Optical soliton perturbation with quadratic-cubic nonlinearity by traveling wave hypothesis

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This paper obtains optical soliton solutions with quadratic-ccubic nonlinearity. The governing nonlinear Schrödinger's equation is integrated with traveling wave hypothesis. Bright and singular soliton solutions are retrieved along with constraint conditions for their existence.

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1. Introduction

Optical solitons form the basic fabric in telecommunication industry. These soliton molecules carry loads of information and transfer data through optical fibers, metamaterials and other forms of waveguides across transcontinental and trransoceanic distances. These fibers and metamaterials appear with several forms of nonlinearity [1-5]. The most commonly studied forms are Kerr law, power law parabolic law, dual-power law, logarithmic nonlinear law and others [5]. This paper studies optical solitons with a very new form of nonlinearity that was proposed during 2011 and was later studied during 2017 [1, 4]. It is referred to as cubic-quintic nonlinear form. In previous works, exact soliton solutions and conservation laws are reported [4].

This paper is an extended version of the results that are already known for the unperturbed NLSE. The governing nonlinear Schrödinger's equation (NLSE) with quadratic-cubic nonlinearity will be studied in presence of perturbation terms that are of Hamiltonian type. This will prevent the destruction of integrability of the perturbed NLSE. Traveling wave hypothesis will be the integrability criteria of the governing model. This will yield bright, singular and combo 1-soliton solutions to perturbed NLSE, depending on the sign of the discriminant, as will be seen. The details of these analysis are detailed in the following sections.

2. The governing equation

The dimensionless form of the governing equation for perturbed NLSE with quadratic-cubic nonlinearity in presence of Hamiltonian perturbations is given by

The dimensionless form of the governing equation for perturbed NLSE with quadratic-cubic nonlinearity in presence of Hamiltonian perturbations is given by

$$\begin{aligned} &iq_t + aq_{xx} + \left(b_1|q| + b_2|q|^2\right)q \\ &= i\left\{\alpha q_x + \lambda\left(|q|^2 q\right)_x + \theta\left(|q|^2\right)_x q\right\}. \end{aligned} \tag{1}$$

In (1) $q(x,t)$ represents the complex-valued wave profile where x and t are spatial and temporal variables respectively. The first term on left hand side is temporal evolution while the coefficient of a is the group velocity dispersion. The two nonlinear terms on the left-hand side with coefficients b_1 and b_2 are with quadratic and cubic forms respectively. On the right-hand side α is the coefficient of inter-modal dispersion. This arises from the fact that the group velocity of light propagating through a waveguide depends, in addition to optical frequency, (also known as chromatic dispersion), on the propagation mode involved. The coefficient of λ is the self-steepening term that avoids the formation of shock waves. Finally, θ is the coefficient of nonlinear dispersion. This paper will integrate (1) to extract soliton solutions by the aid of traveling wave hypothesis.

3. Mathematical analysis

In order to get started, the following hypothesis is selected:

$$q(x,t) = g(s)e^{i\phi(x,t)}, \tag{2}$$

where $g(s)$ represents the shape of the pulse and

$$s = x - vt, \tag{3}$$

and the phase component is defined as

$$\phi(x,t) = -\kappa x + \omega t + \theta_0. \tag{4}$$

Here, κ is the soliton frequency, ω is the wave number of the soliton and θ_0 is the phase constant. Also, in (3), v represents the speed of the soliton. Substituting (2) into (1) and decomposing into real and imaginary parts, give

$$\begin{aligned} &ag''(s) + (b_2 - \lambda\kappa) g^3(s) + b_1 g^2(s) - \\ &(\omega + \alpha\kappa + a\kappa^2) g(s) = 0, \end{aligned} \tag{5}$$

and

$$v + \alpha + 2a\kappa + (3\lambda + 2\theta) g^2(s) = 0, \tag{6}$$

respectively. In (5), $g'(s) = dg/ds$ and $g''(s) = d^2g/ds^2$. From (6), setting the coefficients of the linearly independent functions to zero gives the speed of the soliton as:

$$v = -\alpha - 2a\kappa \tag{7}$$

and the constraint condition

$$3\lambda + 2\theta = 0. \tag{8}$$

Next, multiplying both sides of (5) by $g'(s)$ and integrating with respect to s gives

$$\begin{aligned} 6a\left(\frac{dg}{ds}\right)^2 &= 3(\lambda\kappa - b_2) g^4(s) - \\ &4b_1 g^3(s) + 6(\omega + \alpha\kappa + a\kappa^2) g^2(s). \end{aligned} \tag{9}$$

Separating variables in (9) and integrating yields the following three cases:

Case-1: (Bright soliton)

$$g(s) = \frac{A_1}{D_1 + \cosh[B(x - vt)]} \tag{10}$$

whenever

$$9(\omega + \alpha\kappa + a\kappa^2)(\lambda\kappa - b_2) - 2b_1^2 < 0. \tag{11}$$

