Hidden hyperchaotic attractor in a novel simple memristive neural network

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Potential applications of memristors in low-power processors, ultra-dense memories, programmable analog integrated circuits, and especially neural networks, have been reported recently. This paper introduces a novel simple neural network having a memristive synaptic weight. Fundamental behavior of the proposed neural network is investigated through numerical simulations and circuital implementation. It is very interesting that this memristive neural network can exhibit hyperchaos although it possesses no equilibrium points.

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1. Introduction

Neural networks have been applied in diverse applications including optimization, system control, signal processing, associative memory, or pattern recognition [1-4]. As is well known, chaos can help neural network to escape the local minimum or support memory storage and retrieval [5,6]. As a result, chaotic dynamics in neural networks have been studied in various researches [7-10].

In neurocomputing, Hopfield type neural network plays an important role [11-13]. Although it is relatively simple, it can describe brain dynamics and provide a model for understanding human memory [11, 14-16]. In practice the realization of synaptic weights in Hopfield neural network is a difficult issue. However, this difficulty can be solved by using the memristor, the fourth circuit element in addition to resistor, capacitor and inductor [17-19]. In this perspective, memristor is a prime candidate to replicate the behavior of neuron’s synapse [20-24]. The peculiar features of the memristor promise to create complex dynamics in such neural networks. For example, memristive circuit based on cellular nonlinear networks can display chaos [25] or a small memristive neural network with a line of equilibrium can demonstrate hyperchaos [26]. It is worth noting that the last example belongs to a new category of systems [27]. According to a new classification of chaotic dynamics [28-30], there are two types of attractor: self-excited attractor and hidden attractor. A self-excited attractor has a basin of attraction that is excited from unstable equilibria. In contrast, hidden attractor cannot be found by using a numerical method in which a trajectory started from a point on the unstable manifold in the neighborhood of an unstable equilibrium [31]. The subject of discovering systems with hidden attractors has received considerable attention in the research community because of both academic significance and practical importance [32-37].

Motivated by intrinsic nonlinear characteristics of memristor, the simplicity and practical application of Hopfield type neural network, a simple Hopfield memristive neural network is proposed and studied in this paper. The paper is organized as follows. In the next section, the model of the new memristive neural network is introduced. Its basic dynamics are discovered in section 3 through numerical simulations such as phase portraits, Lyapunov exponents, bifurcation diagram, Poincaré map and limit cycles. Section 4 presents the circuital implementation of the introduced memristive neural network. Finally, the conclusive remarks are drawn in the last section.

2. Model of the new simple memristive neural network

A Hopfield neural network including n neurons [11, 26] can be described by circuital equations of each neuron

$$C_i \dot{x}_i = -\frac{x_i}{R_i} + \sum_{j=1}^{n} w_{ij} v_j + I_i, \qquad (1)$$

where the state x_i of the i-th neuron is the voltage on capacitor C_i, R_i is the membrane resistance between the inside and outside of the neuron, while I_i is the input bias current. The matrix $\mathbf{W}=(w_{ij})$ is synaptic weight matrix which indicates the strength of connection between neurons. Hence the input of each neuron comes from external inputs and inputs from other neurons. It is noting that the voltage input from the j-th neuron v_j [11, 26] is given by

$$v_j = \tanh\left(x_j\right). \tag{2}$$

It is easy to see that, the synaptic weight w_{ij} describes the admittance of the resistor between the j-th neuron and the i-th neuron. For simplicity, a simple Hopfield neural network with only three neurons is considered in this work (see Fig. 1). Different from conventional Hopfield neural networks, a flux-controlled memristor [17, 38] is used as a synaptic weight in this new neural network. Here the flux-controlled memristor is described by

$$\begin{cases} i_M = W\left(\varphi\right) v_M \\ \dot{\varphi} = v_M, \end{cases} \tag{3}$$

where v_M is the voltage across the memristor, i_M is the current through the memristor. $W\left(\varphi\right)$ is the memductance which is defined as

$$W\left(\varphi\right) = \frac{dq\left(\varphi\right)}{d\varphi} = a\varphi^2, \tag{4}$$

where q and φ are the charge and magnetic flux while a is a parameter.

