

Optical soliton perturbation with extended tanh function method

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This paper applies extended tanh-function algorithm to obtain soliton solutions. The governing equation is the perturbed nonlinear Schrödinger's equation with Kerr law nonlinearity. Both topological and singular soliton solutions are obtained. There are other solutions that fall out as a by-product. These are singular periodic solutions and complexiton solutions to the model. The constraint conditions are listed for the solitons to exist.

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1. Introduction

Optical solitons is one of the most important and fascinating areas of research in the field of nonlinear optics. Soliton molecules form the basic fabric in information transfer across trans-oceanic and trans-continental distances. Therefore it is imperative to take a deeper look at this engineering marvel from a mathematical perspective. In the past, several mathematical tricks and techniques have been applied to the governing model to extract exact soliton solution. What is noticeably missing so far is the accountability for perturbation terms. The integrability aspect of solitons is going to be studied, in this paper, with several perturbation terms.

There are various integration algorithms that were applied in earlier studies to the model, without any perturbation terms [1-15]. These are simplest equation method [9, 10], traveling wave hypothesis [1] ansatz approach [12-14], Lie symmetry analysis [15] and several others. This paper will apply the mechanism of extended tanh-function approach to retrieve topological and singular 1-soliton solution to the model with perturbation terms. There are constraint conditions that will be naturally revealed for the existence of the solution. Additionally, as a by-product, few other solutions of the model will naturally fall out although these are not important in nonlinear optics community.

1.1. The model

This paper is motivated by the desire to extend the extended tanh-function method [4] for perturbed nonlinear Schrödinger's equation (NLSE) [1,2] as follows:

$$q_z = ia_1 q_{tt} + ia_2 q|q|^2 + a_3 q_{ttt} + a_4 (q|q|^2)_t + a_5 q(|q|^2)_t + a_6 q_t \quad (1)$$

which describes propagation of ultrashort pulses in nonlinear optical fibers, where the complex function $q = q(z, t)$ is slowly varying envelope of the electric field, the subscripts z and t are spatial and temporal partial derivative in retarded time coordinates. a_1, a_2, a_3, a_4, a_5 and a_6 are the real parameters related to the group velocity, self-phase modulation, third order dispersion, self-steepening and self-frequency shift arising from stimulated Raman scattering respectively.

The paper is arranged as follows. In Section 2, we describe briefly the extended tanh-function method. In Section 3, we apply this method to perturbed NLSE. Concluding remarks are given in Section 4.

2. Extended tanh-function method: an overview

Consider a general nonlinear evolution equation (NLEE) in the form

$$P(u, u_x, u_t, u_{xx}, u_{xt}, \dots) \quad (2)$$

Using the wave complex variable $\xi = ik(x - ct)$ carries Eq. (2) into the following ordinary differential equation (ODE):

$$Q(u, iku', -ikcu', -k^2u'', \dots) \quad (3)$$

where prime denotes the derivative with respect to the same variable ξ .

Then, the solution of Eq. (3) we are looking for is expressed in the form of a finite series of tanh functions

$$u(x, t) = u(\xi) = \sum_{l=0}^M a_l F^l \quad (4)$$

where $F^l = \tanh^l(\xi)$, M is a positive integer that can be determined by balancing the highest order derivative with the highest nonlinear terms in equation. Here, k, c, a_0, \dots, a_M are parameters to be determined. The crucial step of the method is to take full advantage of a Riccati equation that the tanh function satisfies and use its solutions F .

The required Riccati equation is written as

$$F' = b + F^2 \quad (5)$$

where $F' = dF/d\xi$ and b is a constant. The Riccati equation has the general solutions

(a) If $b < 0$

$$F = -\sqrt{-b} \tanh(\sqrt{-b}\xi) \quad (6)$$

$$F = -\sqrt{-b} \coth(\sqrt{-b}\xi)$$

(b) If $b = 0$

$$F = -\frac{1}{\xi} \quad (7)$$

(c) If $b > 0$

$$F = \sqrt{b} \tan(\sqrt{b}\xi) \quad (8)$$

$$F = -\sqrt{b} \cot(\sqrt{b}\xi)$$

Substituting Eq. (4) into Eq. (3) by using Eq. (5) yields a set of algebraic equations for F^l , and all coefficients of

F^l have to vanish. After separated algebraic equations, we can find coefficients k, c, b, a_0, \dots, a_M .

