

On some eccentricity based topological indices of nanostar dendrimers

RASHID FAROOQ*, MEHAR ALI MALIK

School of Natural Sciences, National University of Sciences and Technology, Islamabad, Pakistan

Let G be a molecular graph. The distance between two vertices of G is the length of a shortest path connecting these two vertices. The eccentricity of a vertex u in G is the largest distance between u and any other vertex of G . In this paper, we consider some infinite families of nanostar dendrimers and compute their eccentric-connectivity index and total-eccentricity index. Furthermore, we also compute some eccentricity based Zagreb indices of these nanostar dendrimers.

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1. Introduction

Chemical graph theory is a branch of mathematical chemistry which has an important effect on the development of the chemical sciences. A topological index is a numerical value associated with the chemical constitution of a certain chemical compound aiming to correlate various physical and chemical properties, or some biological activity in it. In an exact phrase, if G denotes the class of all finite graphs then a topological index is a function $Top: \Omega \rightarrow \mathbb{R}$ such that for any $G, H \in \Omega$, $Top(G) = Top(H)$ if G and H are isomorphic.

Carbon nanostructures have found many potential industrial applications such as energy storage, gas sensors, biosensors, nanoelectronic devices and chemical probes [10], just to name a few. Carbon allotropes such as carbon nanocones and carbon nanotubes have been proposed as possible molecular gas storage devices [4, 14]. The nanostar dendrimers are macromolecules that appear to be photon funnels. These macromolecules are used in the formation of nanotubes, micro and macrocapsules, nanolatex, coloured glasses, chemical sensors, modified electrodes, etc. [2, 6]. Recently, many researchers focused to conjecture various topological indices of nanostructures by using computational tools. The reader is referred to [3, 11, 12] for further details.

Let G be an n -vertex molecular graph with vertex set $V(G) = \{v_1, v_2, \dots, v_n\}$ and edge set $E(G)$. The vertices of G correspond to atoms and an edge between two vertices corresponds to the chemical bond between these vertices. A (v_1, v_n) -path on n vertices is defined as a graph with vertex set $\{v_i : 1 \leq i \leq n\}$ and edge set $\{v_i v_{i+1} : 1 \leq i \leq n-1\}$. The length of a path is the

number of edges in it. The distance $d(u, v)$ between two vertices $u, v \in V(G)$ is defined as the length of the shortest (u, v) -path in G . For a given vertex $u \in V(G)$, the eccentricity $\varepsilon(u)$ is defined as the largest distance between u and any other vertex v in G .

The Wiener index [13] is the first reported distance based topological index defined as half sum of the distances between all the pairs of vertices in a graph. Another distance based topological index of a graph G is the eccentric-connectivity index $\xi(G)$ defined as

$$\xi(G) = \sum_{u \in V(G)} d(u)\varepsilon(u). \quad (1)$$

When the vertex degrees are not taken into account, we obtain the total-eccentricity index of the graph G defined by

$$\zeta(G) = \sum_{u \in V(G)} \varepsilon(u). \quad (2)$$

For a k -regular graph G , these two quantities are related as $\xi(G) = k\zeta(G)$. For further detail on these and some other important indices, see [1, 8, 15].

Zagreb indices have been introduced more than thirty years ago by Gutman and Trinajstić [9]. They are defined as

$$M_1(G) = \sum_{v \in V(G)} (d(v))^2, \\ M_2(G) = \sum_{uv \in E(G)} d(u)d(v).$$

Some new versions of Zagreb indices of a molecular graph G defined by Ghorbani and Hosseinzadeh [7] are

expressed in terms of eccentricity as follows:

$$M_1^*(G) = \sum_{uv \in E(G)} [\varepsilon(u) + \varepsilon(v)], \tag{3}$$

$$M_1^{**}(G) = \sum_{v \in V(G)} (\varepsilon(v))^2, \tag{4}$$

$$M_2^*(G) = \sum_{uv \in E(G)} \varepsilon(u)\varepsilon(v). \tag{5}$$

For each $u \in V(G)$, we observe that $\varepsilon(u)$ appears exactly $d(u)$ times in the sum (3). Therefore

$$\begin{aligned} M_1^*(G) &= \sum_{uv \in E(G)} (\varepsilon(u) + \varepsilon(v)) \\ &= \sum_{u \in V(G)} d(u)\varepsilon(u) \\ &= \xi(G). \end{aligned}$$

Thus it is insignificant to compute M_1^* .

