Laser scattering characteristics of dust particle and photoelectric detection signal to noise ratio model in dust concentration testing system

HANSHAN LI^{*}, HANG JING

School of Electronic and Information Engineering, Xi'an Technological University, Xi'an, 710021, China

To improve the measurement accuracy and photoelectric detection ability in dust concentration testing system, this paper proposes using volume scattering function to represent the particle scattering field, analyzes light source and particles scattering characteristics, and the influence of relative humidity on particles scattering characteristics, establishes a new detection sensitivity model and signal to noise ratio (SNR) model of dust concentration photoelectric detection system, deduces the contribution degree of all factors on the sensitivity and SNR. Through calculation and analysis, gets the contribution degree of relative humidity, scale parameter and refractive index on dust concentration sensor detection sensitivity and detection SNR; obtains the effect of the particle volume scattering function distribution on incident light wavelength, particle size, and particle refractive index, gives the changing conditions of the particle volume scattering function distribution direction, incident light wavelength, particle refractive index.

(Received October 31, 2016; accepted June 7, 2017)

Keywords: Photoelectric detection, Dust particle, Laser scattering, Dust concentration

1. Introduction

With the development of industry, dust detection has become research hotspots in many areas. Laser detection method of dust concentration has a lot of advantages like simple operation, high real-time facilities, which makes a wide application. Aim at testing principles and methods at home and abroad, some researcher put forward MESA method, laser transmission method, dual branch method and other methods [1]. These methods are vulnerable to many factors, such as the source incident light polarization direction, particle size, moisture, particle refractive index, the change of these factors along with the scattering angle affect light scattering, which reduce detection system sensitivity and SNR, it affects the validity and accuracy of measurement results [2-3]. Therefore, this paper analyze contribution degree of light scattering yield from these factors, establish dust concentration sensor detection sensitivity model and detection SNR model, research the contribution degree of these factors on dust concentration sensor detection sensitivity and detection SNR, which has great significance in improving measurement accuracy.

2. Laser optical measuring method on dust concentration and analysis of influence factors

2.1. Laser optical measuring method of dust concentration

The effect of particles and laser will be occurring scattering phenomenon. Install the detector at a scattering

angle, based on scattering theory, it is inverse dust concentration after obtaining laser scattering intensity data. According to the angle between detector and laser source, it can be divided into a forward scattering, side scattering and back scattering. The forward scattering is because the angle between the detector and light source is within a range of \pm 60 °, side scattering is within the range of \pm (60 °-120 °), and back scattering is in the range of \pm (120 °-180 °), as shown in Fig. 1.



Fig. 1. Measurement diagram of laser scattering

According to Mie theory [4], when a linearly polarized laser whose wavelength is λ and light intensity is I_0 parallel exposure to isotropic spherical particles whose particle size is d, when the scattering angle is θ , the distance from the scattering is r, the light intensity can be expressed by formula (1).

$$I(\theta) = \frac{\pi^4 d^6}{8\lambda^4} \left(\frac{m^2 - 1}{m^2 + 2}\right) (1 + \cos^2 \theta) I_0 \qquad (1)$$

In (1), θ is the scattering angle, $I(\theta)$ is the scattering intensity in direction of θ , I_0 is the incident light intensity, d is the particle diameter, m is the refraction index. Generally, scattering is coherent, the scattering of different kinds of particles can be added, thus, assume the dust particle concentration is N, the formula (1) turns to formula (2).

$$I(\theta) = N \frac{\pi^4 d^6}{8\lambda^4} \left(\frac{m^2 - 1}{m^2 + 2}\right) (1 + c \circ \hat{s} \theta) I_0$$
(2)

The relationship between dust particle concentration and mass concentration C is as follows:

$$N = \frac{6C}{\pi \rho d^3} \tag{3}$$

Based on formula (3), the light intensity will change into formula (4).

$$I(\theta) = \frac{3\pi^3 d^3}{4\rho\lambda^4} \left(\frac{m^2 - 1}{m^2 + 2}\right) \left(1 + \cos^2\theta\right) I_0 \cdot C \qquad (4)$$

2.2. Analysis of influencing factors

Through the above analysis, using light scattering measurement to measure dust concentration, the scattering of light and particles are the main factors affecting the measurement. From formula (2), the influencing factors of laser source are the incident light wavelength λ , the polarization direction of the incident light φ and the polarization of the incident light.