3. Application to perturbed NLSE

Since $q = q(z, t)$ in Eq. (1) is a complex function, then we can suppose that

$$q(\xi, t) = U(\xi)e^{i(\alpha z + \beta t)}, \quad \xi = ik(z - ct) \quad (9)$$

where α, β, k and c are constants, all of them are to be determined.

Thus, by using Eq. (9) we have

$$q_z = i(kU' + \alpha U)e^{i(\alpha z + \beta t)}$$

$$q_t = i(-kcU' + \beta U)e^{i(\alpha z + \beta t)}$$

$$q_{tt} = (-k^2c^2U'' + 2\beta kcU' - \beta^2U)e^{i(\alpha z + \beta t)}$$

$$q_{ttt} = i(k^3c^3U''' - 3\beta k^2c^2U'' + 3\beta^2kcU' - \beta^3U)e^{i(\alpha z + \beta t)}$$

$$(q|q|^2)_t = i(\beta U^3 - 3kcU^2U')e^{i(\alpha z + \beta t)}$$

$$q(|q|^2)_t = -2ikcU^2U'e^{i(\alpha z + \beta t)} \quad (10)$$

Substituting the travelling wave variables (9) into Eq. (1), we have

$$\begin{aligned} & -(\alpha + a_1\beta^2 + a_3\beta^3 - a_6\beta)U + (a_2 + a_4\beta)U^3 \\ & + k(2a_1c\beta + 3a_3c\beta^2 - a_6c - 1)U' \\ & - k^2c^2(a_1 + 3\beta a_3)U'' + a_3k^3c^3U''' \\ & - kc(3a_4 + 2a_5)U^2U' = 0 \end{aligned} \quad (11)$$

Balancing U''' and U^2U' gives

$$M + 3 = 2M + M + 1 \Leftrightarrow M = 1$$

Therefore, we may choose

$$U(\xi) = b_0 + b_1F(\xi) \quad (12)$$

Substituting Eq. (12) into Eq. (11) using (5) yields a set of algebraic equations for $b_0, b_1, \alpha, \beta, c, k, b$:

$F^4(\xi)$ coeff.:

$$6a_3k^3c^3b_1 - kc(3a_4 + 2a_5)b_1^3 = 0 \quad (13)$$

$$b_1 = \pm ck \sqrt{\frac{6a_3}{3a_4 + 2a_5}} \quad (14)$$

$F^3(\xi)$ coeff.:

$$(a_2 + a_4\beta)b_1^3 - 2kc(3a_4 + 2a_5)b_0b_1^2 - 2k^2c^2(a_1 + 3\beta a_3)b_1 = 0$$

$F^2(\xi)$ coeff.:

$$k(2a_1c\beta + 3a_3c\beta^2 - ca_6 - 1)b_1 + 3(a_2 + a_4\beta)b_0b_1^2 + 8a_3k^3c^3bb_1 - kc(3a_4 + 2a_5)b_0^2b_1 - kc(3a_4 + 2a_5)bb_1^3 = 0$$

$F^1(\xi)$ coeff.:

$$3(a_2 + a_4\beta)b_0^2b_1 - 2k^2c^2(a_1 + 3\beta a_3) - (\alpha + a_1\beta^2 + a_3\beta^3 - a_6\beta)b_1 - 2kc(3a_4 + 2a_5)bb_0b_1^2 = 0$$

$F^0(\xi)$ coeff.:

$$-(\alpha + a_1\beta^2 + a_3\beta^3 - a_6\beta)b_0 - kc(3a_4 + 2a_5)bb_0^2b_1 + (a_2 + a_4\beta)b_0^3 + k(2a_1c\beta + 3a_3c\beta^2 - ca_6 - 1)bb_1 + 2a_3k^3c^3b^2b_1 = 0$$