In this paper, we consider some families of nanostar dendrimers and compute their eccentric-connectivity index, total-eccentricity index and some eccentricity based Zagreb indices. Ashrafi and Saheli [5] obtained the eccentric-connectivity index of the nanostar dendrimers $D_1[n]$ (see Fig. 1) as follows.

$$\xi(D_1[n]) = 504n \times 2^n + 504 \times 2^n - 360n + 1047. \tag{6}$$

However, equation (6) does not provide correct results for the aforesaid index. We correct this result and show in Theorem 3.1 that the eccentric-connectivity index of the nanostar dendrimers $D_1[n]$ is the following:

$$\xi(D_1[n]) = 252n \times 2^n - 33 \times 2^n - 90n + 75. \tag{7}$$

Furthermore, we discuss the eccentricity based Zagreb indices M_1^{**} and M_2^* of these nanostar dendrimers.

2. Some families of nanostar dendrimers

The first type of nanostar dendrimers is $D_1[n]$ shown in Fig. 1. The order and size of $D_1[n]$ nanostar dendrimers are $24 + 36(n-1)$ and $27 + 42(n-1)$, respectively. The second type of nanostar dendrimers is denoted by $D_2[n]$ and is shown in Fig. 2. The order and size of $D_2[n]$ are $120 \times 2^n - 108$ and $140 \times 2^n - 127$, respectively.

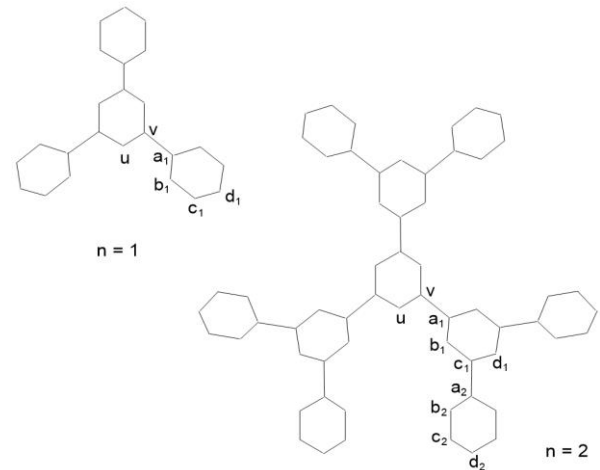


Fig. 1. $D_1[n]$ with $n = 1$ and 2 .

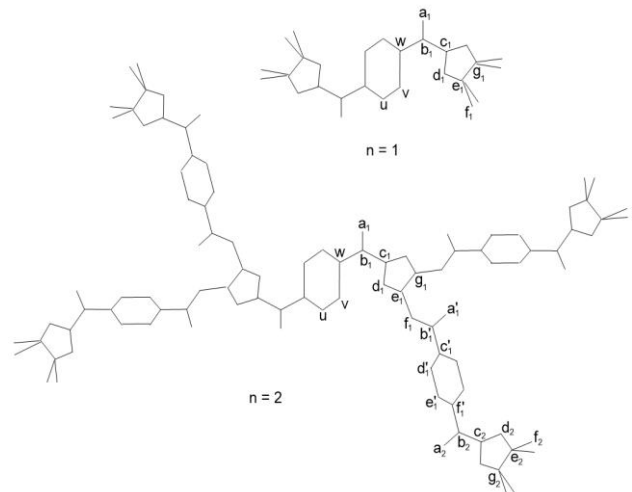


Fig. 2. $D_2[n]$ with $n = 1$ and $n = 2$.

3. The eccentric-connectivity index of nanostar dendrimers

In this section, we compute the eccentric-connectivity index of the nanostar dendrimers $D_1[n]$ and $D_2[n]$ shown in Fig. 1 and Fig. 2, respectively.

Theorem 3.1 *The eccentric-connectivity index of $D_1[n]$ is given by*

$$\xi(D_1[n]) = 252n \times 2^n - 33 \times 2^n - 90n + 75.$$

Proof. Using symmetry of the nanostar dendrimer $D_1[n]$ we use only one branch of $D_1[n]$ as labeled in Fig. 1. We take one representative from a set of vertices which have same degree and eccentricity. These representatives are labelled by u, v, a_i, b_i, c_i, d_i for $1 \leq i \leq n$, and shown in Table 1 along with their degrees, eccentricities and frequencies.

