optics community. This equation is known to support solitons or soliton solutions for various kinds of nonlinearity.The nonlinearities that are studied in this paper are the Kerr law, power law, parabolic law and dual-power law.

The aim of the present study is to get the different optical solitons of the generalized NLSE with various nonlinearity by using Jacobi elliptic functions.

2. Mathematical analysis

We consider the dimensionless form of the generalized NLSE in the form [2]

$$i\left(u^m\right)_t + a\left(u^m\right)_{xx} + bF\left(|u|^2\right)u^m = 0, \tag{1}$$

where x represents the non-dimensional distance along the fiber, t represents time in dimensionless form while complex valued function u represents the wave profile. Also F is a real-valued algebraic function with the condition $F\left(|u|^2\right)u : C \to C.$ Considering the complex plane C as a two-dimensional linear space R^2, the function $F\left(|u|^2\right)u$ is k times continuously differentiable, so that $F\left(|u|^2\right)u \in \cup_{m,n=1}^{\infty} C^k\left((-n,n)\times(-m,m);R^2\right).$ Since u is a complex valued function, the solution equation of (1) can be written in the form

$$u(x,t) = P(x,t)e^{i\phi}. \tag{2}$$

Now we choose the phase as

$$\phi(\text{x,t})=-\kappa\text{x}+\text{wt}+\theta, \tag{3}$$

where κ represents the soliton frequency, w is the soliton wave number and θ is the phase constant. From equation (2), we have

$$\left(u^m\right)_t = \left(mP^{m-1}\frac{\partial P}{\partial t} + imwP^m\right)e^{im\phi}, \tag{4}$$

$$\left(u^m\right)_{xx} = \begin{pmatrix} mP^{m-1}\dfrac{\partial^2 P}{\partial x^2} - 2im^2\kappa P^{m-1}\dfrac{\partial P}{\partial x} \\ +m(m-1)P^{m-2}\left(\dfrac{\partial P}{\partial x}\right)^2 - m^2\kappa^2 P^m \end{pmatrix} e^{im\phi}. \tag{5}$$

Substituting (4) and (5) into (1) and equating the real and imaginary parts yields

$$mP^{m-1}\frac{\partial P}{\partial t} - 2am^2\kappa P^{m-1}\frac{\partial P}{\partial x} = 0, \tag{6}$$

and

$$\begin{aligned} &mwP^m - bF\left(P^2\right)P^m - amP^{m-1}\frac{\partial^2 P}{\partial x^2} \\ &-am(m-1)P^{m-2}\left(\frac{\partial P}{\partial x}\right)^2 + am^2\kappa^2 P^m = 0. \end{aligned} \tag{7}$$

Eq. (6) can be written in the form

$$\frac{\partial P}{\partial t} - 2am\kappa\frac{\partial P}{\partial x} = 0, \tag{8}$$

For some function *f* and provided that *v* is the velocity of the soliton, P is written of the form of travelling wave type as

$$P = f\left(x - vt\right) \tag{9}$$

Thus, (8) leads to

$$v = -2am\kappa \tag{10}$$

The result for the velocity of the soliton, given by (10), is true for all types of nonlinearity in question.

The generalized NLSE will be considered for the following four forms of nonlinearity.

2.1 Kerr Law

The Kerr law nonlinearity is the case when $F(s) = s.$ This kind of nonlinearity emerge from the fact that a light wave in an optical fiber faces nonlinear responses. For Kerr law nonlinearity, the considered generalized NLSE is given by

$$i\left(u^m\right)_t + a\left(u^m\right)_{xx} + b\left|u\right|^2 u^m = 0. \tag{11}$$