Solving the nonlinear systems of equations (13) we get the following cases:

Case-I: For

$b =$

$$\frac{a_1^2ca_4^2 + 6ca_4^2a_3a_6 + 6a_3a_4^2 + 6ca_1a_2a_3a_4 - 9ca_3^2a_2^2}{12k^2c^3a_4^2a_3^2}$$

$$\alpha = -\frac{a_2(a_6a_4^2 + a_1a_2a_4 - a_3a_2^2)}{a_4^3}$$

$$\beta = -\frac{a_2}{a_4}$$

$$b_0 = \mp \frac{\sqrt{6}(a_1a_4 - 3a_2a_3)}{6a_4\sqrt{a_3(3a_4 + 2a_5)}}$$

where k and c are arbitrary constants.

Using Eqs. (6)-(8) and Eqs. (12)-(14), we have

$$U(\xi) = \mp \frac{\sqrt{6}}{a_4\sqrt{a_3(3a_4 + 2a_5)}} \left\{ \frac{a_1a_4 - 3a_2a_3}{6} + \sqrt{\frac{9ca_3^2a_2^2 - a_1^2ca_4^2 - 6ca_4^2a_3a_6 - 6a_3a_4^2 - 6ca_1a_2a_3a_4}{12c}} \right. \\ \left. \times \tanh\left(\sqrt{\frac{9ca_3^2a_2^2 - a_1^2ca_4^2 - 6ca_4^2a_3a_6 - 6a_3a_4^2 - 6ca_1a_2a_3a_4}{12k^2c^3a_4^2a_3^2}}\xi\right) \right\} \quad (15)$$

$$U(\xi) = \mp \frac{\sqrt{6}}{a_4\sqrt{a_3(3a_4 + 2a_5)}} \left\{ \frac{a_1a_4 - 3a_2a_3}{6} + \sqrt{\frac{9ca_3^2a_2^2 - a_1^2ca_4^2 - 6ca_4^2a_3a_6 - 6a_3a_4^2 - 6ca_1a_2a_3a_4}{12c}} \right. \\ \left. \times \coth\left(\sqrt{\frac{9ca_3^2a_2^2 - a_1^2ca_4^2 - 6ca_4^2a_3a_6 - 6a_3a_4^2 - 6ca_1a_2a_3a_4}{12k^2c^3a_4^2a_3^2}}\xi\right) \right\} \quad (16)$$

where $c \left\{ \begin{matrix} a_1^2ca_4^2 + 6ca_4^2a_3a_6 + 6a_3a_4^2 \\ + 6ca_1a_2a_3a_4 - 9ca_3^2a_2^2 \end{matrix} \right\} < 0$.

$$U(\xi) = \pm \frac{\sqrt{6}}{a_4\sqrt{a_3(3a_4 + 2a_5)}} \left\{ \frac{a_1a_4 - 3a_2a_3}{6} - \sqrt{\frac{a_1^2ca_4^2 + 6ca_4^2a_3a_6 + 6a_3a_4^2 + 6ca_1a_2a_3a_4 - 9ca_3^2a_2^2}{12c}} \right. \\ \left. \times \tan\left(\sqrt{\frac{a_1^2ca_4^2 + 6ca_4^2a_3a_6 + 6a_3a_4^2 + 6ca_1a_2a_3a_4 - 9ca_3^2a_2^2}{12k^2c^3a_4^2a_3^2}}\xi\right) \right\} \quad (17)$$

