Optical focused electromagnetic field of a perfect electromagnetic conductor elliptical reflector in un magnetized plasma environment

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A theoretical investigation has been carried out to analyze the optical focusing electromagnetic field by a perfect electromagnetic conductor elliptical reflector in un-magnetized plasma environment. The presented analysis and formulations are general for any perfect conductor elliptical cylinder (PEC, PMC, or PEMC). The co-polarized and the cross-polarized components of the focal region electromagnetic field expressions have been obtained using Maslov's method. The electromagnetic field behaviors at Caustic or focal point for the cases of PEMC, PEC (perfect electric conductor) and PMC (perfect magnetic conductor) elliptical reflector have been computed and compared. The behavior of the electromagnetic field behaviors at Caustic or focal point with the variation of the admittance parameter, plasma frequency and effective collision frequency for the co-polarized and the cross polarized fields is also investigated. The comparisons of the computed results of the presented formulations with the published results of some special cases confirm the accuracy of the presented analysis.

(Received December 16, 2014; accepted January 21, 2015)

Keywords: Perfect electromagnetic conductor, Elliptical reflector, Maslov's method, Plasma

1. Introduction

In recent years the study of analysis of electromagnetic waves focused in the focal region by reflectors [1] is imperative in the current arena of advanced technologies for microwave, millimeter-wave, and optical device applications. The analysis of focal region filed is useful for optical spectroscopy, medical treatment and hyperthermia [2]. The image field may be also useful to generate images of the human body with the help of radio frequency [3]. Such type of analysis is very significant to find out the suitable parameters of the plasma which affect the reflection, absorption, and transmission of the electromagnetic energy [4]. When cylindrical and spherical metallic structures, as reflector antennas for example, enter into the Earth's atmosphere with high velocities, plasma layer forms on their antennas surfaces [5]. Thus, usually antennas of space vehicles are on the surface in contact with the plasma layer and this layer affects the radiation characteristics of antennas [6-9]. In this contest study of analysis of reflector in plasma environment is necessary.

It has been demonstrated theoretically that the PEMC material acts as a perfect reflector of electromagnetic waves but it differs from the PEC and the PMC in that the reflected waves have a cross polarized components [10]. The perfect electromagnetic conductor (PEMC) is a generalized form of both the perfect electric conductor (PEC) and the perfect magnetic conductor (PMC) [11]. Many researchers carried out research on PEMC materials [12, 13].

The geometrical optics (GO) approximation is well known technique. However, in many problems, such as describing fields in the vicinity of caustic, ART or geometrical optics (GO) does not provide satisfactory results [14]. To overcome the defect of GO Maslov's method is used. Maslov's method is a combination of asymptotic ray theory (ART) and Fourier transform method [15]. This technique has been used to study analysis of high frequency field in focal region successfully by many authors [16-18].

Many types of reflectors have been investigated in the open literature recently for focused electromagnetic wave in the focal region [19-21]. To generate an aberration free incident wave becomes a new stimulating job in focusing systems since aberrations are always increased greatly when the aperture of collimator is enlarged as a necessary illumination. However, the size of a source in an elliptical reflector won't be changed with the aperture of the reflector, and this kind of illuminator can be easily obtained by a general small aperture objective due to its two foci. In this system the feed is placed at or near one focus, and the second focus is placed at the region we desire to illuminate.

In this paper, we consider a long perfect electromagnetic conductor elliptical reflector in un-magnetized plasma environment. The electromagnetic field intensity in the caustic or focal region of PEMC elliptical reflector has been derived using Maslov's method. The effects of some physical parameters such as the plasma and wave frequencies and the admittance parameter on the focal regions field have been studied. The results of the presented formulations have been compared with the published results of some special cases which confirm the accuracy of the presented analysis.