We assume that for Eq. (11) *P* is in the form

$$P(x,t) = Asn^p\left(\xi,\ell\right), \tag{12}$$

with

$$\xi = B\left(x - vt\right), \tag{13}$$

where *A* is the amplitude, *B* is the inverse width of the soliton and ℓ is the modulus of jacobi elliptic function. The derivatives of Eq. (12) are as follows:

$$\frac{\partial P}{\partial x} = pABcn\left(\xi,\ell\right)dn\left(\xi,l\right)sn^{p-1}\left(\xi,\ell\right), \tag{14}$$

$$\begin{aligned}\left(\frac{\partial P}{\partial x}\right)^2 &= -p^2\left(\ell^2+1\right)A^2B^2sn^{2p}\left(\xi,\ell\right)\\ &+p^2A^2B^2sn^{2p-2}\left(\xi,\ell\right) + p^2\ell^2A^2B^2sn^{2p+2}\left(\xi,\ell\right),\end{aligned} \tag{15}$$

and

$$\begin{aligned}\frac{\partial^2 P}{\partial x^2} &= \left(p-1\right)pAB^2sn^{p-2}\left(\xi,\ell\right)\\ &-p\left(p+\ell-\ell^2+p\ell^2\right)AB^2sn^p\left(\xi,\ell\right)\\ &+p\ell\left(1+p\ell\right)AB^2sn^{p+2}\left(\xi,\ell\right).\end{aligned} \tag{16}$$

Substituting (15) and (16) into (7) gives

$$\begin{aligned}&mwA^m sn^{mp}\left(\xi,\ell\right)\\ &-am\left(p-1\right)pA^mB^2sn^{mp-2}\left(\xi,\ell\right)\\ &+amp\left(p+\ell-\ell^2+p\ell^2\right)A^mB^2sn^{mp}\left(\xi,\ell\right)\\ &-amp\ell\left(1+p\ell\right)A^mB^2sn^{mp+2}\left(\xi,\ell\right)\\ &+am\left(m-1\right)p^2\left(\ell^2+1\right)A^mB^2sn^{mp}\left(\xi,\ell\right)\\ &-am\left(m-1\right)p^2A^mB^2sn^{mp-2}\left(\xi,\ell\right)\\ &-am\left(m-1\right)p^2\ell^2A^mB^2sn^{mp+2}\left(\xi,\ell\right)\\ &+am^2\kappa^2A^m sn^{mp}\left(\xi,\ell\right)\\ &-bA^{m+2}sn^{(m+2)p}\left(\xi,\ell\right) = 0.\end{aligned} \tag{17}$$

From (17), setting the coefficients $\left(m+2\right)p$ and $mp+2$ equal to one another gives

$$\left(m+2\right)p = mp+2, \tag{18}$$

$$p = 1. \tag{19}$$

Also noting that the functions $sn^{mp+j}\left(\xi,\ell\right)$ for j=-2,0 are linearly independent, their respective coefficients in (17) must vanish. Therefore, this yields

$$w = aB^2\left[\ell^2\left(1-m\right)-\ell-m\right]-am\kappa^2, \tag{20}$$

and

$$B = \left[\frac{-bA^2}{a\ell m\left(1+\ell m\right)}\right]^{\frac{1}{2}}. \tag{21}$$

Hence, for Kerr law nonlinearity, the jacobi elliptic function solution of the generalized NLSE is given by

$$u_1\left(x,t\right) = Asn\left[B\left(x-vt\right),\ell\right]e^{i\left(-\kappa x+wt+\theta\right)}, \tag{22}$$

where the relation between the amplitude *A* and the inverse width *B* is given in (21), the wave number *w* is given by (20) and the relation between the frequency and velocity of the soliton is given by (10). When the modulus $\ell \to 1$ in (22), we obtain following dark optical soliton solution

$$u_{1,1}\left(x,t\right) = A\tan h\left[B_1\left(x-vt\right)\right]e^{i\left(-\kappa x+w_1t+\theta\right)}. \tag{23}$$