$$U(\xi) = \mp \frac{\sqrt{6}}{a_4\sqrt{a_3(3a_4 + 2a_5)}} \left\{ \frac{a_1a_4 - 3a_2a_3}{6} + \sqrt{\frac{a_1^2ca_4^2 + 6ca_4^2a_3a_6 + 6a_3a_4^2 + 6ca_1a_2a_3a_4 - 9ca_3^2a_2^2}{12c}} \right. \\ \left. \times \cot\left(\sqrt{\frac{a_1^2ca_4^2 + 6ca_4^2a_3a_6 + 6a_3a_4^2 + 6ca_1a_2a_3a_4 - 9ca_3^2a_2^2}{12k^2c^3a_4^2a_3^2}}\xi\right) \right\} \quad (18)$$

where $c \left\{ \begin{matrix} a_1^2ca_4^2 + 6ca_4^2a_3a_6 + 6a_3a_4^2 \\ + 6ca_1a_2a_3a_4 - 9ca_3^2a_2^2 \end{matrix} \right\} > 0$.

$$U(\xi) = \mp \left(\frac{\sqrt{6}(a_1a_4 - 3a_2a_3)}{6a_4\sqrt{a_3(3a_4 + 2a_5)}} + ck \sqrt{\frac{6a_3}{3a_4 + 2a_5}} \frac{1}{\xi} \right) \quad (19)$$

where

$$a_1^2ca_4^2 + 6ca_4^2a_3a_6 + 6a_3a_4^2 + 6ca_1a_2a_3a_4 = 9ca_3^2a_2^2$$

Thus, the exact solutions of Eq. (1) can be written as

Type-1: When $c \left\{ \begin{matrix} a_1^2ca_4^2 + 6ca_4^2a_3a_6 + 6a_3a_4^2 \\ + 6ca_1a_2a_3a_4 - 9ca_3^2a_2^2 \end{matrix} \right\} < 0$,

we have

$$q(z,t) = \mp \frac{\sqrt{6}}{a_4\sqrt{a_3(3a_4+2a_5)}} \left\{ \frac{a_1a_4-3a_2a_3}{6} + \sqrt{\frac{9ca_3^2a_2^2 - a_1^2ca_4^2 - 6ca_4^2a_3a_6 - 6a_3a_4^2 - 6ca_1a_2a_3a_4}{12c}} \right. \\ \left. \times \tanh \left(\sqrt{\frac{9ca_3^2a_2^2 - a_1^2ca_4^2 - 6ca_4^2a_3a_6 - 6a_3a_4^2 - 6ca_1a_2a_3a_4}{12k^2c^3a_4^2a_3^2}} (ik(z-ct)) \right) \right\} \\ \times e^{-i \left\{ \left(\frac{a_2(a_6a_4^2+a_1a_2a_4-a_3a_2^2)}{a_4^3} \right) z + \left(\frac{a_2}{a_4} \right) t \right\}}$$

(20)

which represents a topological 1-soliton solution to (1) and

$$q(z,t) = \mp \frac{\sqrt{6}}{a_4\sqrt{a_3(3a_4+2a_5)}} \left\{ \frac{a_1a_4-3a_2a_3}{6} + \sqrt{\frac{9ca_3^2a_2^2 - a_1^2ca_4^2 - 6ca_4^2a_3a_6 - 6a_3a_4^2 - 6ca_1a_2a_3a_4}{12c}} \right. \\ \left. \times \coth \left(\sqrt{\frac{9ca_3^2a_2^2 - a_1^2ca_4^2 - 6ca_4^2a_3a_6 - 6a_3a_4^2 - 6ca_1a_2a_3a_4}{12k^2c^3a_4^2a_3^2}} (ik(z-ct)) \right) \right\} \\ \times e^{-i \left\{ \left(\frac{a_2(a_6a_4^2+a_1a_2a_4-a_3a_2^2)}{a_4^3} \right) z + \left(\frac{a_2}{a_4} \right) t \right\}}$$

(21)

which represents a singular 1-soliton solution to (1).

