On the Ediz eccentric connectivity index of a graph

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If G is a connected graph with vertex set V, then the Ediz eccentric connectivity index of G, ${}^{E}\xi^{c}(G)$, is defined as $\sum_{v\in V}\frac{S_v}{ec(v)}$ where S_v is the sum of degrees of all vertices adjacent to vertex v and $ec(v)$ is its eccentricity . In this paper we investigate some mathematical properties of the Ediz eccentric connectivity index.

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1. Introduction

A topological index is a numerical descriptor of the molecular structure derived from the corresponding (hydrogen-depleted) molecular graph. Various topological indices are widely used for quantitative structure-property relationship (QSPR) and quantitative structure-activity relationship (QSAR) studies [1-4].

Recently the Ediz eccentric connectivity index was defined and computed for nanostar dendrimers in [5] . In this paper we propose to investigate some mathematical properties of this novel connectivity index.

2. Definitions and some examples

Consider a simple connected graph G, and let $V(G)$ and $E(G)$ denote its vertex and edge sets, respectively. $|V(G)|=n$ is called the order of G. The distance between u and v in $V(G)$, $d_G(u,v)$, is the length of a shortest u-v path in G. If no ambiguity is possible, the subscript G may be omitted. The eccentricity, $ec(u)$ of a vertex $u\in V(G)$ is the maximum distance between u and any other vertex in G. The diameter of G, d, is defined as the maximum value of the eccentricities of the vertices of G. Similarly, the radius of G, r , is defined as the minimum value of the eccentricities of the vertices of G. A central vertex of G is any vertex whose eccentricity is equal to the radius of G. And S_v is the sum of degrees of all vertices u , adjacent to vertex v. Finally, the degree of a vertex $v\in V(G)$, $\deg(v)$ is the number of edges incident to v. We define the *Ediz eccentric connectivity index* ${}^{E}\xi^{c}(G)$ of G as;

$$ {}^{E}\xi^{c}(G)=\sum_{v\in V}\frac{S_v}{ec(v)}. $$

For special classes of graphs we compute the following useful values for our parameter, using from definition.

$$ {}^{E}\xi^{c}(K_n)=n(n-1)^2. $$

$$ {}^{E}\xi^{c}(K_{m,n})=\frac{1}{2}mn(m+n). $$

For the star, cycle and path of order n,

$$ {}^{E}\xi^{c}(S_n)=\frac{n(n+2)}{2}. $$

$$ {}^{E}\xi^{c}(C_n)=\frac{4n}{\lfloor n/2\rfloor}. $$

$$ {}^{E}\xi^{c}(P_n)=\begin{cases} 2\left(\frac{2}{n-1}+\frac{3}{n-2}+\frac{4}{n-3}+\frac{4}{n-4}+\ldots+\frac{4}{\lfloor n/2\rfloor+1}\right)+\frac{4}{\lfloor n/2\rfloor} & \text{for } n \text{ is } odd. \\ 2\left(\frac{2}{n-1}+\frac{3}{n-2}+\frac{4}{n-3}+\frac{4}{n-4}+\ldots+\frac{4}{n/2}\right) & \text{for } n \text{ is } even. \end{cases} $$

3. Results

In this section, we give lower and upper bounds for the Ediz eccentric connectivity index of connected graphs in terms of graph invariants such as the number of vertices (n), the radius (r), the diameter (d) and the minimum degree (δ).

Theorem 1. *Let* $G=(V,E)$ *be a connected graph of order n. Then,* ${}^{E}\xi^{c}(G)\leq n.(n-1)^{2}$ *with equality if and only if* G *is a complete graph.*

Proof. It is obvius that the only graph with its diameter *d=1* and the sum S_v equals its maximum is a complete graph. We can directly write from the definition of the *Ediz eccentric connectivity index* for *n* vertex complete graph K_n in which its vertex set is $V=\{1,2,...,n\}$;

$$ {}^{E}\xi^{c}(K_n)=\sum_{v\in K_n}\frac{S_v}{ec(v)}=S_1+S_2+...+S_n $$

$$ =(n-1).(n-1)+(n-1).(n-1)+...+(n-1).(n-1)=n.(n-1)^{2}. $$

Theorem 2. *Let* $G=(V,E)$ *be a connected graph of order n, minimum degre* δ *and diameter d . Then* $,\frac{n.\delta}{d}<{}^{E}\xi^{c}(G)$.

Proof. Let the vertex set is $V(G)=\{1,2,...,n\}$. From the definition of the *Ediz eccentric connectivity index* can be written for any connected graph *G*,

$$ {}^{E}\xi^{c}(G)=\sum_{i=1}^{n}\frac{S_i}{ec(i)}=\frac{S_1}{ec(1)}+\frac{S_2}{ec(2)}+...+\frac{S_n}{ec(n)} $$

$$ >\frac{\delta}{d}+\frac{\delta}{d}+...+\frac{\delta}{d}=\frac{n.\delta}{d}. $$

Theorem 3. *Let* $G=(V,E)$ *be a k-regular graph of order n, radius r and diameter d . Then,* $\frac{nk^{2}}{d}\leq{}^{E}\xi^{c}(G)\leq\frac{nk^{2}}{r}$ *with equality from the both side if and only if* G *is a complete graph.*

Proof. Since the every vertex of *V(G)* adjacent to exactly *k* vertex and for any neighbouring vertex $S_v=k^{2}$, the desired result is acquired from the definiton.

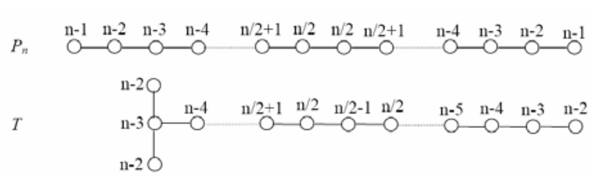

Fig. 1. The graphs P_n *and T and their vertices' eccentricities when n is odd.*

Theorem 4. Let T be a tree of order n, $n\geq 2$. Then ${}^{E}\xi^{c}\left(P_n\right)\leq^{E}\xi^{c}\left(T\right)$.

Proof. Firstly, if $d=n-1$, then $T=P_n$ and the theorem is true. The same is true when $n=2$ or 3. Call α the number of end vertices of T : clearly $\alpha\geq 3$, if T is not a path. So assume that $n\geq 4$, $d\leq n-2$. If n is odd then from the Fig. 1 and the definition, we can write;

$$ {}^{E}\xi^{c}(T)-{}^{E}\xi^{c}(P_n)=\frac{10}{n-2}-\frac{4}{n-1}-\frac{1}{n-3}>0\cdot $$

Clearly, the above difference become even greater if the number of pendent vertices are increasing. If n is even then the proof is made similarly.

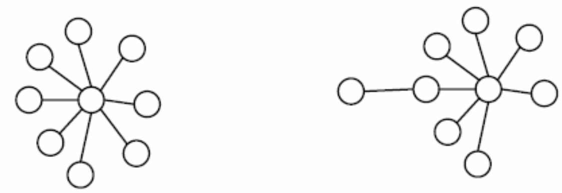

Fig. 2. The star graph S_{n-1} *and the maximal tree T which is not a star.*

Theorem 5. *Let T be a tree of order n,* $n\geq 2$. *Then* ${}^{E}\xi^{c}\left(T\right)\leq^{E}\xi^{c}\left(S_{n-1}\right)$.

Proof. Firstly, if $d=2$, then $T=S_n$ and the theorem is true. Assume that $n\geq 4$ and $d\geq 3$. From the Fig. 2. and the definition the desired result is acquired.

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