2. Methods and formulation

Consider a perfect electromagnetic conductor (PEMC) geometry of an elliptical reflector placed in plasma medium as depicted in Fig. 1. The plasma medium is considered as an un-magnetized, incompressible and isotropic medium. The density of the medium is kept uniform. The motion of electrons in plasma is only considered because mass of ion is much greater as compared to the mass of electron ($M \gg m$). Equation of surface of elliptical reflector is given as

$$\zeta = a\sqrt{1-\frac{\xi^2}{b^2}} \tag{1}$$

where a and b are semi minor axes of the focal length of the PEMC elliptical reflector. The coordinates of a point on the PEMC elliptical reflector $P(\xi, \zeta)$ are defined as [7]

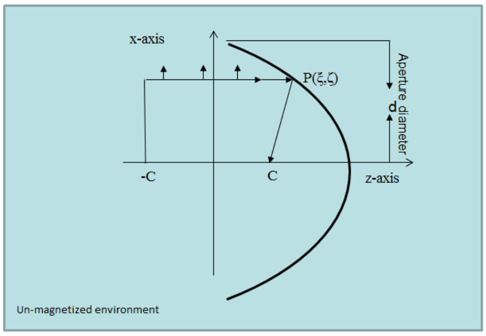

Fig. 1. PEMC Elliptical reflector placed un-magnetized plasma environment.

Now, we consider a monochromatic electromagnetic wave incident on the PEMC elliptical reflector from, parallel to its symmetry axis

$$\boldsymbol{E_{0i}} = E_i \hat{e}_x \exp(-jkz) \tag{2}$$

$$\boldsymbol{H_{0i}} = Z^{-1} E_i \hat{e}_y \exp(-jkz) \tag{3}$$

where $Z = \sqrt{\mu_0 / \varepsilon_p \epsilon_0}$ is intrinsic impedance of plasma medium. In above equations, k represents the wave number in un-magnetized plasma medium. Its value is $k_o(\varepsilon_p)^{1/2}$ as given in [7-8]; k_o is the wave number in free space and $\varepsilon_p = 1 - \frac{i\omega_o{}^2}{(v+i\omega)\omega}$ is the relative permittivity of plasma medium, which is the function of (electron) plasma frequency $\omega_o = \sqrt{\frac{ne^2}{m\varepsilon_o}}$ effective collision frequency v and the incident field frequency ω. As a plane wave from source point $-C$ is made incident on the PEMC elliptical reflector, the wave vector of the reflected wave can be worked out by using Snell's law whose mathematical expression is described in the following form

$$\mathbf{P^r} = \mathbf{P^i} - 2(\mathbf{P^i}.\mathbf{n})\mathbf{n} \tag{4}$$

where $\mathbf{P^i}$ is wave number of incident wave and $\mathbf{n}$ is unit normal to the surface can be written as

$$\mathbf{n} = \hat{e}_x \cos\alpha + \hat{e}_z \sin\alpha \tag{5}$$

By applying Snell's law the wave vector $\mathbf{P^r}$ of the wave reflected is given by [11]

$$\mathbf{P^r} = -\hat{e}_x \sin 2\alpha - \hat{e}_z \cos 2\alpha \tag{6}$$

The Jacobian associated with wave reflected by the parabolic plasma layer is obtained

$$\mathrm{J}(\tau) = \frac{\mathrm{D}(\tau)}{\mathrm{D}(0)} = 1 - \frac{\tau}{R_2} \tag{7}$$

The fields expressions for the reflected rays in geometric optics are obtained as under [13-18]

$$\mathbf{E}^r(\mathrm{r}) = \boldsymbol{E_{r0}}(x,z)[1-\frac{\tau}{R_2}]^{-\frac{1}{2}} exp[-jk(\Psi_0+\tau)] \tag{8}$$

where $\Psi_0 = 2a - \sqrt{(z-c)^2+x^2}$, τ is parameter along the ray from coordinates of point on PEMC elliptical reflector. It is observed that the GO field becomes infinity at the Caustic points as is expected when $J(t) = 0$. We can derive the expression which is valid at the focal point using the Maslov's method. The exact location of focal or caustic point may also be obtained at $J(t) = 0$. By using Maslov's method the focal regions a valid field expression is as [9-18].

