

Investigation of optical properties of one-dimensional photonic crystals by coupled mode theory

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Due to existence of photonic bandgap in photonic crystals, the propagation of light wave is restricted, which has opened the enormous area to design and fabricate the efficient photonic devices for advanced communication systems. This article reports the simulated investigation of optical parameters of one-dimensional photonic crystals (1DPCs) by using pertinent concept of coupled mode theory (CMT). The result reveals that the forward and backward electric field amplitude exponentially decrease inside the grating along x-direction, which is due to the imaginary term involved in wavevector. The forward and backward field intensity shows the oscillatory behavior in a photonic device. The oscillation occurs due to variation in group velocity of photons in reflectors. The group velocity dependence on frequency has been unveiled, it has been observed that group velocity vanishes at the forbidden band edges i.e. 0.389 and 0.681, while it becomes superluminal from 0.521 to 0.532. The group velocity becomes superluminal inside the photonic bandgap due to the evanescent photons.

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1. Introduction

Presently, the propagation of electromagnetic waves in photonic crystals has become the subject of extensive research due to its remarkable properties [1, 2]. These structures are based on periodic variations of dielectric constants, with a length scale comparable to wavelength in the medium. According to quantum theory, the energy spectrum of an electron in a solid consists of bands separated by gaps [3, 4]. The energy bands in semiconductor controls the motion of charge carriers through the crystal. However, in case of photonic crystals, the periodic layers of different refractive indices control the momentum of photons. The one-dimensional (1D) photonic crystal is a Bragg reflector that has been studied for a long time and most widely used in optical devices. Generally, the reflectors are classified as metallic and dielectric reflector. The metallic reflector has the capabilities to reflect the light over a wide range of frequencies but at higher frequencies it causes power loss due to absorption. However, a photonic crystal has minimum loss, high mechanical robustness and high reflectivity in a certain range of frequencies because of the transparent material are used to fabricate it. Due to the unique properties of photonic crystals, it has numerous potential applications [5-8].

Porous silicon presents very interesting optical passive properties. Its different orientations and sizes of the pores offer versatile optical properties for applications in many fields, from optical devices to chemical and biological sensors. Thus, porous silicon becomes an ideal candidate for the fabrication of photonic crystals. One-dimensional photonic crystals can be inexpensively fabricated using porous silicon multilayer structure [9-13]. The precise

values of the refractive indices for constituent layers depend on its porosity and the porosity can be engineered by considering the current density and etching time during the anodization process.

In present work, we employed coupled mode theory for the analysis of the optical properties of one-dimensional photonic crystal [14]. A weak perturbation can be effectively and accurately handled by using coupled mode theory. In case of coupled mode theory, the grating serve as a perturbation, which couples the forward and backward waves at an optical axis. In this method, we consider that the amplitudes do not change abruptly and the second derivative of amplitudes with respect to optical axis is dropped resulting in the amplitudes being represented by a first order linear differential equation with constant coefficients.

Here, we present the simulation work on the investigation of optical properties of One-Dimensional Photonic Crystals by adopting the Coupled Mode Theory (CMT). In section second, the mathematical approach is presented for computation of reflectance, electric field distribution, Bloch wavevector and group velocity. The obtained results and discussions are presented in section third; section fourth, concludes the paper.

2. Mathematical approach

Considering the coupling between forward and backward propagating electromagnetic waves to obtain, the coupled mode equations along x- direction and are given as

$$\frac{dA}{dx} = kB \exp(j2\Delta\beta x) \quad (1)$$

$$\frac{dB}{dx} = kA \exp(-j2\Delta\beta x) \quad (2)$$

where, k and βx are the coupling coefficient and Bragg resonance respectively. We are interested to investigate the electric field distribution and reflection of light through the grating. Further we have considered that the light is incident from left hand side of grating at $x=0$ so the boundary condition is applied and given as

$$B(L) = 0$$

After applying above condition and doing some mathematical manipulations one can write the expressions for forward and backward field amplitudes and are represented as

$$A(x) = \frac{A_0(\exp(j\Delta\beta x))}{j\Delta\beta \sinh SL + S \cosh SL} ((j\Delta\beta \sinh(S(L-x)) + S \cosh(S(L-x))) \quad (3)$$

$$B(x) = \frac{A_0(-ik \exp(-j\Delta\beta x))}{j\Delta\beta \sinh SL + S \cosh SL} \sinh(S(L-x)) \quad (4)$$

with
$$S = \sqrt{k - \Delta\beta^2}$$

The parameters of above equation are defined as

$$k = \frac{\pi\Delta n}{\lambda} \quad (5)$$

and

$$\Delta\beta = 2\pi m_{eff} \left(\frac{1}{\lambda} - \frac{1}{\lambda_B} \right) \quad (6)$$

where, k , λ and λ_B are the coupling coefficient, input wavelength and Bragg wavelength respectively.