Table 1. The representatives of vertices of $D_1[n]$ with their degrees, eccentricities and frequencies of occurrence, for $1 \leq i \leq n$.

Representatives	Degree	Eccentricity	Frequency
u	2	$3n + 4$	3
v	3	$3n + 3$	3
a_i	3	$3n + 3i + 1$	$3 \times 2^{i-1}$
b_i	2	$3n + 3i + 2$	$6 \times 2^{i-1}$
$c_i (i \neq n)$	3	$3n + 3i + 3$	$6 \times 2^{i-1}$
c_n	2	$6n + 3$	$6 \times 2^{n-1}$
d_i	2	$3n + 3i + 4$	$3 \times 2^{i-1}$

Using Table 1, we can write the eccentric-connectivity index of $D_1[1]$ as follows.

$$\begin{aligned} \xi(D_1[1]) &= \sum_{u \in V(D_1[1])} d(u)\varepsilon(u) \\ &= 2 \times 3 \times (3 + 4) + 3 \times 3 \times (3 + 3) \\ &\quad + 3 \times 3 \times (3 + 3 + 1) + 2 \times 6 \times (3 + 3 + 2) \\ &\quad + 2 \times 6 \times (6 + 3) + 2 \times 3 \times (3 + 3 + 4) \\ &= 252 \times 2 - 33 \times 2 - 90 + 75. \end{aligned}$$

The eccentric-connectivity index of $D_1[n]$ for $n \geq 2$, can be written as follows.

$$\begin{aligned} \xi(D_1[n]) &= 2 \times 3(3n + 4) + 3 \times 3(3n + 3) + 2 \times 6(6n + 3)2^{i-1} \\ &\quad + \sum_{i=1}^n [3 \times 3(3n + 3i + 1)2^{i-1} + 2 \times 6(3n + 3i + 2)2^{i-1}] \\ &\quad + 2 \times 3(3n + 3i + 4)2^{i-1} + \sum_{i=1}^{n-1} (3 \times 6(3n + 3i + 3)2^{i-1}) \end{aligned}$$

After simplification, the eccentric-connectivity index $\xi(D_1[n])$ can be written as:

$$\xi(D_1[n]) = 252n \times 2^n - 33 \times 2^n - 90n + 75.$$

This completes the proof.

Next theorem gives eccentric-connectivity index of $D_2[n]$ nanostar dendrimers.

Theorem 3.2 The eccentric-connectivity index of $D_2[n]$ is given by

$$\xi(D_2[n]) = 1600n \times 2^n - 1670 \times 2^n - 1000n + 1762.$$

Proof. Using symmetry of the nanostar dendrimer $D_2[n]$ we use only one branch of $D_2[n]$ as labeled in Fig. 2. We take one representative from a set of vertices which have same degree and eccentricity. These representatives are labelled by $u, v, w, a_i, b_i, c_i, d_i, e_i, f_i, g_i$ for $1 \leq i \leq n$ and $a_{i'}, b_{i'}, c_{i'}, d_{i'}, e_{i'}, f_{i'}$ for $1 \leq i' \leq n - 1$, and are shown in Table 1, Table 2 and Table 3 along with their degrees, eccentricities and frequencies.

Table 2. The representatives of vertices of $D_2[n]$ with their degrees, eccentricities and frequencies of occurrence, for $1 \leq i \leq n$.

Representatives	Degree	Eccentricity	Frequency
u	2	$10n - 3$	2
v	2	$10n - 3$	2
w	3	$10n - 2$	2
a_i	1	$10n + 10i - 10$	2^i
b_i	3	$10n + 10i - 11$	2^i
c_i	3	$10n + 10i - 10$	2^i
d_i	2	$10n + 10i - 9$	2^{i+1}
$e_i (i \neq n)$	3	$10n + 10i - 8$	2^{i+1}
e_n	4	$20n - 8$	2^{n+1}
$f_i (i \neq n)$	2	$10n + 10i - 7$	2^{i+1}
f_n	1	$20n - 7$	2^{n+2}

Table 3. The representatives of vertices of $D_2[n]$ with their degrees, eccentricities and frequencies of occurrence, for $1 \leq i \leq n-1$ and $n \geq 2$.