Here, B_1 and w_1 is in the form

$$w_1 = am\left(-2B_1^2-\kappa^2\right), \tag{24}$$

$$B_1 = \left[\frac{-bA^2}{am\left(1+m\right)}\right]^{\frac{1}{2}}. \tag{25}$$

To get another jacobi elliptic function solution of generalized NLSE with Kerr law nonlinearity, we use the following function

$$P(x,t) = Acn^{p}[\xi,\ell]. \quad (26)$$

For (26), one obtains

$$\frac{\partial P}{\partial x} = -pABsn(\xi,\ell)dn(\xi,l)cn^{p-1}(\xi,\ell), \quad (27)$$

$$\begin{aligned}\left(\frac{\partial P}{\partial x}\right)^2 &= p^2(2\ell^2-1)A^2B^2cn^{2p}(\xi,\ell) \\ &-p^2(\ell^2-1)A^2B^2cn^{2p-2}(\xi,\ell) \\ &-p^2\ell^2A^2B^2sn^{2p+2}(\xi,\ell),\end{aligned} \quad (28)$$

and

$$\begin{aligned}\frac{\partial^2 P}{\partial x^2} &= -(p-1)p(\ell^2-1)AB^2cn^{p-2}(\xi,\ell) \\ &+p(-p+\ell-\ell^2+2p\ell^2)AB^2cn^{p}(\xi,\ell) \\ &-p\ell(1+p\ell)AB^2cn^{p+2}(\xi,\ell).\end{aligned} \quad (29)$$

Thus, Eq. (7) reduces

$$\begin{aligned}&mwA^mcn^{mp}(\xi,\ell) \\ &+am(p-1)p(\ell^2-1)A^mB^2cn^{mp-2}(\xi,\ell) \\ &-amp(-p+\ell-\ell^2+2p\ell^2)A^mB^2cn^{mp}(\xi,\ell) \\ &+amp\ell(1+p\ell)A^mB^2cn^{mp+2}(\xi,\ell) \\ &-am(m-1)p^2(2\ell^2-1)A^mB^2cn^{mp}(\xi,\ell) \\ &+am(m-1)p^2(\ell^2-1)A^mB^2cn^{mp-2}(\xi,\ell) \\ &+am(m-1)p^2\ell^2A^mB^2cn^{mp+2}(\xi,\ell) \\ &+am^2\kappa^2A^mcn^{mp}(\xi,\ell) \\ &-bA^{m+2}cn^{(m+2)p}(\xi,\ell)=0.\end{aligned} \quad (30)$$

From (30), setting the coefficients $(m+2)p$ and $mp+2$ equal to one another gives the same value of p which is in (19). The functions $cn^{mp+j}(\xi,\ell)$, for $j=-2,0$ are linearlyindependent, this yields

$$\begin{aligned}w &= aB^2\left[\ell^2+\ell-1+(m-1)(2\ell^2-1)\right] \\ &-am\kappa^2,\end{aligned} \quad (31)$$

$$B=\left[\frac{bA^2}{a\ell m(1+\ell m)}\right]^{\frac{1}{2}}. \quad (32)$$

So, for Kerr law nonlinearity, another jacobi elliptic function solution of the generalized NLSE is given by

$$u_2(x,t) = Acn\left[B(x-vt),\ell\right]e^{i(-\kappa x+wt+\theta)}, \quad (33)$$

where the relation between the amplitude A and the inverse width B is given in (32), the wave number w is given by (31) and the relation between the frequency and velocity of the soliton is given by (10). When the modulus $\ell\to 1$ in (33), we obtain following bright optical soliton solution

$$u_{2,1}(x,t) = A\operatorname{sech}\left[B_1(x-vt)\right]e^{i(-\kappa x+w_1 t+\theta)}, \quad (34)$$

where

$$w_1 = am\left(B_1^2-\kappa^2\right), \quad (35)$$

$$B_1=\left[\frac{bA^2}{am(1+m)}\right]^{\frac{1}{2}}. \quad (36)$$

2.2 Power Law

This law of nonlinearity arises in nonlinear plasmas and the context of nonlinear optics. For the case of power law nonlinearity, where $F(s)=s^n$, so that the generalized NLSE reduces to

$$i\left(u^m\right)_t + a\left(u^m\right)_{xx} + b|u|^{2n}u^m = 0. \quad (37)$$

To obtain the solution of this equation, the starting assumption for the form of P stays the same as in (12). So, substituting (15) and (16) into (7), then setting the coefficients $(m+2n)p$ and $mp+2$ equal to one another in the obtained equation