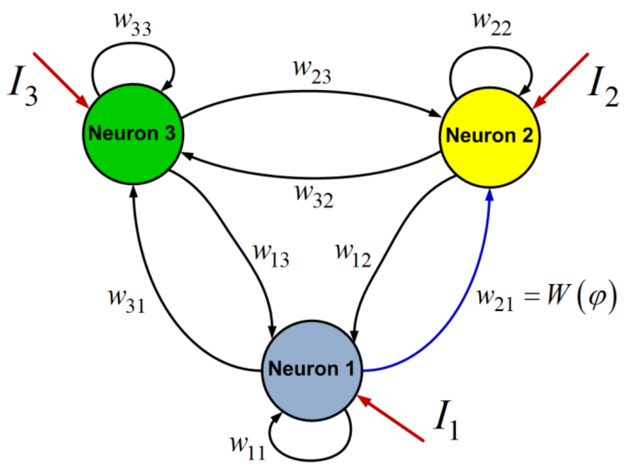


Fig. 1. A simple neural network with the presence of a memristive synaptic weight.

Based on circuital equations of each neuron (1) and the used memristor (3), let C_i = 1, R_i = 1, the novel memristive neural network in Fig. 1 is characterized by the following equations

$$\begin{cases} \dot{x}_i = -x_i + \sum_{j=1}^{3} w_{ij} v_j + I_i \\ \dot{\varphi} = \tanh\left(x_1\right), \end{cases} \tag{5}$$

with i = 1, 2, 3 and the synaptic weight matrix is given by

$$\mathbf{W} = \left(w_{ij}\right) = \begin{bmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ w_{31} & w_{32} & w_{33} \end{bmatrix} = \begin{bmatrix} 1.6 & 2 & 1 \\ a\varphi^2 & 1.5 & 0 \\ 3 & -2 & 1 \end{bmatrix}. \tag{6}$$

In particular, the input bias current term is

$$\mathbf{I} = \left[I_1, I_2, I_3\right]^T = \left[0, 0, b\right]^T \tag{7}$$

where b is the parameter which indicates the input current at the third neuron. The presence of an input bias current makes new neural network (5) different from the 4D memristive neural network in [26] or the memristive circuit based on cellular nonlinear networks [25].

3. Dynamics of the proposed memristive neural network

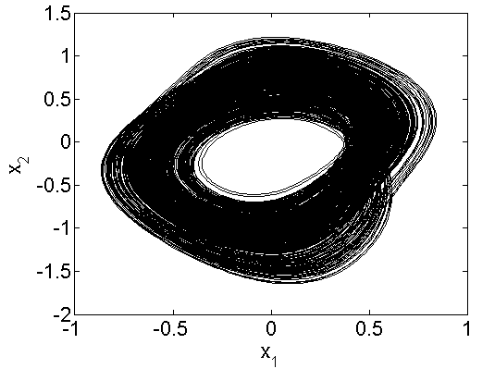
When b = 0, the memristive neural network (5) has the line equilibrium E(0, 0, 0, φ). Moreover, neural network (5) is hyperchaotic for different values of a. For example, when a = –0.05, b = 0, and the initial conditions $\left(x_1(0), x_2(0), x_3(0), \varphi(0)\right) = (0, 0.01, 0.01, 0)$, hyperchaos is obtained due to the fact that neural network (5) has more than one positive Lyapunov exponents $\lambda_1 = 0.029$, $\lambda_2 = 0.0088$, $\lambda_3 = 0$, and $\lambda_4 = -0.1151$. In this case, neural network (5) is similar to the studied system in [26], hence it will not be discussed here.