As different spectral components are associated with the propagation of light at different speeds, so the dispersion of such multilayer structure plays a significant role in the propagation of waves. Fig. 1 shows a structure of 1D photonic crystal composed of two distinct refractive index layer n_1 and n_2 with lattice constant Λ . The photonic band dispersion relation derived by using coupled mode theory can be represented as

$$K = \frac{m\pi}{\Lambda} \pm i \sqrt{k^2 - \left(\frac{N_{Eff}}{c} \right) (\omega - \omega_0)^2 \cos^2 \theta} \quad (7)$$

This equation shows the relationship between the local frequency and wavevector for 1D photonic crystal. Here, Λ , N_{Eff} and θ are the lattice constant, effective refractive index and incidence angle respectively.

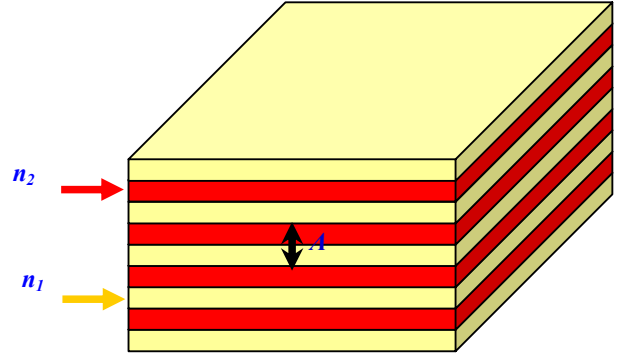


Fig. 1. One-dimensional photonic crystal composed of alternate layers of refractive indices n_1 and n_2 respectively with lattice constant $\Lambda = d_1 + d_2$.

Further, the group velocity can be expressed by taking the differentiation of local frequency with respect to the wavevector K (equation 7) and given as

$$Vg = \pm \left(\frac{c}{N_{Eff}} \right) \frac{K}{\sqrt{k^2 + K^2}} \quad (8)$$

Similarly, we have derived an equation of the photonic dispersion of 1D photonic crystal by transfer matrix method in order to compare with coupled mode theory, as it is more proficient for high refractive index contrast structure.

The dispersion relation of 1DPC by transfer matrix method (TMM) is represented as

$$K\Lambda = \cos^{-1} \left[\frac{1}{2} (A + D) \right] \quad (9)$$

where, A and D are the coefficients and can be given as

$$A = e^{-ik_1 d_1} \left[\cos(k_2 d_2) - \frac{1}{2} i \left(\frac{k_2}{k_1} + \frac{k_1}{k_2} \right) \sin(k_2 d_2) \right] \quad (10)$$

$$D = e^{ik_1 d_1} \left[\cos(k_2 d_2) + \frac{1}{2} i \left(\frac{k_2}{k_1} + \frac{k_1}{k_2} \right) \sin(k_2 d_2) \right] \quad (11)$$

where n_1 and n_2 are the refractive indices, d_1 and d_2 are the thicknesses of the corresponding layers respectively; c and ω are the speed and frequency of the light and $\Lambda = d_1 + d_2$ is lattice constant.

The perpendicular wavevector components in each layer is given by

$$k_{ix} = \left[\left(n_i \frac{\omega}{c} \right)^2 - \beta^2 \right]^{1/2} \quad (12)$$

here, β is the parallel wavevector component and ω/c is the wave number of electromagnetic wave in the vacuum where ω and c are the angular frequency and speed of light in vacuum.

3. Results and discussion

The simulated reflectivity spectras are depicted in Fig. 2(a) for 1.55 μm wavelength with different grating length. It is revealed that we have maximum reflectivity for an input wavelength equal to Bragg wavelength. This maximum reflectivity at the Bragg wavelength indicates the region of photonic bandgap. Therefore, light is completely reflected in this region however, the reflectivity reduces to zero outside of this region. The sidelobes can be interpreted as a result of weak reflection. The maximum reflectivity can be tuned by taking into account the grating length as depicted in Fig. 2 and the optimal reflection spectra is obtained at grating length 60 μm . Ultimately, the photonic bandgap is a function of coupling coefficient since as coupling coefficient decreases the bandgap becomes narrow. In this way by considering the coupling coefficient, the spectral properties can be designed and accordingly photonic bandgap can be engineered. Nevertheless, it is important to note that the coupled mode theory is suitable for weak refractive index perturbation but for high refractive index contrast, this method is not appropriate. The image Fig. 2(b) demonstrates the intensities of light by darker bands and brighter strips for different grating length. The darker area points out the forbidden band, which restricts the entire incident light, consequently standing waves emerges. However, just outside the band the enhanced intensity can be observed which is due to accumulation of incident and reflected lights.