$$(m+2n)p = mp+2, \quad (38)$$

that gives

$$p=\frac{1}{n}. \quad (39)$$

setting the coefficients of $sn^{mp+j}(\xi,\ell)$ to zero, where $j=-2,0$, since these functions are linearly independent yields

$$\begin{aligned}w &= -\frac{a}{n^2}B^2\left[\ell^2+1+n\left(\ell-\ell^2\right)\right] \\ &-\frac{a}{n^2}B^2(m-1)\left(\ell^2+1\right)-am\kappa^2,\end{aligned} \quad (40)$$

$$B=\left[\frac{-bn^2A^{2n}}{a\ell m(n+\ell m)}\right]^{\frac{1}{2}}. \quad (41)$$

Thus, for power law nonlinearity, the jacobi elliptic function solution of the generalized NLSE is given by

$$u_1(x,t) = Asn^{\frac{1}{n}}\left[B(x-vt),\ell\right]e^{i(-\kappa x+wt+\theta)}, \quad (42)$$

where the free parameters$A$ and $B$are connected as given in (41), while the wave number is given by (40) and the relation between the frequency and velocity of the soliton is given by (10). When the modulus $\ell \to 1$in (42), we obtain following dark optical soliton solution

$$u_{1,1}(x,t) = A\tan h^{\frac{1}{n}}\left[B_1(x-vt)\right]e^{i(-\kappa x+w_1 t+\theta)}, \quad (43)$$

where

$$w_1 = -\frac{2}{n^2}amB_1^2 - am\kappa^2, \quad (44)$$

$$B_1 = \left[\frac{-bn^2A^{2n}}{am(n+m)}\right]^{\frac{1}{2}}. \quad (45)$$

Now, to look for another jacobi elliptic function solution of generalized NLSE with power law nonlinearity, we use the starting assumption for the form of P the same as in (26). So, (7) reduces to a new equation. In this equation, setting the coefficients $(m+2n)p$ and $mp+2$ equal to one another gives the same value of p which is in (39). Again, as in the previous equation, setting the coefficients of thelinearly independent functions $cn^{mp+j}(\xi,\ell)$, for $j=-2,0$ to zero gives

$$w = \frac{a}{n^2}B^2\left[n\ell(1-\ell)+m\left(2\ell^2-1\right)\right] -am\kappa^2, \quad (46)$$

$$B = \left[\frac{bn^2A^{2n}}{a\ell m(n+\ell m)}\right]^{\frac{1}{2}}. \quad (47)$$

Hence, for power law nonlinearity, another solution of Eq. (36) is given by

$$u_2(x,t) = Acn^{\frac{1}{n}}\left[B(x-vt),\ell\right]e^{i(-\kappa x+wt+\theta)}. \quad (48)$$

When the modulus $\ell \to 1$in (46), we obtain following bright optical soliton solution

$$u_{2,1}(x,t) = A\sec h^{\frac{1}{n}}\left[B_1(x-vt)\right]e^{i(-\kappa x+w_1 t+\theta)}, \quad (49)$$

where

$$w_1 = \frac{a}{n^2}m\left(B_1^2 - n^2\kappa^2\right), \quad (50)$$

$$B_1 = \left[\frac{bn^2A^{2n}}{am(n+m)}\right]^{\frac{1}{2}}. \quad (51)$$

2.3 Parabolic Law

Parabolic law nonlinearity appears also in fiber optics. For this kind of nonlinearity, $F(s) = s + k_1 s^2$. In this case, the generalized NLSE is

$$i\left(u^m\right)_t + a\left(u^m\right)_{xx} + b\left(|u|^2 + k_1|u|^4\right)u^m = 0. \quad (52)$$