When $b \neq 0$, it can be noticed that memristive neural network (5) possesses no equilibrium points. Interestingly, when a = –0.05, b = –0.001, and the initial conditions $\left(x_1(0), x_2(0), x_3(0), \varphi(0)\right) = (0, 0.01, 0.01, 0)$, the novel neural network (5) can display hyperchaotic attractor without equilibrium as shown in Fig. 2. In this case, neural network (5) is hyperchaotic because it has two positive Lyapunov exponents $\lambda_1 = 0.0291 > 0$, $\lambda_2 = 0.0098 > 0$, $\left(\lambda_3 = 0, \text{ and } \lambda_4 = -0.1152\right)$. Obviously, this memristive neural network without equilibrium is categorized as a hyperchaotic system with hidden attractor [27-30] because its basin of attraction does not intersect with small neighborhoods of any equilibrium points.

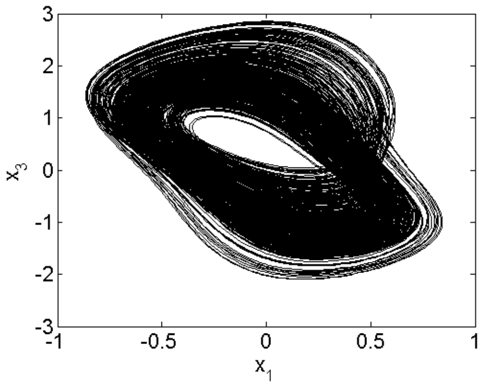
The Kaplan-Yorke fractional dimension [39], which presents the complexity of attractor, is defined by

$$D_{KY} = j + \frac{1}{\left|\lambda_{j+1}\right|} \sum_{i=1}^{j} \lambda_i,$$

where j is the largest integer satisfying $\sum_{i=1}^{j}\lambda_i \geq 0$ and $\sum_{i=1}^{j+1}\lambda_i < 0$. The calculated fractional dimension of memristive neural network (5) when a = –0.05 and b = –0.001 is $D_{KY} = 3.3377 > 3$. Therefore, it indicates a strange attractor. In addition, as it can be seen from the Poincaré map in Fig. 3, memristive neural network (5) has a rich dynamical behavior.



a



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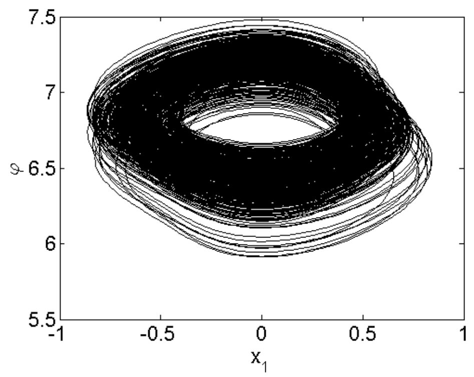

c

Fig. 2. Hyperchaotic attractor in the novel memristive neural network for a = –0.05, and b= –0.001 (a) in the x_1 - x_2 plane, (b) in the x_1 - x_3 plane, and (c) in the x_1 - φ plane.

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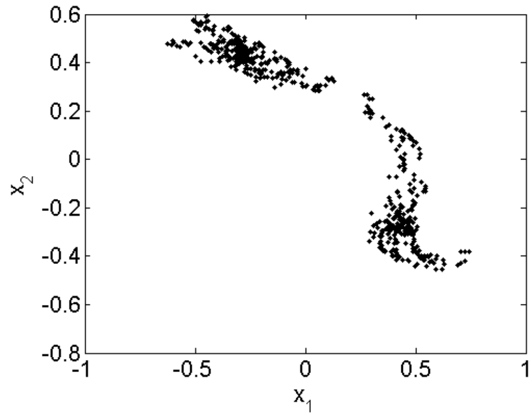

Fig. 3. Poincaré map in the x_1 - x_2 plane, when x_3 = 0.