Type-2: When $c \left\{ \begin{matrix} a_1^2ca_4^2 + 6ca_4^2a_3a_6 + 6a_3a_4^2 \\ + 6ca_1a_2a_3a_4 - 9ca_3^2a_2^2 \end{matrix} \right\} > 0$,

the following singular periodic solutions fall out:

$$q(z,t) = \pm \frac{\sqrt{6}}{a_4\sqrt{a_3(3a_4+2a_5)}} \left\{ \frac{a_1a_4-3a_2a_3}{6} - \sqrt{\frac{a_1^2ca_4^2 + 6ca_4^2a_3a_6 + 6a_3a_4^2 + 6ca_1a_2a_3a_4 - 9ca_3^2a_2^2}{12c}} \right. \\ \left. \times \tan \left(\sqrt{\frac{a_1^2ca_4^2 + 6ca_4^2a_3a_6 + 6a_3a_4^2 + 6ca_1a_2a_3a_4 - 9ca_3^2a_2^2}{12k^2c^3a_4^2a_3^2}} (ik(z-ct)) \right) \right\} \\ \times e^{-i \left\{ \left(\frac{a_2(a_6a_4^2+a_1a_2a_4-a_3a_2^2)}{a_4^3} \right) z + \left(\frac{a_2}{a_4} \right) t \right\}}$$

(22)

and

$$q(z,t) = \mp \frac{\sqrt{6}}{a_4\sqrt{a_3(3a_4+2a_5)}} \left\{ \frac{a_1a_4-3a_2a_3}{6} + \sqrt{\frac{a_1^2ca_4^2 + 6ca_4^2a_3a_6 + 6a_3a_4^2 + 6ca_1a_2a_3a_4 - 9ca_3^2a_2^2}{12c}} \right. \\ \left. \times \cot \left(\sqrt{\frac{a_1^2ca_4^2 + 6ca_4^2a_3a_6 + 6a_3a_4^2 + 6ca_1a_2a_3a_4 - 9ca_3^2a_2^2}{12k^2c^3a_4^2a_3^2}} (ik(z-ct)) \right) \right\} \\ \times e^{-i \left\{ \left(\frac{a_2(a_6a_4^2+a_1a_2a_4-a_3a_2^2)}{a_4^3} \right) z + \left(\frac{a_2}{a_4} \right) t \right\}}$$

(23)

which are most studied in nonlinear optics. Nevertheless, these solutions are listed for the sake of complete analysis of the governing equation.

Type-2: When

$$a_1^2ca_4^2 + 6ca_4^2a_3a_6 + 6a_3a_4^2 + 6ca_1a_2a_3a_4 = 9ca_3^2a_2^2$$

we can obtain the following solution

$$q(z,t) = \mp \left(\frac{\sqrt{6}(a_1a_4-3a_2a_3)}{6a_4\sqrt{a_3(3a_4+2a_5)}} + ck \sqrt{\frac{6a_3}{3a_4+2a_5}} \frac{1}{(ik(z-ct))} \right) \\ \times e^{-i \left\{ \left(\frac{a_2(a_6a_4^2+a_1a_2a_4-a_3a_2^2)}{a_4^3} \right) z + \left(\frac{a_2}{a_4} \right) t \right\}}$$

(24)

Case-II: Here,

$$b = \frac{1}{24a_3^2k^2c^3(a_4+a_5)^2} \left\{ (8ca_4a_5 + 4ca_5^2 + 3ca_4^2)a_1^2 \right. \\ \left. + 6ca_1a_2a_3a_4 + 12a_3a_4^2 + 12ca_3a_6a_4^2 + 12a_3a_5^2 \right. \\ \left. + 12ca_3a_6a_5^2 + 24ca_3a_4a_3a_6 + 24a_3a_4a_5 - 9ca_2^2a_3^2 \right\}$$