In order to better characterize the light scattering distribution of spherical particles, we uses volume scattering function to analyze its characteristic. Volume scattering function of particles is its inherent optical properties, it is independent of the surrounding environment, and it is a physical volume to represent particles scattering characteristics [5]. For particle scattering, when irradiance incident light is *E* per unit, unit volume is *V*, the scattering direction of particles is θ , the definition of volume scattering function $\beta(\theta)$ is the derivative of the scattering intensity *I* to the differential element of volume, it is shown in (5).

$$\beta(\theta) = \frac{dI(\theta)}{EdV} \tag{5}$$

According to the definition, the physical meaning of volume scattering function is scattering intensity per unit of incident irradiance (incident light is monochromatic). $\beta(\theta)$ changes along with θ in the range of $[0,2\pi]$, its unit is $m^{-1}sr^{-1}[6]$.

One-dimensional scattering function of particles is defined, it applies to non-polarized incident light condition and the particle targets with random orientation. The complete volume scattering function of particles should be expressed as $\beta(\theta, \phi, r)$, θ is the scattering angle, ϕ is the azimuth angle, r is the distance between surface and the centroid of particles. For spherical particles, r is a constant, volume scattering function of particles can be abbreviated to $\beta(\theta, \phi)$.

Polarization of particles incident light is different, which will have an impact on the particles light scattering distribution. When the incident light is totally non-polarized, that is to say, vibration probabilities of light in all directions are the same, assume light amplitude in a direction is E_i , at this time, particle volume scattering function $\beta(\theta, \phi)$ can be expressed by (6).

$$\beta(\theta,\phi) = I_s = \int_0^{2\pi} |E_i|^2 (i_2(\theta) \cos^2 \phi + i_1(\theta) \sin^2 \phi) d\phi$$
(6)

In order to simplify the formula (6), it can turn into (7).

$$\beta(\theta, \phi) = \pi |E_i|^2 (i_1(\theta) + i_2(\theta))$$
(7)

In (7), i_1 is the scattering intensity component that parallel to the scattering surface, i_2 is the scattering intensity component that perpendicular to the scattering surface, $\beta(\theta, \phi)$ is known as the scattering function. Therefore, when the incident light is non-polarized, particle volume scattering function is independent of the azimuth angle ϕ , it only depends on the scattering angle and the nature of the particles themselves. When incident light is linearly polarized, particle volume scattering function $\beta(\theta, \phi)$ can be expressed by (8).

$$\beta(\theta,\phi) = I_{I/S}(\phi) + I_{\perp S}(\phi) = i_2 I_{I/I}(\phi) + i_1 I_{\perp I}(\phi)$$
(8)

In (8), I and $I_{\perp s}(\phi)$ represent the components of the scattering light intensity when it is parallel and vertical to the scattering plane, while, $I_{//i}(\phi)$ and $I_{\perp i}(\phi)$ are the components of the incident light intensity when it is parallel and vertical to the scattering plane, the formula (8) can simplify to formula (9).

$$\beta(\theta, \phi) = i_2^2 \left| E_x \cos\phi + E_y \sin\phi \right|^2 + i_1^2 \left| E_x \sin\phi - E_y \cos\phi \right|^2$$
(9)

In (9), E_x is the unit vector when it vibrates along with the electric field (x axis), E_y is along with the magnetic field (y axis). When linearly polarized light polarizes along the x axis, the formula (9) can be simplified to formula (10).

$$\beta(\theta, \varphi) = |E_x|^2 \left(i_2^2 \cos^2 \varphi + i_1^2 \sin^2 \varphi\right) \quad (10)$$

What need to be described is that even though the calculation results of volume scattering function in formula (7) and (10) are infinite outline physical volumes, however, after normalization processing on the calculation results, the obtained normalization volume scattering function can completely represent volume scattering function, it is used to characterize distribution situation of spherical particles scattering field. Thus, the calculation results are still reliable for reference meaning.

3. Laser scattering characteristics of dust particle

3.1. Particle size

According to the relationship between particle size and light wavelength, scattering can be divided into Rayleigh scattering, Mie scattering and the diffraction. When the particle is much smaller than the wavelength, it is known as Rayleigh scattering, when the ratio of the particle radius relative to the wavelength increases to 0.03, the scattering gradually transfers to Mie scattering [7,8].