For a clear view of the nonlinear dynamics of memristive neural network (5), the bifurcation diagram is presented in Fig. 4 by plotting the local maxima of the state variable $x_3(t)$ when changing the value of the parameter b. Furthermore, Lyapunov exponents of neural network (5) have been calculated using the algorithm in [40] and are shown in Fig. 5.

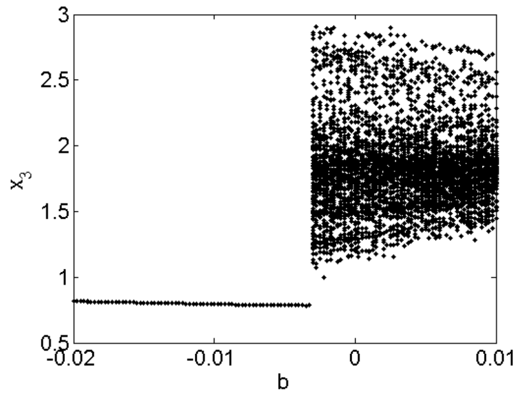

Fig. 4. Bifurcation diagram of $x_{3\max}$ with a = –0.05 and b as varying parameter.

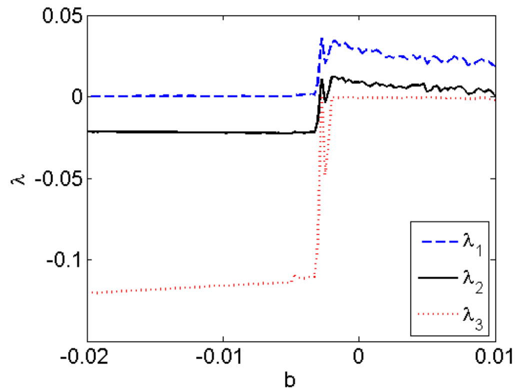

Fig. 5. Three largest Lyapunov exponents λ_1, λ_2, λ_3 (blue dash line, black solid line, and red dot line, respectively) of network (5) versus b for a = –0.05.

Fig. 5 suggests that the system changes from limit cycle to hyperchaos. This kind of transition has been observed in another recent work [41]. As shown in Figs. 4, 5, there are some windows of limit cycle, of chaotic behavior and of hyperchaotic behavior. Fig. 6 illustrates the periodic orbit of memristive neural network (5) for the parameter $b = -0.01$.

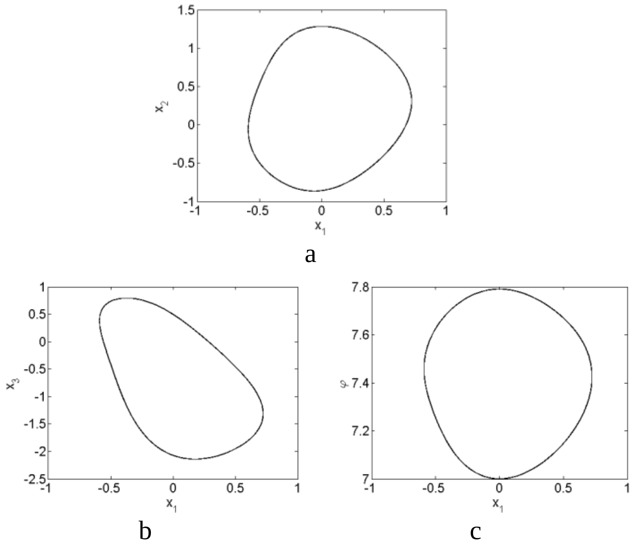

Fig. 6. The periodic orbit of memristive neural network (5) for a = –0.05, and b = –0.01 (a) in the x_1 - x_2 *plane, (b) in the* x_1 - x_3 *plane, and (c) in the* x_1 - φ *plane..*

In order to estimate the influence of different values of the parameter a in the dynamic behavior of neural network (5), its Lyapunov exponents versus the parameter a has been also reported in Fig. 7. Although the positive Lyapunov exponent does not mean chaos every time [42-44], there is no ambiguity on the indication of chaos in our regular work. It is clear that hyperchaos is observed showing the noticeable role of the memristive synaptic weight.