$$\alpha = -\frac{1}{54ca_3^2(a_4 + a_5)^3} \{ (6ca_4a_5^2 + 4ca_5^3)a_1^3 + (36ca_2a_3a_4a_5 + 18ca_2a_3a_5^2)a_1^2 + (18ca_3a_5a_4^2a_6 - 27a_3a_4a_5^2 + 54ca_2^2a_4a_3^2 - 54a_3a_4^2a_5 + 18ca_3a_5^3a_6 - 27a_4^3a_3 + 36ca_3a_4a_5^2a_6)a_1 + 81a_2a_3^2a_4^2 + 81a_3^2a_5^2a_2 - 54ca_2^2a_3^2 + 108ca_2a_3^2a_4a_5a_6 + 54ca_2a_3^2a_4^2a_6 + 162a_3^2a_2a_4a_5 + 54ca_2a_3^2a_5^2a_6 \}$$

$$\beta = -\frac{3a_1a_4 + 2a_1a_5 - 3a_2a_3}{6a_3(a_4 + a_5)}$$

$$b_0 = 0$$

$$b_1 = \pm ck \sqrt{\frac{6a_3}{3a_4 + 2a_5}} \tag{25}$$

where k and c are arbitrary constants.

Using Eqs. (6)-(8), Eq. (12) and Eq. (25), we obtain

$$U(\xi) = \mp \frac{1}{2(a_4 + a_5)} \sqrt{\frac{9ca_3^2a_2^2 - H}{ca_3(3a_4 + 2a_5)}} \times \tanh\left(\sqrt{\frac{9ca_3^2a_2^2 - H}{24a_3^2k^2c^3(a_4 + a_5)^2}} \xi\right) \tag{26}$$

$$U(\xi) = \mp \frac{1}{2(a_4 + a_5)} \sqrt{\frac{9ca_3^2a_2^2 - H}{ca_3(3a_4 + 2a_5)}} \times \coth\left(\sqrt{\frac{9ca_3^2a_2^2 - H}{24a_3^2k^2c^3(a_4 + a_5)^2}} \xi\right) \tag{27}$$

$$U(\xi) = \pm \frac{1}{2(a_4 + a_5)} \sqrt{\frac{H - 9ca_3^2a_2^2}{ca_3(3a_4 + 2a_5)}} \times \tan\left(\sqrt{\frac{H - 9ca_3^2a_2^2}{24a_3^2k^2c^3(a_4 + a_5)^2}} \xi\right) \tag{28}$$

$$U(\xi) = \mp \frac{1}{2(a_4 + a_5)} \sqrt{\frac{H - 9ca_3^2a_2^2}{ca_3(3a_4 + 2a_5)}} \times \cot\left(\sqrt{\frac{H - 9ca_3^2a_2^2}{24a_3^2k^2c^3(a_4 + a_5)^2}} \xi\right) \tag{29}$$

and

$$U(\xi) = \mp ck \sqrt{\frac{6a_3}{3a_4 + 2a_5}} \frac{1}{\xi} \tag{30}$$

where, it was assumed

$$H = (8ca_4a_5 + 4ca_5^2 + 3ca_4^2)a_1^2 + 6ca_1a_2a_3a_4 + 12a_3a_4^2 + 12a_3a_5^2 + 12ca_3a_6a_5^2 + 12ca_3a_6a_4^2 + 24ca_3a_4a_5a_6 + 24a_3a_4a_5 \tag{31}$$

Thus, the solutions of the perturbed NLSE is of the following three types:

Type-1: When $9c^2a_3^2a_2^2 > cH$,

$$q(z, t) = \mp \frac{1}{2(a_4 + a_5)} \sqrt{\frac{9ca_3^2a_2^2 - H}{ca_3(3a_4 + 2a_5)}} \times \tanh\left(\sqrt{\frac{9ca_3^2a_2^2 - H}{24a_3^2k^2c^3(a_4 + a_5)^2}} (ik(z - ct))\right) \times \exp[-i\{(\frac{1}{54ca_3^2(a_4 + a_5)^3} \{ (6ca_4a_5^2 + 4ca_5^3)a_1^3 + (36ca_2a_3a_4a_5 + 18ca_2a_3a_5^2)a_1^2 + (18ca_3a_5a_4^2a_6 - 27a_3a_4a_5^2 + 54ca_2^2a_4a_3^2 - 54a_3a_4^2a_5 + 18ca_3a_5^3a_6 - 27a_4^3a_3 + 36ca_3a_4a_5^2a_6)a_1 + 81a_2a_3^2a_4^2 + 81a_3^2a_5^2a_2 - 54ca_2^2a_3^2 + 108ca_2a_3^2a_4a_5a_6 + 54ca_2a_3^2a_4^2a_6 + 162a_3^2a_2a_4a_5 + 54ca_2a_3^2a_5^2a_6 \})z + (\frac{3a_1a_4 + 2a_1a_5 - 3a_2a_3}{6a_3(a_4 + a_5)}t)\}] \tag{32}$$

