# Influence of the continuous variation route of delay feedback time on the dynamics of semiconductor lasers with optoelectronic feedback

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The dynamics of a semiconductor laser with delayed optoelectronic feedback are theoretically investigated when the delay time is continuously varied. The results show that, for a fixed feedback time, the output dynamics state of the semiconductor laser with delayed optoelectronic feedback is dependent on the continuous variation route of delay feedback time; the bistability can be obtained for a given varying range of the delay feedback time.

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## 1. Introduction

The nonlinear dynamics of semiconductor lasers (SLs) [1-20] have been extensively studied due to their wide applications in laser physics, optical communications and other fields. Optoelectronic feedback (OEF), as an external perturbation added to SLs, has gained considerable attention due to its convenience to be electrically controlled and its insensitivity to optical phase variation. In resent years, the nonlinear dynamics of SLs with OEF have been investigated both theoretically and experimentally [6-20]. To our knowledge, most of relevant results, obtained by either numerical simulations or experiments, are based on resetting the initial values when the delay time is changed, and therefore the connection among the output dynamics at different delay time  $\tau$  is erased. In Ref. [11], under the case of considering the history of how the delay time was varied to reach a value  $\tau$ , we have experimentally investigated the influence of the continuous variation route of the delay time on the dynamics of a SL with OEF, and the bistabilities or multistabilities have been observed. In this paper, based on the rate equations of SLs with OEF and after considering the varying route of the delay time, the dynamics of SLs with OEF has been theoretically investigated, the simulated results is similar to the experimental observation and the bistability has been obtained for a given varying range of delay feedback time.

# 2. Theory

The nonlinear behavior of a SL with delayed OEF can be described by the following coupled-rate equations:

$$\frac{dS}{dt} = -\gamma_c S + \Gamma g S \tag{1}$$

$$\frac{dN}{dt} = \frac{J}{ed} \left[ 1 + \xi \frac{S(t-\tau) - S_0}{S_0} \right] - \gamma_s N - gS \quad (2)$$

where *S* is the intracavity photon density, *N* is the carrier density, *J* is the biased current density, *S*<sub>0</sub> is the free-running intracavity photon density, *g* is the optical gain coefficient,  $\tau$  is the feedback delay time,  $\xi$  is the dimensionless feedback parameter,  $\gamma_c$  is the cavity photon decay rate,  $\gamma_s$  is the spontaneous carrier decay rate,  $\Gamma$  is the confinement factor, *e* is the electronic charge constant, and *d* is the active layer thickness.

For the convenience of numerical simulation, the dimensionless variables are defined as  $\tilde{s} \equiv (S - S_0)/S_0$ ,

$$\tilde{n} \equiv (N - N_0)$$
, and  $\tilde{J} \equiv (J/ed - \gamma_s N_0)/\gamma_s N_0$  with respect to

the free-running values  $S_0$  and  $N_0$ . As a result, the numerical calculations are performed on the following normalized dimensionless rate equations:

$$\frac{d\tilde{s}}{dt} = \frac{\gamma_c \gamma_n}{\tilde{J} \gamma_s} \tilde{n}(\tilde{s}+1) - \gamma_p \tilde{s}(\tilde{s}+1)$$
(3)

$$\frac{d\tilde{n}}{dt} = \gamma_s \xi(1+\tilde{J})\tilde{s}(t-\tau) - \gamma_s \tilde{n} - \gamma_s \tilde{J}\tilde{s} - \gamma_n \tilde{n}(1+\tilde{s}) + \frac{\gamma_s \gamma_p}{\gamma_c} \tilde{J}\tilde{s}(\tilde{s}+1)$$
(4)

where the details of other parameters is the same as those in Ref. [8].

The rate equations (3)-(4) can be solved numerically with four-order Runge-Kutta method. During the process of the simulation, it is assumed that the delay time begins with a minimum value, and then is gradually increased up to a maximum value while the laser is operating without being intermitted. Similarly, the delay time is supposed to be decreased gradually to the minimum in the same manner. This method is called the continuous time approach, and the initial conditions are given only in the beginning of the simulation. Unlike the usual method called the brute force approach, the initial values are reset for every time when the delayed feedback time is changed [10]. During the calculation, the parameters are [7]:  $\gamma_c$ =2.4×10<sup>11</sup>s<sup>-1</sup>,  $\gamma_s$  =1.458×10<sup>9</sup>s<sup>-1</sup>,  $\gamma_n$  = 3 $\tilde{j}$  ×10<sup>9</sup>s<sup>-1</sup>,  $\gamma_p$  = 3.6 $\tilde{j}$  ×10<sup>9</sup>s<sup>-1</sup>, and  $\tilde{j}$  =1/3. With above given parameters, the relaxation resonance frequency of the laser in the free-running condition is calculated as  $f_r = 2.47 GHz$ . For convenience, the normalized photon density  $\hat{S} = S/S_0 = \tilde{s} + 1$  is used in following calculations.