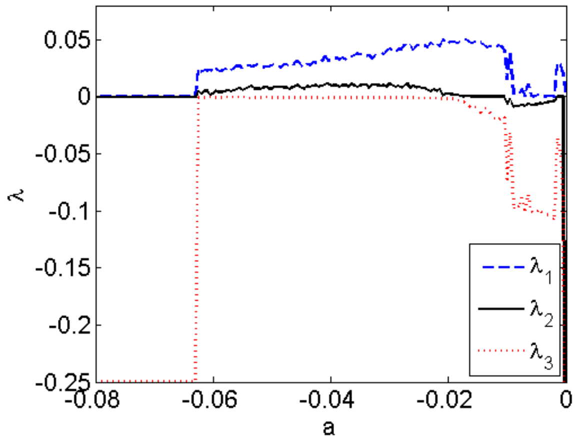

Fig. 7. Three largest Lyapunov exponents λ_1, λ_2, λ_3 *(blue dash line, black solid line, and red dot line, respectively) of neural network (5) versus a for b = –0.001.*

4. Circuit implementation of the memristive neural network

Implementation of nonlinear systems by using electronic circuits provides another effective approach for investigating dynamics of such systems. In fact, this rigorous and inexpensive approach has been used for experimental characterization of the modeled dynamics [45] or emulating complex business cycles [46]. Moreover, circuital realization of a theoretical model plays a vital role in practical chaos-based applications, such as secure communications, random numbers generator, or path planning for autonomous robots [47-49]. Therefore, in this section, a circuital realization of memristive neural network (5) is presented to illustrate the feasibility and correctness of the theoretical model.

The designed circuit is shown in Fig. 8 where the state variables x_1, x_2, x_3, and φ of memristive neural network (5) are the voltages across the capacitors C_1, C_2, C_3, and C_4, respectively.

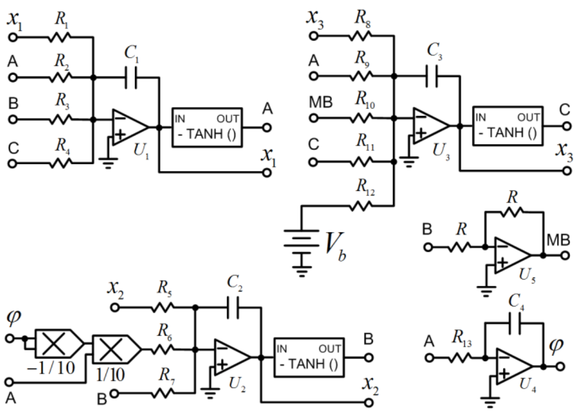

Fig. 8. Circuital schematic of the proposed memristive neural network without equilibrium (5).

By using Kirchhoff's circuit laws, the circuital equations of the designed circuit in Fig. 8 are derived as follows:

$$\begin{cases} \dot{x}_1 = -\dfrac{1}{R_1C_1}x_1 + \dfrac{1}{R_2C_1}\tanh(x_1) + \dfrac{1}{R_3C_1}\tanh(x_2) \\ \qquad + \dfrac{1}{R_4C_1}\tanh(x_3) \\ \dot{x}_2 = -\dfrac{1}{R_5C_2}x_2 - \dfrac{1}{100R_6C_2}\varphi^2\tanh(x_1) \\ \qquad + \dfrac{1}{R_7C_2}\tanh(x_2) \\ \dot{x}_3 = -\dfrac{1}{R_8C_3}x_3 + \dfrac{1}{R_9C_3}\tanh(x_1) - \dfrac{1}{R_{10}C_3}\tanh(x_2) \\ \qquad + \dfrac{1}{R_{11}C_3}\tanh(x_3) - \dfrac{1}{R_{12}C_3}V_b \\ \dot{\varphi} = \dfrac{1}{R_{13}C_4}\tanh(x_1). \end{cases} \tag{8}$$