and

$$q(z, t) = \mp \frac{1}{2(a_4 + a_5)} \sqrt{\frac{9ca_3^2a_2^2 - H}{ca_3(3a_4 + 2a_5)}} \times \coth\left(\sqrt{\frac{9ca_3^2a_2^2 - H}{24a_3^2k^2c^3(a_4 + a_5)^2}} (ik(z - ct))\right) \times \exp[-i\{(\frac{1}{54ca_3^2(a_4 + a_5)^3} \{ (6ca_4a_5^2 + 4ca_5^3)a_1^3 + (36ca_2a_3a_4a_5 + 18ca_2a_3a_5^2)a_1^2 + (18ca_3a_5a_4^2a_6 - 27a_3a_4a_5^2 + 54ca_2^2a_4a_3^2 - 54a_3a_4^2a_5 + 18ca_3a_5^3a_6 - 27a_4^3a_3 + 36ca_3a_4a_5^2a_6)a_1 + 81a_2a_3^2a_4^2 + 81a_3^2a_5^2a_2 - 54ca_2^2a_3^2 + 108ca_2a_3^2a_4a_5a_6 + 54ca_2a_3^2a_4^2a_6 + 162a_3^2a_2a_4a_5 + 54ca_2a_3^2a_5^2a_6 \})z + (\frac{3a_1a_4 + 2a_1a_5 - 3a_2a_3}{6a_3(a_4 + a_5)}t)\}] \tag{33}$$

respectively represent topological and singular 1-soliton solution to (1), where H is given by (31).

Type-2: For $9c^2a_3^2a_2^2 < cH$, the following singular periodic solutions are recovered:

$$\begin{aligned}
q(z, t) = & \pm \frac{1}{2(a_4 + a_5)} \sqrt{\frac{H - 9ca_3^2 a_2^2}{ca_3(3a_4 + 2a_5)}} \\
& \times \tan \left(\sqrt{\frac{H - 9ca_3^2 a_2^2}{24a_3^2 k^2 c^3 (a_4 + a_5)^2}} (ik(z - ct)) \right) \\
& \times \exp \left[-i \left\{ \left(\frac{1}{54ca_3^2 (a_4 + a_5)^3} \{ (6ca_4 a_5^2 + 4ca_5^3) a_1^3 \right. \right. \right. \\
& + (36ca_2 a_3 a_4 a_5 + 18ca_2 a_3 a_5^2) a_1^2 + (18ca_3 a_5 a_4^2 a_6 \\
& - 27a_3 a_4 a_5^2 + 54ca_2^2 a_4 a_3^2 - 54a_3 a_4^2 a_5 + 18ca_3 a_5^3 a_6 \\
& - 27a_4^3 a_3 + 36ca_3 a_4 a_5^2 a_6) a_1 + 81a_2 a_3^2 a_4^2 + 81a_3^2 a_5^2 a_2 \\
& - 54ca_2^2 a_3^2 + 108ca_2 a_3^2 a_4 a_5 a_6 + 54ca_2 a_3^2 a_4^2 a_6 \\
& + 162a_3^2 a_2 a_4 a_5 + 54ca_2 a_3^2 a_5^2 a_6 \} \} z \\
& \left. + \left(\frac{3a_1 a_4 + 2a_1 a_5 - 3a_2 a_3}{6a_3 (a_4 + a_5)} t \right) \right]
\end{aligned} \quad (34)$$