Dust particles are a part of atmosphere; it is the realm of aerosol. Aerosol characteristics are often closely related to the water content in the atmosphere. Usually, when the relative humidity is greater than about 35%, aerosol can absorb moisture and grow; when the relative humidity is greater than 60%-70%, the aerosol absorption of water vapor is even more pronounced [9]. The adsorption of water makes not only the aerosol size increases, but also its composition is different from the dry aerosol particles, then changes laser scattering properties of aerosol.

When the relative humidity is δ , we use ζ to denote the ratio of wet aerosol particle radius $r(\delta)$ and dry aerosol particle.

$$\varsigma = \frac{r(\delta)}{r} = \left(1 - \delta\right)^{-\frac{1}{\mu}} \tag{11}$$

In (11), μ is a constant coefficient, capacity of aerosol humidity absorption is poor for the clean air in the city, $\mu = 5.8$; capacity of aerosol humidity absorption is strong for marine air, $\mu = 3.9$; capacity of aerosol humidity absorption is somewhere between clean air and marine air for polluting atmosphere, $\mu = 4.4$. If the relative humidity is known, the ratio of wet aerosol particle radius and dry aerosol particle can be gain by formula (11).

3.2. Index refraction of particle

According to Maxwell's electromagnetic wave equation, refractive index of the medium can be expressed.

$$n = \frac{c}{v} = \sqrt{\varepsilon_r \mu_r} \tag{12}$$

In (12), v is the speed in medium, and c is the speed of light in vacuum. For non-dissipative dielectric, its refractive index is a real number, and for optical absorption substances such as conductor or dissipation dielectric, the refractive index needs to be represented by a complex number.

$$m = n\left(1 + i\eta\right) \tag{13}$$

In (13), η is absorption index. Dust particles are in the atmosphere, which belongs to the dissipative media, it has absorption of light. Therefore, the difference of refractive index of particles will affect the particle light scattering.

4. Sensitivity model and SNR model in dust concentration sensor detection

4.1. Sensitivity model in dust concentration sensor detection

According to Mie scattering and angle scattering method, the system sensitivity can be defined as the ratio of light intensity variation ΔI and concentration variation ΔC , it can also be regarded as light intensity variation under concentration variation per unit [10], the bigger the light intensity variation ΔI is, the higher the sensitivity becomes. The formula of system sensitivity is as follows.

$$S_C = \frac{\Delta I}{\Delta C} = \frac{3\pi^3 d^3}{4\rho\lambda^4} \left(\frac{m^2 - 1}{m^2 + 2}\right) \left(1 + \cos^2\theta\right) I_0 \quad (14)$$

Due to difference of relative humidity will affect the particle size variation, so the sensitivity model under humidity condition changes into formula (15).

$$S_C = \frac{\Delta I}{\Delta C} = \frac{3\pi^3 \varsigma^3 d^3}{4\rho \lambda^4} \left(\frac{m^2 - 1}{m^2 + 2}\right) \left(1 + \cos^2 \theta\right) I_0 \quad (15)$$

From the formula (15), the system sensitivity increases along with the increment of particle size d, refractive index m and humidity ζ under a certain input intensity and a certain angle, while, the system sensitivity decreases along with the increment of wavelength λ . If the scattering angle is less than 90⁰, the system sensitivity decreases with the increment of the scattering angle; if the scattering angle is greater than 90⁰, the sensitivity increases along with the scattering angle increases.

4.2. SNR model in dust concentration sensor detection

The noise sources of photoelectric detector consists of thermal noise and shot noise, due to the concentration signals are low frequency signals, so there is 1/f noise, the total noise model is as follows.

$$I_n = \sqrt{\frac{4kTf}{R_L} + 2q(I_d + I_b)f + k_f \frac{I^c \Delta f}{f^a}} \quad (16)$$

In (16), $2qI_{d}\Delta f$ is shot noise caused by dark current by the detector itself, $2qI_{b}\Delta f$ is shot noise caused by light electric current of the signal and background light, $4kT\Delta f/R_{L}$ is the thermal noise of the detector, $k_{f}I^{c}\Delta f/f^{a}$ is the 1/f noise [11].