The power supplies are ±15 volts while the values of circuit components in Fig. 8 are selected as follows: $R_1 = R_4 = R_5 = R_8 = R_{11} = R_{13} = R = 10\text{k}\Omega$, $R_2 = 6.25\text{k}\Omega$, $R_3 = R_{10} = 5\text{k}\Omega$, $R_6 = 2\text{k}\Omega$, $R_7 = 6.667\text{k}\Omega$, $R_9 = 3.333\text{k}\Omega$, $R_{12} = 1\text{M}\Omega$, $C_1 = C_2 = C_3 = C_4 = 10\text{nF}$, and $V_b = 0.1\text{V}_{\text{DC}}$.

Comparing circuital system (8) with the theoretical model (5), indeed the circuit in Fig. 8 emulates the memristive neural network (5). The designed circuit is implemented in the electronic simulation package OrCAD and the obtained results are reported in Figs. 9, 10.

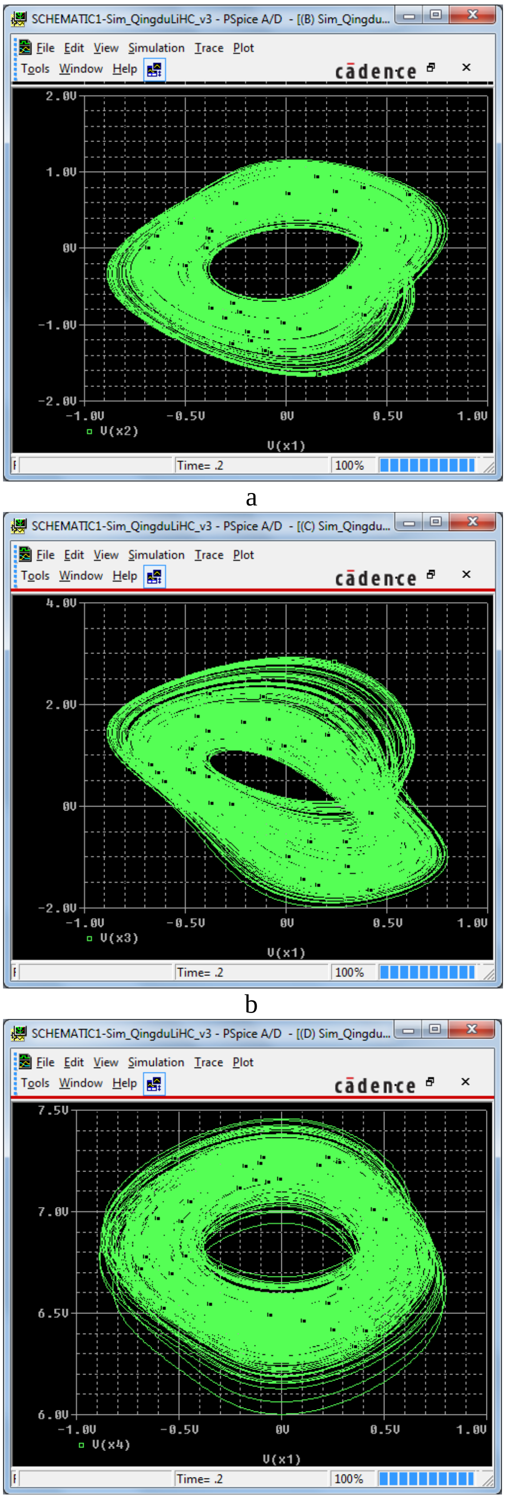

Fig. 9. Hyperchaotic attractor of the designed electronic circuit obtained from OrCAD (a) in the x_1 - x_2 plane, (b) in the x_1 - x_3 plane, and (c) in the x_1 - φ plane.