$$\mathrm{E}^r(\mathrm{r}) = \sqrt{\frac{\mathrm{k}}{\mathrm{j}2\pi}} \int_{-\infty}^{\infty} \left[\frac{D(\tau)}{D(0)} \frac{\partial(\mathrm{p_x})}{\partial(\mathrm{x})}\right]^{-\frac{1}{2}} \exp\{-\mathrm{jk}[\Psi_0 + \tau - x(p_x,\mathrm{z})\, p_x + p_x x]\}\, \mathrm{d}p_x \tag{9}$$

To obtain initial value of reflected field $\boldsymbol{E_{r0}}(x,z)$ on PEMC elliptical reflector we applied PEMC boundary conditions. The boundary conditions at the surface of the PEMC

$$\left.\begin{aligned} \boldsymbol{n} \times (\boldsymbol{H} + M\boldsymbol{E}) = 0 \\ \boldsymbol{n} \cdot (\boldsymbol{D} - M\boldsymbol{B}) = 0 \end{aligned}\right\} \tag{10}$$

where $\boldsymbol{n}$ is the unit normal, and M is the admittance parameter characterizing the PEMC. The values M=0 and $M \to \pm\infty$, correspond to PMC and PEC respectively. The electromagnetic energy cannot enter into the PEMC region because that the pointing vector is complex and imaginary [19]. Using above boundary conditions we get

$$\boldsymbol{E_{r0}}(x,z) = \left[\frac{1-(\mathrm{M}\eta_0)^2}{(\mathrm{M}\eta_0)^2+1}\hat{e}_x - \frac{2\mathrm{M}\eta_0}{(\mathrm{M}\eta_0)^2+1}\hat{e}_y\right]E_i \qquad (11)$$

Using above equations the co polarized and cross polarized field components in the region are

$$\frac{E_x(x,z)}{E_i} = \frac{1}{4j\pi}\int_{-X}^{X}\frac{1-(\mathrm{MZ})^2}{(\mathrm{MZ})^2+1}\frac{1}{\sqrt{R_1R_2}}\sqrt{\frac{b^2}{b^2-\xi^2}}\exp\{-jk[\rho\cos(2\alpha-\theta)-2ka]\}\,d\alpha \qquad (12)$$

$$\frac{E_x(x,z)}{E_i} = -\frac{1}{4j\pi}\int_{-X}^{X}\frac{2MZ}{(\mathrm{MZ})^2+1}\frac{1}{\sqrt{R_1R_2}}\sqrt{\frac{b^2}{b^2-\xi^2}}\exp\{-jk[\rho\cos(2\alpha-\theta)-2ka]\}\,d\alpha \qquad (13)$$

where $X = tan^{-1}(a/2c)$ is the angle which subtends the aperture.

3. Numerical result and discussion

In this paper, optical focused electromagnetic fields from a perfect electromagnetic conductor elliptical reflector in un-magnetized plasma environment is obtained at normal incident from primary focal point $-C$ around the second focal point C. Then the effects of admittance parameter, electron density in plasma and effective collision frequency of plasma on the optical focused fields are discussed. The incident frequency of electromagnetic wave is taken as $f = 1\times10^9$Hz. The numerical results are compared with the published literature to check the correctness of analytical calculations and also as well the working of software pack. By using MATHEMATICA software, the computations are made of equations (11) and (12). Fig. 2 shows the comparison at $MZ = \infty$, between, focused fields from PEMC elliptical reflector in un-magnetized plasma environment our study and previous study [14]. When plasma medium is replaced by the dielectric free space then the present work transforms into the published work as in [14] and good agreement is found between analytical and then numerical calculations. It is also observed from the comparison PEMC elliptical reflector in un-magnetized plasma environment and PEMC elliptical reflector placed in free space that field intensity decreases 4 times in un-magnetized as compared to free space environment.