Representatives	Degree	Eccentricity	Frequency
a_i'	1	$10n + 10i - 5$	2^{i+1}
b_i'	3	$10n + 10i - 6$	2^{i+1}
c_i'	3	$10n + 10i - 5$	2^{i+1}
d_i'	2	$10n + 10i - 4$	2^{i+2}
e_i'	2	$10n + 10i - 3$	2^{i+2}
f_i'	3	$10n + 10i - 2$	2^{i+1}

Using the data given in Table 2, the eccentric-connectivity index of $D_2[1]$ can be written as

$$\begin{aligned} \xi(D_2[1]) &= \sum_{u \in V(D_2[1])} d(u)\varepsilon(u) \\ &= 2 \times 2 \times (10 - 3) + 2 \times 2 \times (10 - 3) \\ &\quad + 3 \times 2 \times (10 - 2) + 1 \times 2 \times (10 + 10 - 10) \\ &\quad + 3 \times 2 \times (10 + 10 - 11) + 3 \times 2^2 \times (10 + 10 - 8) \\ &\quad + 4 \times 2^2 \times (20 - 8) + 2 \times 2^2 \times (10 + 10 - 7) \\ &\quad + 1 \times 2^3 \times (20 - 7) \\ &= 1600 \times 2 - 1670 \times 2 - 1000 + 1762. \end{aligned}$$

Using the data given in Table 2 and Table 3, the eccentric-connectivity index of $D_2[n]$ for $n \geq 2$, can be written as

$$\begin{aligned} \xi(D_2[n]) &= 2 \times 2 \times (10n - 3) + 2 \times 2 \times (10n - 3) \\ &\quad + 3 \times 2 \times (10n - 2) \\ &\quad + \sum_{i=1}^n [1 \times 2^i \times (10n + 10i - 10) \\ &\quad + 3 \times 2^i \times (10n + 10i - 11) \\ &\quad + 3 \times 2^i \times (10n + 10i - 10) \\ &\quad + 2 \times 2^{i+1} \times (10n + 10i - 9) \\ &\quad + 3 \times 2^{i+1} \times (10n + 10i - 8) \\ &\quad + 4 \times 2^{n+1} \times (10n + 10n - 8) \\ &\quad + 2 \times 2^{i+1} \times (10n + 10i - 7) \\ &\quad + 1 \times 2^{n+2} \times (10n + 10n - 7)] \\ &\quad + \sum_{i=1}^{n-1} [1 \times 2^{i+1} \times (10n + 10i - 5) \\ &\quad + 3 \times 2^{i+1} \times (10n + 10i - 6) \end{aligned}$$

$$\begin{aligned} &+ 3 \times 2^{i+1} \times (10n + 10i - 5) \\ &+ 2 \times 2^{i+2} \times (10n + 10i - 4) \\ &+ 2 \times 2^{i+2} \times (10n + 10i - 3) \\ &+ 3 \times 2^{i+1} \times (10n + 10i - 2)]. \end{aligned}$$

This expression can be simplified to the following form:

$$\xi(D_2[n]) = 1600n \times 2^n - 1670 \times 2^n - 1000n + 1762,$$

which completes the proof.

The next two results can easily be seen from Table 1 and Table 2-3, respectively.

Corollary 3.3 The total-eccentricity index of nanostar dendrimers $D_1[n]$ is given by

$$\zeta(D_1[n]) = (108n - 9) \times 2^n - 36n + 30.$$

Corollary 3.4 The total-eccentricity index of $D_2[n]$ nanostar dendrimers is given by

$$\zeta(D_2[n]) = (720n - 736) \times 2^n - 440n + 770.$$

4. The Zagreb-eccentricity indices of nanostar dendrimers

In this section, we compute eccentricity based Zagreb indices M_1^{**} and M_2^* defined respectively by (4) and (5) of the nanostar dendrimers $D_1[n]$ and $D_2[n]$.

Table 4. The edge partition of $D_1[n]$ with respect to the representatives of pairs of end-vertices and their frequency of occurrence. The eccentricities are taken from Table 1.