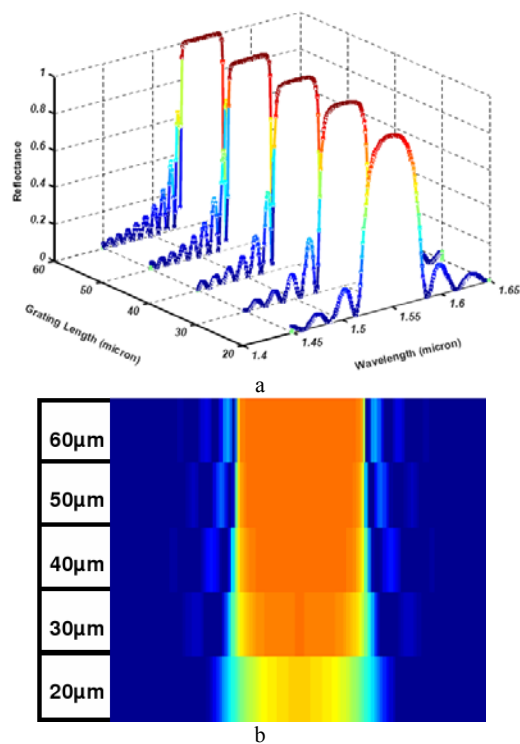


Fig. 2. Reflection spectras as a function of wavelength for different grating length (Fig. 2(a)) and image figure of light intensities 2(b).

Fig. 3 shows the forward and backward field intensities inside the structure as a function of grating length. From this figure, it reveals that both forward and backward electric field amplitudes are exponentially decreasing inside the grating along x-direction, which is due to the imaginary term involved in wavevector.

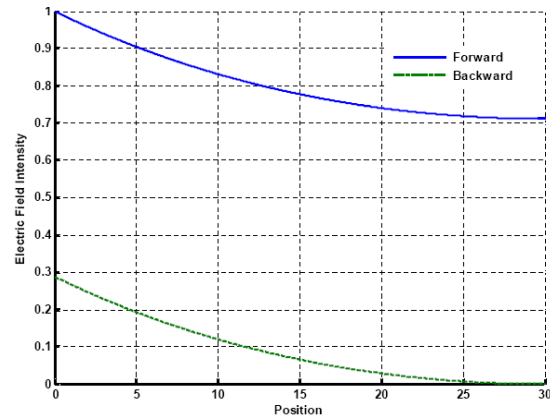


Fig. 3. Electric field Intensity for forward and backward waves with respect to the grating length.

Similarly, Fig. 4(a) & 4(b) depicts the 3D images of forward and backward field intensities as function of grating length inside the structure at slightly shifted wavelength. At slightly shifted wavelength, the forward and backward field amplitude exponentially goes to its minimum value in range 0-4.44 μm along x-axis. Hence, in this region the reflection will be exponentially increased but lower than at Bragg wavelength. Further, in the range of 4.44-10.56 μm , the transmission is dominant and this behavior repeated as the oscillation in field amplitudes along the propagating direction (x-axis). Since we are out of resonance, it is not possible to maintain the higher reflectivity for a longer time. Therefore, from these result it is observed that maximum reflectivity is obtained due to the field decay at the Bragg wavelength and outside this wavelength reflectivity tends to zero.

Fig. 5 shows the 1D photonic bandgap dispersion for both real and imaginary values of wavevector with respect to the normalized frequency. As depicted in this result up to some values of frequency of incidence light, the propagation of photons are allowed however, at a particular value of angular frequency the wavevector becomes complex which contains real and imaginary part and hence, forbidden bandgap is emerged in which reflectivity attain the maximum value. Fig. 6 shows the one-dimensional photonic band structure obtained by transfer matrix method theory in order to compare with coupled mode analysis. As obvious, the range of forbidden frequency band has minimum variation as compared to the range obtained by coupled mode theory. The normalized forbidden frequency range obtained by TMM is 0.420-0.667 and 0.389 to 0.681 by CMT.

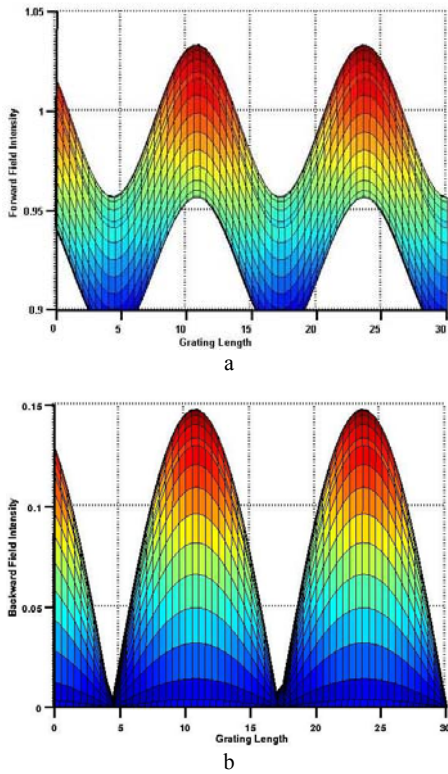


Fig. 4. Forward and backward electric field intensity (Fig. 4(a) and 4(b)) as a function of grating length at slightly shifted wavelength.