## 3. Results and discussions

In Fig.1, we give the bifurcation diagram of the SL with OEF obtained by usual method for the feedback strength  $\xi = -0.060$  and the delay time  $\tau$  increased from 8 ns to 20ns. Because the initial feedback within the time range of  $[0-\tau]$  is always returned to zero for every feedback delay time, a determined output state of SL with OEF should be obtained. For different feedback delay time, diverse dynamic states such as regular pulsing (RP), quasiperiodic pulsing (QP), and chaos can be obtained.



Fig.1. Bifurcation diagram of the extrema of the peak series obtained by usual method with the feedback strengths  $\xi = -0.060$ , where the delay time  $\tau$  varies from 8ns to 20ns.



Fig.2. Bifurcation diagram of the extrema of the peak series obtained by continuous time approach with the feedback strengths  $\xi$ = -0.06, where (a)  $\tau$  increased from 17.45 ns to 17.90 ns, (b)  $\tau$  decreased from 17.90 ns to 17.45 ns.

Fig. 2 shows the bifurcation diagram of the SL with OEF obtained by continuously varying the delay time for  $\xi = -0.06$  and the variation range of delay time is between 17.45 ns and 17.90 ns, where Fig. 2(a) and Fig. 2(b) are obtained by the feedback delay time is increased and decreased, respectively. From Fig. 2(a), it can be observed that when  $\tau$  increases from 17.45 ns to 17.67 ns, there is only one kind of dynamic state (the RP state). However, when the delay time is decreased (see Fig.2 (b)), the region that  $\tau$  is within 17.67 ns~17.49 ns shifts to the QP state, and the range of which is 0.18 ns approximately. Therefore, within the region of the  $\tau$  varied between 17.49 ns and 17.67 ns, the bistability appears.





Fig. 3. Power spectra, phase portraits, and time series obtained by continuous time method for a fixed τ =17.49 ns but different varying route. Left column: τ reaches 17.49 ns by increasing from 17.45 ns; Right column: τ reaches 17.49 ns by decreasing from 17.90 ns.

Fig. 3 gives two different dynamical states for the delay time is expired by different varying route to reach a fixed feedback delay time 17.49 ns. The output dynamical states are characterized by the time series, the power spectra, and the phase portraits. For the delay time reaches 17.49 ns by increasing from 17.45 ns, the output is the RP, the power spectrum has only one fundamental pulsing frequency at  $f_1 = 2.42 GH_z$ , which is close to relaxation resonance frequency  $f_r$  of the laser. However, the output of SL is QP if the delay time reaches 17.49 ns by decreasing from 17.90 ns, and several frequency spikes appear near the resonance frequency of the laser. Therefore, the output exits two different states for a fixed feedback delay time, i. e., bistability appears. The reason that induces the bistability in a SL with OEF may be as follows: Assuming the varying range of  $\tau$  is  $[\tau_b, \tau_e]$  and the calculation step of  $\tau$  is  $\Delta \tau$ , the dynamics of SL with feedback time  $\tau_b$  can be obtained under setting initial feedback  $S(t-\tau_b)=0$  for  $0 \le t \le \tau_b$ . For calculation the dynamics of SL with  $\tau_m$  ( $\in$ ( $\tau_b$ ,  $\tau_e$ ]), the initial feedback for  $0 \le t \le \tau_m$  is not zero but determined by the tail output time series of SL with  $\tau_b \pm \Delta \tau$  ('+' and '-' for  $\tau$  reaches to  $\tau_m$ after decreasing or increasing, respectively). Under this circumstance, the history of the variation of the delay time will affect the output dynamics of SL with OEF. Through continuously varying the delay time along different routes, the bistability or multistability may be observed depending on the varying range of delay time.

### 4. Conclusion

In conclusion, we theoretically investigated the influence of the continuous variation route of delay feedback time on the dynamics of SLs with OEF. At a fixed delay time, for different varying route to reach this delay time, the different output dynamics of SL with OEF may be observed. The simulated results can support partially the experiment results [11]. We believe that this work will be helpful for further understanding the nonlinear dynamics of the SLs with OEF.

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