Because the photoelectric detector itself has thermal noise, shot noise, 1/f noise and other signal noise, it makes the detection output signal are not only target signal I_s but also noise signal I_N . Due to hardware limitation, light variation that detectors can sense is limited. Only when light variation meets $\Delta Q > Q_{\min}$, in another word, $\Delta I > I_{\min}$, the signals can output. The signal I_{\min} that reflects tiny concentration C_{\min} does not output, therefore, signals that reflects the concentration should be expressed by (17).

$$I_{s} = I_{0} - I - I_{n} + I_{\min}$$
(17)

According to the definition of the signal to noise ratio (SNR), it can be described by formula (18).

$$SNR = \frac{I_s}{I_n} = \frac{I - I_0 - I_n + I_{\min}}{I_n}$$
 (18)

Based on formula (16) and (17), formula (18) can turn into formula (19).

$$SNR = \frac{N\frac{\pi^{4}d^{6}}{8\lambda^{4}} \left(\frac{m^{2}-1}{m^{2}+2}\right) (1+\cos^{2}\theta) I_{0} - I_{0} - \sqrt{\frac{4kTf}{R_{L}} + 2q(I_{d}+I_{b})f + k_{f}\frac{I^{c}\Delta f}{f^{a}}} + I_{min}}{\sqrt{\frac{4kTf}{R_{L}} + 2q(I_{d}+I_{b})f + k_{f}\frac{I^{c}\Delta f}{f^{a}}}}$$

(19)

If we take into account humidity condition, Signal-to-noise ratio changes into formula (20).

$$SNR = \frac{N\frac{\pi^{4}\varsigma^{6}d^{6}}{8\lambda^{4}} \left(\frac{m^{2}-1}{m^{2}+2}\right) (1+\cos^{2}\theta) I_{0} - I_{0} - \sqrt{\frac{4kTf}{R_{L}} + 2q(I_{d}+I_{b})f + k_{f}\frac{I^{c}\Delta f}{f^{a}}} + I_{m}}{\sqrt{\frac{4kTf}{R_{L}} + 2q(I_{d}+I_{b})f + k_{f}\frac{I^{c}\Delta f}{f^{a}}}}$$
(20)

In (20), under a certain input intensity and a certain

angle, I_n and I_{min} are determined values, SNR is proportional to the refractive index *m*, particle size *d*, humidity coefficient ζ , and it is inversely proportional to the wavelength λ . If the scattering angle is less than 90⁰, the system SNR decreases with the increment of the scattering angle, the SNR increases along with the scattering angle increases, when the scattering angle is greater than 90⁰. The sensitivity and signal to noise ratio become the maximum when the scattering angle is 0[°] and 180[°], so in the process of testing, scattering angle can be chosen to be 0[°] and 180[°] to improve accuracy.

5. Calculation and analysis

5.1. Sensitivity calculation and analysis of dust concentration sensor detection

When the average particle size is $10\mu m$, the incident light wavelength is 650nm, detection circuit magnification is 35 times, the density is $\rho = 7.7g/m^3$, the Fig. 2 gives the relationship of detection sensitivity, relative humidity and the refractive index on dust concentration sensors.



Fig. 2. The calculation results on dust concentration sensor detection sensitivity and relative humidity and the refractive index

In Fig. 2, the system sensitivity increases along with the increment of refractive index and relative humidity. Due to the increase of relative humidity, the particle diameter will increase. According to the scale parameter of $\alpha = \pi d/\lambda$, the increment of particle diameter and reduce of wavelength will make the particle size parameter increases. The effect situation of relative humidity can reflect the effect situation of the particle diameter and wavelength on the sensitivity. So, the sensitivity can be improved by increasing the input light intensity and reducing the input wavelength of light, and the larger the measuring particle size of the dust particles is, the greater the sensitivity becomes, the higher the precision of the system becomes. In addition, particles become larger in humidity condition, the sensitivity becomes higher for precise detection.

5.2. SNR calculation and analysis of dust concentration sensor detection

Based on Fig. 2, the minimum acceptable luminous flux of detector is 30lm, according to formula (20), Fig. 3 gives the relationship of detection SNR, relative humidity and the refractive index on dust concentration sensors.



Fig. 3. The calculation results on SNR and relative humidity and the refractive index

In Fig. 3, with the increment of the refractive index and relative humidity, the SNR gradually increases, and this situation is the same as sensitivity, the sensitivity is proportional to the SNR, the higher the system sensitivity is, the higher the SNR becomes. From Fig. 2 and Fig. 3, when the relative humidity, particle size parameter and the refractive index increase, the sensitivity and the SNR also increase. Therefore, the SNR can be improved by increasing the input light intensity and reducing the input wavelength of light, and the larger the measuring particle size of the dust particles is, the greater the SNR becomes, the higher the precision of the system becomes. In addition, particles become larger in humidity condition, the SNR becomes higher for precise detection.