Here the constant k_1 binds the two nonlinear terms in the parabolic law. We assume that for Eq. (52), P is in the form

$$P(x,t) = A\left[D_1 + sn(\xi,\ell)\right]^p, \quad (53)$$

where the constant D_1 and the unknown index p will be determined. The required derivatives can obtained from (52). Substituting these derivatives in the Eq. (7), and then setting the exponents $mp+1$ and $(m+2)p$ equal to one another

$$mp+2 = (m+2)p \quad (54)$$

that gives

$$p = \frac{1}{2}. \quad (55)$$

It needs to be noted that the same value of the exponent p is yielded when the exponents $mp+2$ and $(m+4)p$ are equated. The functions $\left[D_1 + sn(\xi,\ell)\right]^{mp+j}$, for $j=-2,-1,0,1,2$ are linearly independent this yields,

$$w = \frac{aB^2}{4}\left[\ell^2 - 2\ell - 1 - (m-1)\left(\ell^2+1\right) +6\ell(\ell+2)D_1^2 + 6\ell^2(m-1)D_1^2 -6\ell(\ell+1)D_1^3\right] - am\kappa^2, \quad (56)$$

$$B = 2A\left[-\frac{b}{a\ell mD_1(\ell(4m-6)-6)}\right]^{\frac{1}{2}}, \quad (57)$$

and

$$D_1 = \left[\frac{(m-1)\left(\ell^2+1\right)+\ell^2+\ell}{\ell(2\ell m-3\ell+1)}\right]^{\frac{1}{2}}. \quad (58)$$

Thus, the jacobi elliptic function solution in a parabolic law media for the generalized NLSE is given by

$$u_1(x,t) = A\left[D_1 + sn(\xi,\ell)\right]^{\frac{1}{2}}e^{i(-\kappa x+wt+\theta)}, \quad (59)$$

where the relation between the amplitude and the inverse width is given in (57) while the wave number and constant D_1 are respectively given by (56) and (58). When the modulus $\ell \to 1$ in (59), we obtain following dark optical soliton solution

$$u_{1,1}(x,t) = A\left[D_{1,1} + \tan h\left[B_1(x - vt)\right]\right]^{\frac{1}{2}} .e^{i(-\kappa x + w_1 t + \theta)}, \quad (60)$$

where

$$w_1 = \frac{aB_1^2}{2}\left(-m + 3D_{1,1}^2(m+2) - 6D_{1,1}^3\right) - am\kappa^2, \quad (61)$$

$$B_1 = A\sqrt{\frac{-b}{amD_{1,1}(m-3)}}, \quad (62)$$

and

$$D_{1,1} = \sqrt{\frac{m}{m-1}}. \quad (63)$$

Now, we use the starting assumption as

$$P(x,t) = A\left[D_1 + cn(\xi,\ell)\right]^p. \quad (64)$$

The required derivatives can be obtained from (64). Similarly, substituting these derivatives in the Eq. (7), and then setting the coefficients $mp+1$ and $(m+2)p$ equal to one another gives the same value of p which is in (55). It is worthy of notethat the same value of p is obtained on equating $mp+2$ and $(m+4)p$. The functions $\left[D_1 + cn(\xi,\ell)\right]^{mp+j}$, for $j=-2,-1,0,1,2$ are linearly independent, this yields

$$w = \frac{aB^2}{4}\left[2\ell - 1 + (m-1)\left(2\ell^2 + 6\ell^2 D_1 - 1\right) - 6\ell D_1^2\right] - am\kappa^2, \quad (65)$$

$$B = A\left[\frac{2b}{am\left(1 - 2\ell^2(m-1)D_1\right)}\right]^{\frac{1}{2}}, \quad (66)$$

and

$$D_1 = \left[\frac{\ell(1-2\ell) + (m-1)\left(2\ell^2 - 1\right)}{\ell(1-\ell) + 2\ell^2(m-1)}\right]^{\frac{1}{2}}. \quad (67)$$