End-vertices of edges	Eccentricities	Frequency of edges
$[u, v]$	$[3n+4, 3n+3]$	6
$[a_1, v]$	$[3n+4, 3n+3]$	3
$[a_i, b_i]$	$[3n+3i+1, 3n+3i+2]$	3×2^i
$[b_i, c_i]$	$[3n+3i+2, 3n+3i+3]$	3×2^i
$[c_i, d_i]$	$[3n+3i+3, 3n+3i+4]$	3×2^i
$[c_i, a_{i+1}]$	$[3n+3i+3, 3n+3(i+1)+1]$	3×2^i

In the next theorem we compute M_1^{**} defined by equation (4) of the nanostar dendrimer $D_1[n]$.

Theorem 4.1 The second Zagreb-eccentricity index of $D_1[n]$ is given by

$$M_1^{**}(D_1[n]) = 648n^2 \times 2^n - 108n \times 2^n + 345 \times 2^n - 108n^2 + 180n - 270.$$

Proof. From Table 1, we compute the second Zagreb-eccentricity index of the nanostar dendrimers $D_1[n]$ as follows:

$$\begin{aligned} M_1^{**}(D_1[n]) &= \sum_{v \in V} [\mathcal{E}(v)]^2 \\ &= 3 \times (3n+4)^2 + 3 \times (3n+3)^2 \\ &\quad + \sum_{i=1}^n [3 \times 2^{i-1} (3n+3i+1)^2 \\ &\quad + 6 \times 2^{i-1} (3n+3i+2)^2 \\ &\quad + 6 \times 2^{i-1} (3n+3i+3)^2 \\ &\quad + 3 \times 2^{i-1} (3n+3i+4)^2] \\ &= 252n \times 2^n - 33 \times 2^n - 90n + 75. \end{aligned}$$

The proof is complete.

Theorem 4.2 The third Zagreb-eccentricity index of $D_1[n]$ is given by

$$M_2^*(D_1[n]) = (756n^2 - 198n + 408) \times 2^n$$

$$-135n^2 + 225n - 336.$$

Proof. From Table 4, we compute the third Zagreb-eccentricity index of the nanostar dendrimers $D_1[n]$ as follows:

$$\begin{aligned} M_2^*(D_1[1]) &= \sum_{uv \in E} \mathcal{E}(u)\mathcal{E}(v) \\ &= 6 \times (3+4)(3+3) + 3 \times (3+4)(3+3) \\ &\quad + 3 \times 2(3+3+1)(3+3+2) \\ &\quad + 3 \times 2(3+3+2)(3+3+3) + \\ &\quad + 3 \times 2(3+3+3)(3+3+4) \\ &= (756 - 198 + 408) \times 2 - 135 + 225 - 336. \end{aligned}$$

For $n \geq 2$, we have

$$\begin{aligned} M_2^*(D_1[n]) &= 6 \times (3n+4)(3n+3) + 3 \times (3n+4)(3n+3) \\ &\quad + \sum_{i=1}^n (3 \times 2^i (3n+3i+1)(3n+3i+2) \\ &\quad + 3 \times 2^i (3n+3i+2)(3n+3i+3) \\ &\quad + 3 \times 2^i (3n+3i+3)(3n+3i+4)) \\ &\quad + \sum_{i=1}^{n-1} 3 \times 2^i (3n+3i+3)(3n+3(i+1)+1) \\ &= (756n^2 - 198n + 408) \times 2^n \\ &\quad - 135n^2 + 225n - 336. \end{aligned}$$

This gives the required result.

Table 5. The edge partition of $D_2[n]$ with respect to the representatives of pairs of end-vertices and their frequency of occurrence. The eccentricities are taken from Table 1, Table 2 and Table 3, where $1 \leq i \leq n$.