5.3. Calculation analysis on the contribution of influence factors to particle scattering field

5.3.1. Influence of incident light polarization direction on particle scattering distribution function

Under the condition of the linear polarization incident light wavelength is 532 nm, the spherical particles radius is 50 nm, 200 nm and refractive index is 1.59, whose scattering function with scattering angle changes, in which incident light along the x axis linear polarization. Therefore, the polarization angle ϕ is the angle between incident light polarization direction and the projection of the scattering vector on the incident light vibration plane, it can be described by Fig. 4.



Fig. 4. The particle scattering light vector distribution

In Fig. 4, the polarized light intensity of plane line is I_0 , the light spreads forward along z axis, the vibration direction of electric field and magnetic field respectively parallel to the x axis and y axis. Scattering angle θ is the angle between scattering light vector and the positive direction of z axis, azimuth angle ϕ is the angle between the x axis and the projection of scattering vector in xoy plane.

In Fig. 4, it can be seen that the polarization angle φ is the azimuth angle ϕ . Changes of the incident light polarization direction can be represented by changes of the polarization angle. Fig. 5 is the scattering function distribution changes with the incident light polarization direction under the condition that the particle radius is 50 nm.

In Fig. 5, under the condition of different incident light polarization directions, when the particle radius is 50nm, the differences of volume scattering function distribution curve are large. When the polarization angle is 0°, volume scattering function curve in the side-scattering areas exists a "depression" obviously, and side-scattering has the minimum value with a small intensity. The minimum intensity increases gradually along with the increment of polarization angle, the "depression" phenomenon gradually disappears, and side-scattering becomes stronger. Therefore, when the particle radius is 50nm, the influence of incident light polarization direction on the volume scattering function distribution mainly reflects on the side-scattering intensity, and side-scattering intensity varies with different polarization directions. As the particle size increases, the particle volume scattering distribution function changes. When the particle radius is 200 nm, volume scattering function distribution curve cannot clearly show the effect of different incident light polarization direction in normal coordinate. Therefore, we use semi-logarithmic coordinate to show this kind of particles, the result is shown in Fig. 6.



Fig. 5. Scattering function distribution changes with the incident light polarization direction under the condition that the particle radius is 50 nm



Fig. 6. Scattering function distribution changes with the incident light polarization direction under the condition that the particle radius is 200 nm

In Fig. 6, it can be seen that the scattering angle increases with the reduction of volume scattering function when the particle radius is 200 nm, function intensity mainly focuses on forward scattering, the influence of changing incident light polarization direction on volume scattering function mainly reflects in the side-scattering region and back-scattering region (scattering angle in the range of 80° - 160°).

5.3.2. The effect of incident light wavelength on the particle volume scattering function

When the incident light is linear polarization, the polarization angle is $\varphi = 0^{\circ}$, the spherical particle radius is 100 nm and refractive index is 1.59, volume scattering function changes because of scattering angle variation, the distribution curve is shown in Fig. 7, we use semi-logarithmic coordinate to show it, different curves correspond to different incident light wavelengths, and these incident light are all in the visible band.



Fig. 7. Particle volume scattering function changes with the incident light wavelength under the condition that the incident light wavelength reduces from 600 nm to 800 nm

In Fig. 7, when incident light wavelength becomes larger, distribution curve of particle volume scattering function exists the minimum value in the side-scattering region as the spherical particle radius is 100 nm. With the increment of scattering angle, particle volume scattering function firstly gradually reduces to the minimum, then it will increases, corresponding, these particles have high intensity of forward scattering and backward scattering, and the side-scattering is weak. With the reduction of the incident light wavelength, the minimum that appears in the scattering region will gradually increases, and those corresponding scattering angle increases, meanwhile the particle backward scattering intensity decreases.

5.3.3. The influence of particle size on light scattering

As the incident light is linear polarization and wavelength is 532 nm, when the polarization angle is $\varphi = 0^{\circ}$, spherical particle refractive index is 1.59, and volume scattering function changes along with scattering angle variation, the distribution curve is shown in Fig. 8, we use semi-logarithmic coordinate to show it, different curves represent different particle radius.