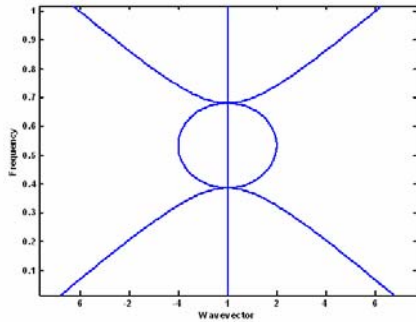


Fig. 5. Photonic band dispersion obtained by coupled mode theory.

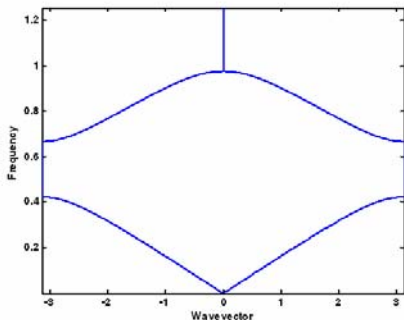
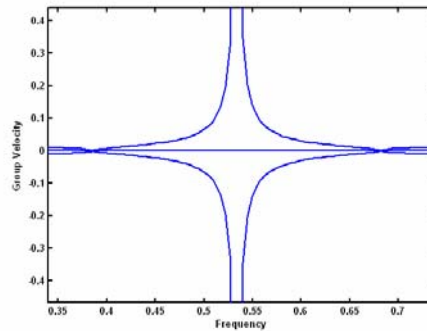
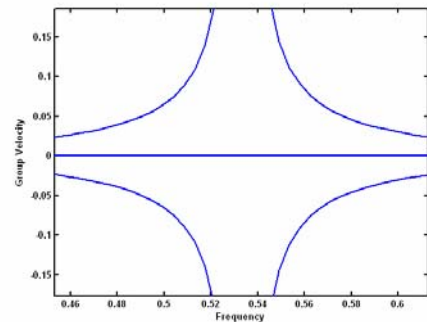


Fig. 6. Photonic band dispersion obtained by transfer matrix theory.

The light propagation is possible outside the forbidden band, which is due to the small velocities and subsequently less than the speed of light in such medium. The reduction in the group velocity can be explained in term of multiple reflections at the interfaces. Fig. 7 shows the variation of both real and imaginary values of group velocity with respect to the frequency and Fig. 7(b) is enlarged image of Fig. 7(a) in order to observe the sharp increment of group velocity inside the bandgap. As shown in the Fig. 7 (a), the group velocity approaches to zero at the bandedges and further it becomes superluminal inside the bandgap as it is expected. From this figure, it is revealed that the group velocity vanishes at the forbidden bandedges (i.e. at normalized frequencies 0.389 and 0.681), where photon are localized. Further, there is a rapid increment of group velocity at the center of the bandgap due to evanescent appearance of the light waves. This delocalization of the photons at superluminal group velocity occurs at normalized frequency range from 0.521 to 0.532 as shown in Fig. 7(b). The group velocity slows down which involves the concept of photons that are localized in the center of the stop band and as a result of it a large bandwidth of forbidden band is required to enfold the distribution of photon momentum. However, inside a photonic bandgap more number of photons become evanescent instead of localization as the imaginary values of its wavevector is increased which results in the group velocity to be increased.



a



b

Fig. 7. Group velocity as a function of normalized frequency (Fig. 7(a)) and while Fig. 7 (b) is the enlarged image of the same.

4. Conclusions

The coupled mode theory is suitable to apply on a Bragg grating periodic structure of porous silicon layers with different dielectric constants. A photon localized in the bandgap and the velocity of photons becomes superluminal as the wave becomes evanescent. Therefore, by increasing the bandwidth of stop band the tunneling of photons can be reduced efficiently. At the bandgap the group velocity oscillates and becomes smaller, which is the indication of light is slowed down while near the centre of bandgap it rapidly increases and goes significantly larger than the speed of light in vacuum and becomes superluminal at the centre. A small value of group velocity indicates that interaction time between the matter systems, radiation mode is high, and it is useful to achieve a threshold-less lasing when an active medium is sandwiched between 1D photonic crystals.

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