So, another jacobi elliptic function solution in a parabolic law media for the generalized NLSE is given by

$$u_2(x,t) = A\left[D_1 + cn(\xi,\ell)\right]^{\frac{1}{2}} e^{i(-\kappa x + wt + \theta)}, \quad (68)$$

where the free parameters A and B are related as seen in (65) , the wave number and constant D_1 are respectively given by (65) and (67). When the modulus $\ell \to 1$ in (68), we obtain following bright optical soliton solution

$$u_{2,1}(x,t) = A\left[D_{1,1} + \sec h\left[B_1(x - vt)\right]\right]^{\frac{1}{2}} .e^{i(-\kappa x + w_1 t + \theta)}, \quad (69)$$

where

$$w_1 = \frac{aB^2}{4}\left[(m-1)(1 + 6D_{1,1}) - 6D_{1,1}^2 + 1\right] - am\kappa^2, \quad (70)$$

$$B_1 = A\left[\frac{2b}{am\left[1 - 2(m-1)D_{1,1}\right]}\right]^{\frac{1}{2}}, \quad (71)$$

and

$$D_{1,1} = \sqrt{\frac{(m-2)}{2(m-1)}}. \quad (72)$$

2.4 Dual-power Law

Thedual-power law nonlinearity appears in photo-voltatic-photo-refractive materials such as $LiNbO_3$ and it is formulated as $F(s) = s^n + k_2 s^{2n}$. This law is the generalization of the parabolic law nonlinearity. Namely, if *n=1*, the dual-power reduces to parabolic law nonlinearity.The generalized NLSE with dual-power law nonlinearity is given by

$$i\left(u^m\right)_t + a\left(u^m\right)_{xx} + b\left(|u|^{2n} + k_2|u|^{4n}\right)u^m = 0, \quad (73)$$

where, the constant k_2 binds the two nonlinear terms in (73). For this law nonlinearity, the starting hypothesis for P is given by

$$P(x,t) = A\left[D_2 + sn(\xi,\ell)\right]^p. \quad (74)$$

Here the constant D_2 and the unknown index p will be determined. Substituting the required derivatives in the Eq. (7), and then equating the coefficients $(m+2n)p$ and $mp+1$ yields

$$(m+2n)p = mp+1, \quad (75)$$

$$p = \frac{1}{2n}. \quad (76)$$

It needs to be noted that the same value of the exponent$p$ is yielded when the exponents $(m+4n)p$ and $mp+2$ are equated. Now, setting the coefficients of $\left[D_2 + sn(\xi,\ell)\right]^{mp+j}$ to zero, for $j=-2,-1,0,1,2$, gives

$$w=\frac{aB^2}{4n^2}\Big[-2n\left(1+2n\ell-2n\ell^2+\ell^2\right)$$
$$+12n\ell\left(2n+\ell\right)D_2^2-12n^2\left(\ell+1\right)D_2^2 \quad (77)$$
$$-\left(m-1\right)\left(\ell^2+1\right)+6\left(m-1\right)\ell^2D_2^2\Big]-am\kappa^2,$$

$$B=\left[\frac{2n^2bA^{2n}}{am\left[\ell\left(3n+4\ell\right)-\ell^2\left(n+2m\right)D_2\right]}\right]^{\frac{1}{2}}, \quad (78)$$

and

$$D_2=\left[\frac{\left(m-1\right)\left(\ell^2+1\right)+2\left(-\ell^2+\ell+1\right)}{\left(m-1\right)\ell^2+\ell\left(7\ell+8\right)}\right]^{\frac{1}{2}}. \quad (79)$$

Thus, the jacobi elliptic function solution for the generalized NLSE with dual power law nonlinearity is given by

$$u_1\left(x,t\right)=A\left[D_2+sn\left(\xi,\ell\right)\right]^{\frac{1}{2n}}e^{i\left(-\kappa x+wt+\theta\right)}, \quad (80)$$

where the relation between the amplitude and the inverse width is given in (78) while the wave number and constant D_2 are respectively given by (77) and (79). If the modulus $\ell\rightarrow 1$, solution (80) become

$$u_{1,1}\left(x,t\right)=A\left[D_{2,1}+\tan h\left[B_1\left(x-vt\right)\right]\right]^{\frac{1}{2n}} \quad (81)$$
$$.e^{i\left(-\kappa x+w_1t+\theta\right)},$$