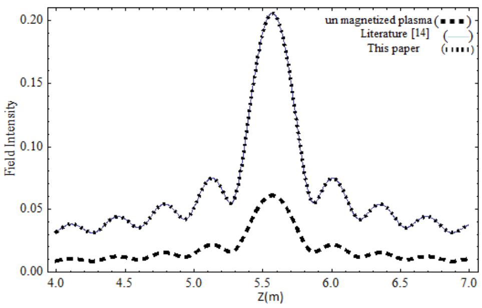

Fig. 2. Comparison of field intensity of PEMC elliptical reflector under special conditions (solid line),elliptical reflector (dashed line)[14] and PEMC elliptical reflector in un-magnetized plasma environment(thick dashed line) at Caustic point.

Fig. 3a and Fig. 3b show the comparison of the co polarized and cross polarized field intensity distribution of PEMC elliptical reflector in un-magnetized plasma environment along z-axis for the different values of admittance parameter respectively. The solid, dotted, thick dashed, dashed and thick dotted lines show the result at $MZ = \infty$, $MZ = 0.0$, $MZ = 0.3$, $MZ = 0.4$ and $MZ = 0.6$, for the co polarized and cross polarized field intensity distribution of PEMC elliptical reflector in un-magnetized plasma environment along z-axis respectively. From Fig. 3a it is observed that at $MZ = \infty$ and $MZ = 0$ co polarized focused the field intensity corresponds to reflect from PEC, PMC elliptical reflectors and at intermediate values of admittance parameter the focused the field intensity corresponds to PEMC elliptical reflectors. It is also observed that field intensity shifts to a smaller value as we increase the admittance parameter. Fig. 3b it is observed that at $MZ = \infty$ and $MZ = 0$ cross polarized fields components disappear and at intermediate values of admittance parameter the cross polarized focused the field intensity decreases with decrease in the values of MZ.

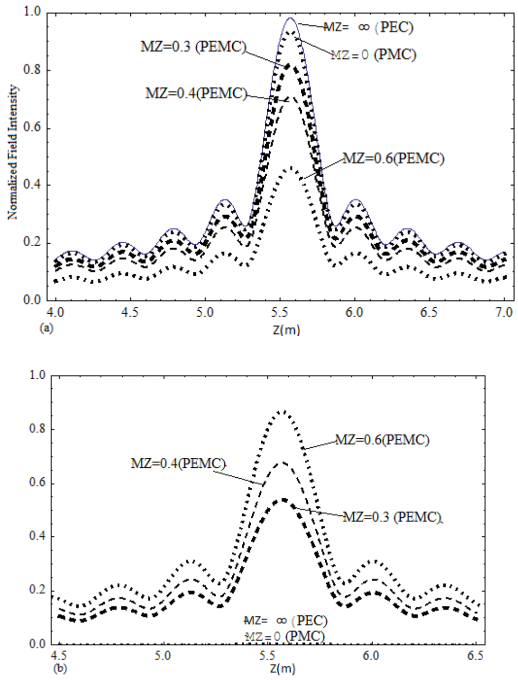

Fig. 3. Normalized field intensity distributions around focal pointC of PEMC elliptical reflector in un-magnetized plasma environment for different values of admittance parameter along z-axis (a) Co polarized field (b) Cross polarized field.

Fig. 4a and Fig. 4b show the comparison of the co polarized and cross polarized field intensity distribution of PEMC elliptical reflector in un-magnetized plasma environment along z-axis around the focal point for the different values of plasma frequencies at $\omega_p = 5.63\times10^8$Hz(solid line), $\omega_p = 7.97\times10^8$Hz (thick dashed line), $\omega_p = 9.76\times10^8$ Hz(dashed line) $\omega_p = 1.12\times10^9$Hz (dotted line), and $\omega_p = 1.26\times10^9$Hz(thick dotted line) respectively. From Fig. 4a and Fig. 4a it is observed that the field intensity shifts to a smaller value as we increase the values of plasma frequency. It is also observed that field intensity of co polarized field component is 5 times greater than the field intensity of cross polarized field component at focal point.