Thus, bright 1-soliton solution is given by

$$q(x,t) = \frac{A_1}{D_1 + \cosh[B(x - vt)]} e^{i(-\kappa x + \omega t + \theta_0)} \tag{12}$$

where the amplitude A_1, the width B and the parameter D_1 are given by

$$A_1 = \frac{6(\omega + \alpha\kappa + a\kappa^2)}{\sqrt{4b_1^2 - 18(\omega + \alpha\kappa + a\kappa^2)(\lambda\kappa - b_2)}}, \tag{13}$$

$$D_1 = \frac{2b_1}{\sqrt{4b_1^2 - 18(\omega + \alpha\kappa + a\kappa^2)(\lambda\kappa - b_2)}}, \tag{14}$$

and

$$B = \sqrt{\frac{\omega + \alpha\kappa + a\kappa^2}{a}}. \tag{15}$$

The width of the soliton B introduces another constraint that is given by

$$a(\omega + \alpha\kappa + a\kappa^2) > 0. \tag{16}$$

The following Fig. 1 is the bright soliton solution to the model. The parameter values chosen in this case are: $a = 1,\ b_1 = 1,\ b_2 = 1,\ \kappa = -0.6,\ \omega = 1,\ \alpha = 1, \lambda = 1$

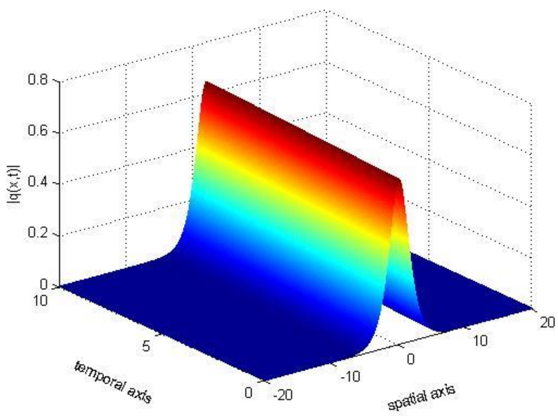

Fig. 1. Bright Soliton

Case-2: (Singular soliton)

$$g(s) = \frac{A_2}{D_2 + \sinh[B(x - vt)]} \tag{17}$$

whenever

$$9\left(\omega+\alpha\kappa+a\kappa^2\right)\left(\lambda\kappa-b_2\right)-2b_1^2>0. \quad (18)$$

In this case, singular 1-soliton solution is

$$q(x,t)=\frac{A_2}{D_2+\sinh[B(x-vt)]}e^{i(-\kappa x+\omega t+\theta_0)} \quad (19)$$

where the parameters A_2 and D_2 are

$$A_2=\frac{6\left(\omega+\alpha\kappa+a\kappa^2\right)}{\sqrt{18\left(\omega+\alpha\kappa+a\kappa^2\right)\left(\lambda\kappa-b_2\right)-4b_1^2}}, \quad (20)$$

and

$$D_2=\frac{2b_1}{\sqrt{18\left(\omega+\alpha\kappa+a\kappa^2\right)\left(\lambda\kappa-b_2\right)-4b_1^2}}, \quad (21)$$

respectively. The parameter B stays the same as in (15) together with the constraint (16).

Case-3: Combo solitons

$$g(s)=\frac{1}{D_3+\cosh[B(x-vt)]-\sinh[B(x-vt)]} \quad (22)$$

whenever

$$9\left(\omega+\alpha\kappa+a\kappa^2\right)\left(\lambda\kappa-b_2\right)-2b_1^2=0, \quad (23)$$

and

$$D_3=\frac{b_1}{3\left(\omega+\alpha\kappa+a\kappa^2\right)}. \quad (24)$$

Finally, the combo 1-soliton solution is

$$g(s)=\frac{1}{D_3+\cosh[B(x-vt)]-\sinh[B(x-vt)]} \quad (25)$$
$$e^{i(-\kappa x+\omega t+\theta_0)}.$$

The parameter B from (15) together with the the constraint (16) still remains valid here.

4. Conclusions

This paper obtained bright, singular and combo optical soliton solution to perturbed NLSE that appars with quadratic-cubic nonlinearity. The perturbation terms are related to inter-modal dispersion, nonlinear dispersion and self-steepening. The traveling wave hypothesis is employed in this paper. The soliton solutions appear with constraint conditions that naturally emerge from the mathematical structure of its parameters such as the amplitude and width. The results of this paper are an extended version of the results that are reported in the past [4]. In fact, when the perturbation terms are turned off, all results of this paper collapse to the ones that are obtained earlier [4]. These extended results will further serve as a foundation stone to study this problem. Later, these results will be extended to optical couplers, optical metamaterials, magneto-optic waveguides and other such devices. Those extensions are presently awaited and will be disseminated soon.

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