End-vertices of edges	Eccentricities	Frequency of edges
$[u, v]$	$[10n - 3, 10n - 3]$	2
$[v, w]$	$[10n - 3, 10n - 2]$	4
$[w, b_i]$	$[10n - 2, 10n - 1]$	2
$[a_i, b_i]$	$[10n + 10i - 10, 10n + 10i - 11]$	2^i
$[b_i, c_i]$	$[10n + 10i - 11, 10n + 10i - 10]$	2^i
$[c_i, d_i]$	$[10n + 10i - 10, 10n + 10i - 9]$	2^{i+1}
$[d_i, e_i]$	$[10n + 10i - 9, 10n + 10i - 8]$	2^{i+1}
$[e_i, f_i] (i \neq n)$	$[10n + 10i - 8, 10n + 10i - 7]$	2^{i+1}
$[e_n, f_n]$	$[10n + 10n - 8, 10n + 10n - 7]$	2^{n+2}
$[e_i, g_i]$	$[10n + 10n - 8, 10n + 10n - 8]$	2^i

Table 6. The edge partition of $D_2[n]$ with respect to the representatives of pairs of end-vertices and their frequency of occurrence. The eccentricities are taken from Table 1, Table 2 and Table 3, where $1 \leq i \leq n - 1$ and $n \geq 2$.

End-vertices of edges	Eccentricities	Frequency of edges
$[a_i, b_i]$	$[10n + 10i - 5, 10n + 10i - 6]$	2^{i+1}
$[b_i, c_i]$	$[10n + 10i - 6, 10n + 10i - 5]$	2^{i+1}
$[c_i, d_i]$	$[10n + 10i - 5, 10n + 10i - 4]$	2^{i+2}
$[d_i, e_i]$	$[10n + 10i - 4, 10n + 10i - 3]$	2^{i+2}
$[e_i, f_i]$	$[10n + 10i - 3, 10n + 10i - 2]$	2^{i+2}
$[f_i, b_i]$	$[10n + 10i - 7, 10n + 10i - 6]$	2^{i+1}
$[f_i, b_{i+1}]$	$[10n + 10i - 2, 10n + 10(i + 1) - 11]$	2^{i+1}

Theorem 4.3 The second Zagreb-eccentricity index of $D_2[n]$ is given by

$$M_1^{**}(D_2[n]) = 14400n^2 \times 2^n - 29440n \times 2^n + 22516 \times 2^n - 4400n^2 + 15400n - 22654.$$

Proof. From Table 2 and Table 3 we derive the second Zagreb-eccentricity index of the nanostar dendrimers $D_2[n]$ as follows:

$$\begin{aligned} M_1^{**}(D_2[1]) &= \sum_{v \in V} [\mathcal{E}(v)]^2 \\ &= 2 \times (10 - 3)^2 + 2 \times (10 - 3)^2 \\ &\quad + 2 \times (10 - 2)^2 + 2 \times (10 + 10 - 10)^2 \\ &\quad + 2 \times (10 + 10 - 11)^2 + 2 \times (10 + 10 - 10)^2 \\ &\quad + 2^2 \times (10 + 10 - 9)^2 \end{aligned}$$

$$\begin{aligned} &+ 2^2 \times (10 + 10 - 8)^2 + 2^2 \times (10 + 10 - 7)^2 \\ &- 2^2 \times (10 + 10 - 7)^2 + 2^3 \times (10 + 10 - 7)^2 \end{aligned}$$

$$= 14400 \times 2 - 29440 \times 2 + 22516 \times 2 - 4400 + 15400 - 22654.$$

For $n \geq 2$, we have

$$\begin{aligned} M_1^{**}(D_2[n]) &= 2 \times (10n - 3)^2 + 2 \times (10n - 3)^2 \\ &\quad + 2 \times (10n - 2)^2 + \\ &\quad \sum_{i=1}^n [2^i \times (10n + 10i - 10)^2 \\ &\quad + 2^i \times (10n + 10i - 11)^2 \\ &\quad + 2^i \times (10n + 10i - 10)^2 \\ &\quad + 2^{i+1} \times (10n + 10i - 9)^2 \\ &\quad + 2^{i+1} \times (10n + 10i - 8)^2 + \end{aligned}$$

$$\begin{aligned}
& 2^{i+1} \times (10n + 10i - 7)^2] \\
& - 2^{n+1} \times (10n + 10n - 7)^2 \\
& + 2^{n+2} \times (10n + 10n - 7)^2 + \\
& \sum_{i=1}^{n-1} [2^{i+1} \times (10n + 10i - 5)^2 \\
& + 2^{i+1} \times (10n + 10i - 6)^2 \\
& + 2^{i+1} \times (10n + 10i - 2)^2] \\
= & 14400n^2 \times 2^n - 29440n \times 2^n + 22516 \times 2^n \\
& - 4400n^2 + 15400n - 22654,
\end{aligned}$$

which is the required result.