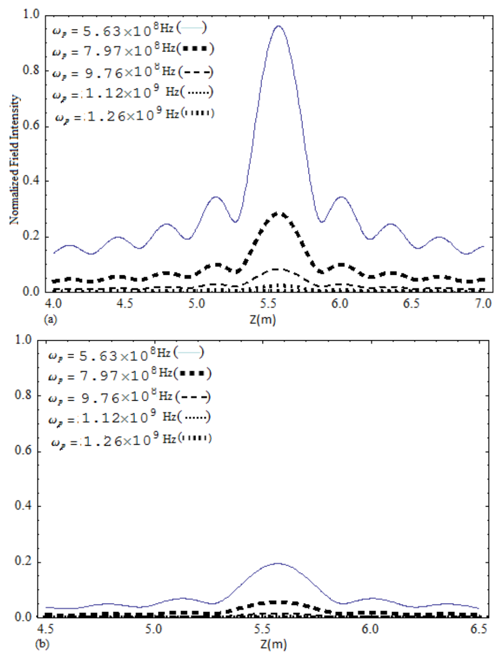

Fig. 4. Normalized field intensity distributions around focal points C of PEMC elliptical reflector in un-magnetized plasma environment for different values of plasma frequency along z-axis (a) Co polarized field (b) Cross polarized field.

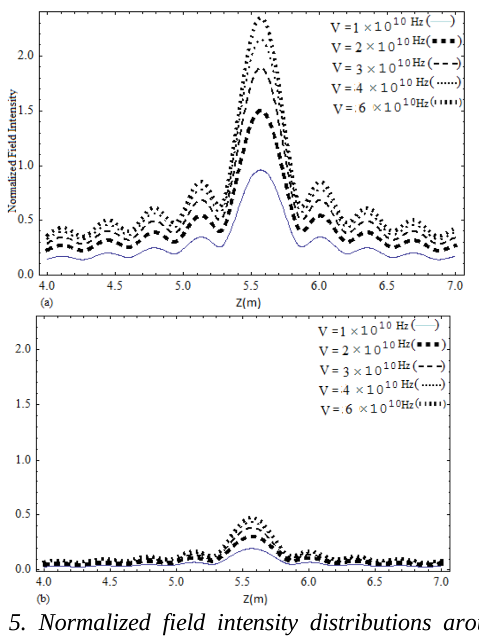

Fig. 5. Normalized field intensity distributions around focal points C of PEMC elliptical reflector in un-magnetized plasma environment for different values of effective collision frequency along z-axis(a) Co polarized field (b) Cross polarized field.

Fig. 5a and Fig. 5b show the comparison of the co polarized and cross polarized field intensity distribution of PEMC elliptical reflector in un-magnetized plasma environment along z-axis around the focal point for the different values of effective collision frequencies at $V = 1 \times 10^{10}$Hz(solid line), $V = 2 \times 10^{10}$Hz (thick dashed line), $V = 3 \times 10^{10}$ Hz(dashed line) $V = 4 \times 10^{10}$Hz (dotted line), and $V = 6 \times 10^{10}$Hz(thick dotted line) respectively. From Fig. 5a and Fig. 5a it is observed that the field intensity shifts to a smaller value as we decrease the values of effective collision frequency. It is also observed that field intensity of co polarized field component is 4 times greater than the field intensity of cross polarized field component at focal point.

4. Conclusion

In this paper, the focusing of electromagnetic field through a PEMC elliptical reflector in un-magnetized plasma environment for normal incidence is analyzed numerically by using Maslov's method. The field distribution is presented in the form of numerical results for different parameters to clarify the focusing behavior of PEMC elliptical reflector in un-magnetized plasma environment. It is observed that the cross polarized field component appeared for PEMC elliptical reflector. It is clear from analysis of co polarized field component that at infinite value of admittance parameter PEMC elliptical reflector approaches to PEC elliptical reflector and at null value of admittance parameter PEMC elliptical reflector approaches to PMC elliptical reflector. In case of cross polarized field component at infinite value and null value of admittance field cross polarized field component disappear. It is also observed that field intensity increases with decrease in plasma frequency and decrease with increase of effective collision frequency. These results can be helpful in the study and designing of optical devices.

Acknowledgment

The authors would like to extend their sincere appreciation to the Deanship of Scientific Research (DSR) at King Saud University for its funding of this research through the Research Group Project no RGP-VPP-312.

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