and

$$\begin{aligned}
q(z, t) = & \mp \frac{1}{2(a_4 + a_5)} \sqrt{\frac{H - 9ca_3^2 a_2^2}{ca_3(3a_4 + 2a_5)}} \\
& \times \cot \left(\sqrt{\frac{H - 9ca_3^2 a_2^2}{24a_3^2 k^2 c^3 (a_4 + a_5)^2}} (ik(z - ct)) \right) \\
& \times \exp \left[-i \left\{ \left(\frac{1}{54ca_3^2 (a_4 + a_5)^3} \{ (6ca_4 a_5^2 + 4ca_5^3) a_1^3 \right. \right. \right. \\
& + (36ca_2 a_3 a_4 a_5 + 18ca_2 a_3 a_5^2) a_1^2 + (18ca_3 a_5 a_4^2 a_6 \\
& - 27a_3 a_4 a_5^2 + 54ca_2^2 a_4 a_3^2 - 54a_3 a_4^2 a_5 + 18ca_3 a_5^3 a_6 \\
& - 27a_4^3 a_3 + 36ca_3 a_4 a_5^2 a_6) a_1 + 81a_2 a_3^2 a_4^2 + 81a_3^2 a_5^2 a_2 \\
& - 54ca_2^2 a_3^2 + 108ca_2 a_3^2 a_4 a_5 a_6 + 54ca_2 a_3^2 a_4^2 a_6 \\
& + 162a_3^2 a_2 a_4 a_5 + 54ca_2 a_3^2 a_5^2 a_6 \} \} z \\
& \left. + \left(\frac{3a_1 a_4 + 2a_1 a_5 - 3a_2 a_3}{6a_3 (a_4 + a_5)} t \right) \right]
\end{aligned} \quad (35)$$

Type-3: Finally, when $9c^2 a_3^2 a_2^2 = cH$, the solution is:

$$\begin{aligned}
q(z, t) = & \mp ck \sqrt{\frac{6a_3}{3a_4 + 2a_5}} \frac{1}{ik(z - ct)} \\
& \times \exp \left[-i \left\{ \left(\frac{1}{54ca_3^2 (a_4 + a_5)^3} \{ (6ca_4 a_5^2 + 4ca_5^3) a_1^3 \right. \right. \right. \\
& + (36ca_2 a_3 a_4 a_5 + 18ca_2 a_3 a_5^2) a_1^2 + (18ca_3 a_5 a_4^2 a_6 \\
& - 27a_3 a_4 a_5^2 + 54ca_2^2 a_4 a_3^2 - 54a_3 a_4^2 a_5 + 18ca_3 a_5^3 a_6 \\
& - 27a_4^3 a_3 + 36ca_3 a_4 a_5^2 a_6) a_1 + 81a_2 a_3^2 a_4^2 + 81a_3^2 a_5^2 a_2 \\
& - 54ca_2^2 a_3^2 + 108ca_2 a_3^2 a_4 a_5 a_6 + 54ca_2 a_3^2 a_4^2 a_6 \\
& + 162a_3^2 a_2 a_4 a_5 + 54ca_2 a_3^2 a_5^2 a_6 \} \} z \\
& \left. + \left(\frac{3a_1 a_4 + 2a_1 a_5 - 3a_2 a_3}{6a_3 (a_4 + a_5)} t \right) \right]
\end{aligned} \quad (36)$$

4. Conclusions

This paper applied extended tanh-function algorithm to retrieve soliton solutions to the perturbed NLSE that is only considered with Kerr law nonlinearity. This method only revealed topological and singular soliton solutions to the model. However, this scheme fails to recover bright 1-soliton solution that is the most important type of soliton in nonlinear optics. There are however several other schemes that are applied to recover such soliton solutions. They are discussed earlier [12-15]. Later, this paper will be extended to study the same perturbed NLSE with non-Kerr law nonlinearity. The results of that research will be published elsewhere. This is just a tip of the iceberg.

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