Theorem 4.4 The third Zagreb-eccentricity index of $D_2[n]$ is given by

$$\begin{aligned}
M_2^*(D_2[n]) = & 16000n^2 \times 2^n - 33400n2^n + 25712 \times 2^n \\
& - 5000n^2 + 17620n - 25914.
\end{aligned}$$

Proof. From Table 5 and Table 6 we derive the third Zagreb-eccentricity index of the nanostar dendrimers $D_2[n]$ as follows:

$$\begin{aligned}
M_2^*(D_2[1]) = & \sum_{uv \in E} \varepsilon(u)\varepsilon(v) \\
= & 2 \times (10 - 3)(10 - 3) + \\
& 4 \times (10 - 3)(10 - 2) + \\
& 2 \times (10 - 2)(10 - 1) \\
& + 2(10 + 10 - 10)(10 + 10 - 11) \\
& + 2(10 + 10 - 11)(10 + 10 - 10) \\
& + 2^2(10 + 10 - 10)(10 + 10 - 9) \\
& + 2^2(10 + 10 - 9)(10 + 10 - 8) \\
& + 2^2(10 + 10 - 8)(10 + 10 - 7) \\
& + 2(10 + 10 - 8)(10 + 10 - 8) \\
& - 2^2(10 + 10 - 8)(10 + 10 - 7) \\
& + 2^3(10 + 10 - 8)(10 + 10 - 7) \\
= & 16000 \times 2 - 33400 \times 2 + 25712 \times 2 \\
& - 5000 + 17620 - 25914.
\end{aligned}$$

For $n \geq 2$, we have

$$\begin{aligned}
M_2^*(D_2[n]) = & 2 \times (10n - 3)(10n - 3) + \\
& 4 \times (10n - 3)(10n - 2) + \\
& 2 \times (10n - 2)(10n - 1) \\
& + \sum_{i=1}^n [2^i (10n + 10i - 10)(10n + 10i - 11)
\end{aligned}$$

$$\begin{aligned}
& + 2^i (10n + 10i - 11)(10n + 10i - 10) \\
& + 2^{i+1} (10n + 10i - 10)(10n + 10i - 9) \\
& + 2^{i+1} (10n + 10i - 9)(10n + 10i - 8) \\
& + 2^{i+1} (10n + 10i - 8)(10n + 10i - 7) \\
& + 2^i (10n + 10i - 8)(10n + 10i - 8) \\
& - 2^{n+1} (10n + 10n - 8)(10n + 10n - 7) \\
& + 2^{n+2} (10n + 10n - 8)(10n + 10n - 7)] \\
& + \sum_{i=1}^{n-1} [2^{i+1} (10n + 10i - 5)(10n + 10i - 6) \\
& + 2^{i+1} (10n + 10i - 6)(10n + 10i - 5) \\
& + 2^{i+2} (10n + 10i - 5)(10n + 10i - 4) \\
& + 2^{i+2} (10n + 10i - 4)(10n + 10i - 3) \\
& + 2^{i+2} (10n + 10i - 3)(10n + 10i - 2) \\
& + 2^{i+1} (10n + 10i - 7)(10n + 10i - 6) \\
& + 2^{i+1} (10n + 10i - 2)(10n + 10(i + 1) - 11)]
\end{aligned}$$

Which can be simplified to

$$\begin{aligned}
M_2^*(D_2[n]) = & 16000n^2 \times 2^n - 33400n2^n + 25712 \times 2^n \\
& - 5000n^2 + 17620n - 25914.
\end{aligned}$$

This gives the required result.

5. Conclusion

In this paper, we consider some families of nanostar dendrimers and compute their eccentric-connectivity index, total-eccentricity index and some eccentricity based Zagreb indices defined by Ghorbani and Hosseinzadeh [7]. We show that the first eccentricity based Zagreb index $M_1^*(G)$ defined in [7] for a molecular graph G is the same as the eccentric-connectivity index $\xi(G)$ of G . We correct the eccentric-connectivity index $\xi(G)$ of the nanostar dendrimers $D_1[n]$ calculated by Ashrafi and Saheli [5]. Furthermore, we compute the eccentricity based Zagreb indices M_1^{**} and M_2^* of these nanostar dendrimers.

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*Corresponding author: farook.ra@gmail.com