where

$$w_1=\frac{aB^2}{4n^2}\Big[-4n+12n\left(2n+1\right)D_{2,1}^2-24n^2D_{2,1}^2 \quad (82)$$
$$-2\left(m-1\right)+6\left(m-1\right)D_{2,1}^2\Big]-am\kappa^2,$$

$$B_1=\left[\frac{2n^2bA^{2n}}{am\left[3n+4-\left(n+2m\right)D_{2,1}\right]}\right]^{\frac{1}{2}}, \quad (83)$$

and

$$D_{2,1}=\sqrt{\frac{2m}{m+14}}. \quad (84)$$

Now, to look for the solution of Eq. (73), the starting hypothesis is given by

$$P\left(x,t\right)=A\left[D_2+cn\left(\xi,\ell\right)\right]^p. \quad (85)$$

Substituting the hypothesis into (7), and then eqauting the coefficients $\left(m+2n\right)p$ and $mp+1$ gives the same value of p which is in (76). We note that the same value of p is obtained on equating $\left(m+4n\right)p$ and $mp+2$. Then, setting the coefficients of $\left[D_2+cn\left(\xi,\ell\right)\right]^{mp+j}$ to zero, for $j=-2,-1,0,1,2$, gives

$$w=\frac{aB^2}{4n^2}\Big[2n\ell\left(\ell-1\right)-1+2\ell^2-6\ell\left(n+\ell-n\ell\right)D_2^2 \quad (86)$$
$$+\left(m-1\right)\left(2\ell^2-1\right)-6\left(m-1\right)\ell^2D_2^2\Big]-am\kappa^2,$$

$$B=\left[\frac{4n^2bA^{2n}}{a\ell mD_2\left(-4\left(2n+\ell\right)+2n\left(\ell+1\right)-4\ell\left(m-1\right)\right)}\right]^{\frac{1}{2}}, \quad (87)$$

and

$$D_2=\left[\frac{2n\left(\ell+1\right)+2\ell^2\left(2-3n\right)}{\ell\left(2n+4\ell-6n\ell\right)}\right]^{\frac{1}{2}}. \quad (88)$$

Thus, finally, another jacobi elliptic function solution for the generalized NLSE with dual power law nonlinearity is given by

$$u_2\left(x,t\right)=A\left[D_2+cn\left(\xi,\ell\right)\right]^{\frac{1}{2n}}e^{i\left(-\kappa x+wt+\theta\right)}, \quad (89)$$

where the A and B are connected as given in Eq. (87), while w and D_2 are respectively given by (86) and (88). When the modulus $\ell\rightarrow 1$ in (89), we obtain following bright optical soliton solution

$$u_{2,1}\left(x,t\right)=A\left[D_{2,1}+\sec h\left[B_1\left(x-vt\right)\right]\right]^{\frac{1}{2n}} \quad (90)$$
$$.e^{i\left(-\kappa x+w_1t+\theta\right)},$$

where

$$w_1=\frac{amB^2}{4n^2}\left(1-6D_{2,1}^2\right)-am\kappa^2, \quad (91)$$

$$B_1=\left[\frac{n^2bA^{2n}}{-am\left(n+m\right)D_{2,1}}\right]^{\frac{1}{2}}, \quad (92)$$

and

$$D_{2,1}=\frac{1}{2}\sqrt{\frac{4-2n}{1-n}}. \quad (93)$$

3. Conclusions

We study the generalized form of the NLSE and we obtain the exact solutions of this equation by using Jacobi elliptic functions in this study. These solutions are going to be extremely useful in the study of Nonlinear Optics, Fluid dynamics, Plasma Physics and many other areas where the NLSE predominantly studied.

There exist twelve Jacobi elliptic functions in the literature. We will use two of them to find new exact solutions of the generalized NLSE with four different forms of nonlinearity. Other jacobi elliptic functions can also be applied to this equation. Furthermore, we note that this method can also be used other nonlinear equations and systems.

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