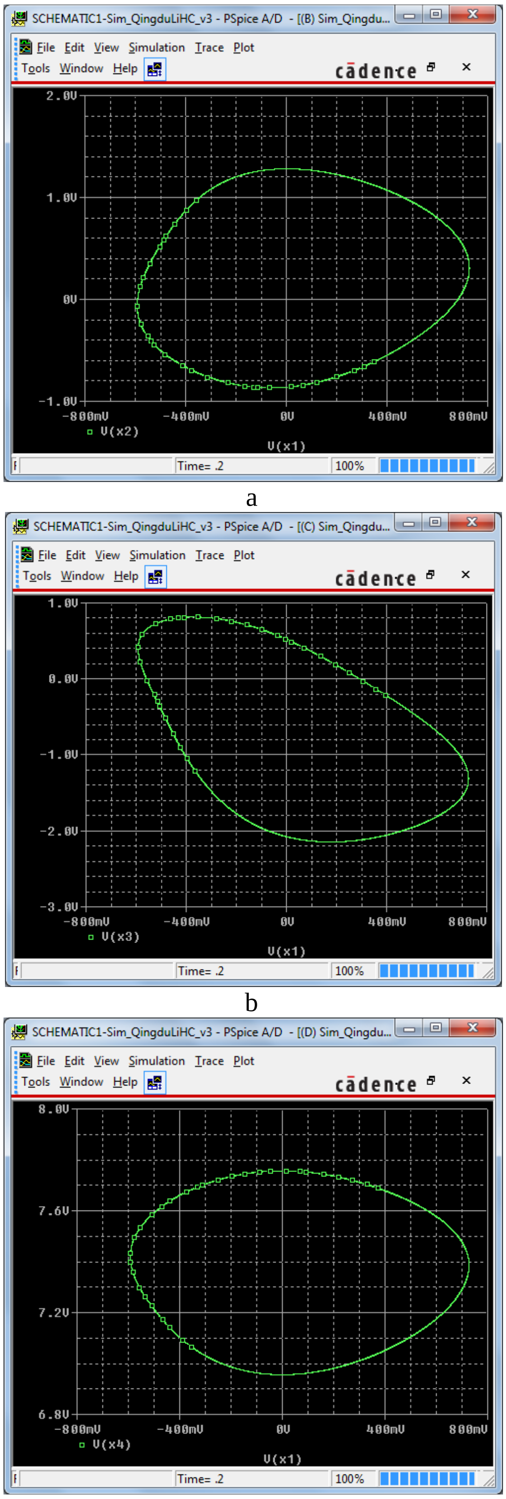

Fig. 10. The periodic orbit of the designed electronic circuit obtained from OrCAD (a) in the x_1 - x_2 plane, (b) in the x_1 - x_3 plane, and (c) in the x_1 - φ plane.

It is easy to see a good agreement between the theoretical attractor (Fig. 2) and the circuital one (Fig. 9). In order to investigate the behavior of the neural network with respect to the input bias term b, the value of resistor R_{12} can be varied using a trimmer. For example, when R_{12} = 100kΩ the behavior of the circuit is a periodic limit cycle (see Fig. 10) corresponding to an implemented value of b = –0.01, which can be compared to the model behavior reported in Fig. 6.

5. Conclusions

A simple neural network with a memristive synaptic weight has been studied in this paper through numerical simulations and circuital implementation. Interestingly, this new memristive neural network is able to show complex behavior, like chaos and hyperchaos in spite of its simple structure. Moreover, such neural network can be considered as a system with hidden attractor because there is no equilibrium. The major difference from the published works [25, 26] is the absence of any equilibrium points in new neural network (5).

When there are more than one memristive synaptic weight are considered, the dimension of the network increases. For example, the neural network with the presence of two memristive synaptic weights becomes a 5D nonlinear system. This could be studied in our next work.

The combination of neural network and memristor leads the proposed network to interesting applications in neural computing as well as chaos-based systems. In addition, hidden attractor exhibited from the memristive neural network will be further explored in the future studies.

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