Fig. 8. Particle volume scattering function changes with the particle size, under the condition that the range of radius from 800 nm to 600 nm

In Fig. 8, it can be seen when the particle size becomes larger, particle scattering will mostly concentrate in the forward scattering region, side-scattering and back-scattering are weak. In other words, the incident light wavelength and particle radius are the individual variation parameters, but the actual influence on scattering distribution is the particle size parameter $\alpha = \pi d/\lambda$. The increment of particle size or the reduction of separate incident light wavelength will lead to the increment of particle size parameter.

5.3.4. The influence of particle refractive index on particle scattering function distribution

Suppose, the incident light is linear polarization and wavelength is 532 nm, when the polarization angle is $\varphi = 0^{\circ}$, spherical particle radius is 200 nm, and volume scattering function with scattering angle changes, the distribution curve is shown in Fig. 9, we use semi-logarithmic coordinate to show it, different curves represent different particle refractive indexes.



Fig. 9. Particle volume scattering function changes with the particle refractive index, under the condition that the range of refractive index from 1.25 to 1.50

In Fig. 9, it can be seen that when the incident light wavelength is 532 nm and particle radius is 200 nm, particle scattering mostly concentrates on the forward scattering region. As the particle refractive index gradually increases from 1.25 to 1.5, intensity changes will reduce to the minimum around 100^{0} , then volume scattering function intensity becomes higher as the angle increases, and the volume scattering function intensity becomes smaller along with the increment of angle around 160^{0} .

6. Conclusions

According to dust detection principle, this paper proposes volume scattering function to characterize particle scattering field using angle scattering, dissects the effect of humidity on particle scattering characteristics, sets up the sensitivity model and SNR model of dust concentration testing system, as well as two models in the condition of humidity, analyzes main influence factors of measurement such as light source and the particle scattering characteristics, as well as the influence of all factors on sensitivity model and SNR model. Through calculation and analysis, obtain the contribution degree of each factor to sensitivity and SNR. The sensitivity increases along with the increment of particle diameter, refractive index m, humidity coefficient, the sensitivity decreases along with the increment of wavelength, SNR is proportional to particle diameter, refractive index, humidity coefficient; SNR is inversely proportional to wavelength. The influence of particle volume scattering function distribution on the scattering angle changes along with incident light polarization direction, incident light wavelength, particle size and particle refractive index. In accordance with the specific experimental object and experimental environment; together with the proposed sensitivity model and SNR model, we can determine the optimal scattering angle, which provides the theoretical basis for accurate measurement.

Acknowledgement

This work has been supported by Project of the National Natural Science Foundation of China (No. 61575155) and Key Industrial Science and Technology Project of Shaanxi Province (No. 2015GY018), and the Program for Innovative Science and Research Team of Xi'an Technological University.

References

- Zhichao Wang, Zhansong Wu, Yingxia Yang, J. Tsinghua Univ. 53(3), 366 (2013).
- [2] M. H. Mahdieh, M. Akbari Jafarabadi, Optics & Laser Technology 44(1), 78 (2012).
- [3] Xiu Guang Li, Houg Dong, S. Radandt, Journal of Northeastern University 25(1), 44 (2004).
- [4] Li Hanshan, Applied Optics 54(7), 1612 (2015).
- [5] Cheng Minxi, He Zhenjiang, Method Optics and Optoelectronic Technology 1(5), 30 (2003).
- [6] Hao Zengzhou, Gong Fang, Pan Delu, Huang Haiqing, Acta Optica Sinica **32**(1), 21(2012).
- [7] Huang Xufeng, Bu Yang, Wang Xiangzhao, Chinese Journal of Lasers 37(12), 3001 (2010).
- [8] J. H. Wu, Y. K. Peng, J. J. Chen, Spectroscopy and Spectral Analysis 30(7), 1815 (2010).
- [9] Mingjun Wang, Xizheng Ke, Xuexia Hua, High Power Laser and Particle Beams 28(11), 10021 (2016).
- [10] E. Ferdinandov, B. Pachedjieva, B. Bonev, Opt. Commun. 27(2), 121 (2007).
- [11] J. C. Ricklin, S. M. Hammel, F. D. Eataon, J. Opt. Fiber Commun. 3(2), 111 (2006).

^{*}Corresponding author: lihanshan269@163.com