Generating a novel hyperchaotic system out of equilibrium

VIET-THANH PHAM[a,*], CHRISTOS VOLOS[b], SAJAD JAFARI[c], XIONG WANG[d]

[a] *School of Electronics and Telecommunications, Hanoi University of Science and Technology, 01 Dai Co Viet, Hanoi, Vietnam*

[b] *Physics Department, Aristotle University of Thessaloniki, GR-54124, Greece*

[c] *Biomedical Engineering Department, Amirkabir University of Technology, Tehran 15875-4413, Iran*

[d] *Department of Electronic Engineering, City University of Hong Kong, Hong Kong SAR, China*

In this paper a new no-equilibrium hyperchaotic system is proposed by adding a tiny perturbation to a known hyperchaotic system with a line equilibrium. The fundamental dynamical properties of the new system are discovered including Lyapunov exponents, bifurcation diagram, Poincaré map and limit cycles. In addition, an electronic circuit emulating the new system is also presented to verify the feasibility of theoretical model. Interestingly, by applying the introduced methodology another new memristor-based hyperchaotic system without equilibrium has also been created.

(Received April 11, 2014; accepted May 15, 2014)

Keywords: Hyperchaos, No equilibrium, Hidden attractor, Lyapunov exponent, Bifurcation diagram, Poincaré map, Memristor

1. Introduction

A simple well-known chaotic flow with five terms and two nonlinearities (Sprott case A) was found in 1994 [1]. This chaotic flow is worth noting because it has no equilibrium. It means that the conventional Shilnikov criteria [2, 3] cannot be applied to prove the chaos in the flow. However, studies on chaotic systems without equilibrium have only been received significant attention recently due to the fact that such systems belong to a class of chaotic system with hidden attractor, which has practical and theoretical importance [4-7]. Remark chaotic systems with no equilibria have been reported in [8, 9]. In addition, hyperchaotic systems without equilibrium have been introduced simultaneously. Wang et al. [10] proposed and analyzed a new no-equilibrium hyperchaotic system with nine terms including three quadratic nonlinearities and two cubic nonlinearities. But the authors only described their system by four first-order ordinary differential equations. By introducing an additional fourth state and combining it into the second equation of a generalized diffusionless Lorenz equations, a no-equilibrium hyperchaotic were obtained [11]. The system has eight terms with two quadratic nonlinearities. In a similar way, Li and Sprott presented a simple equilibrium-free autonomous system with the existence of hyperchaos [12]. Li's system with only seven terms consists of two nonlinearities.

Investigations on no-equilibrium hyperchaotic system with hidden attractors is an attractive topic. Such systems have potential applications, especially in the field of secure communication because there is no limitation of equilibrium.

Motivated by very complex dynamical behaviors of hyperchaotic systems and unusual features of hidden attractors, a novel hyperchaotic system without equilibrium is newly introduced in this paper. Its dynamics is discovered through Lyapunov exponents, bifurcation diagram, Poincaré map and limit cycles. Moreover, the circuit realization of system is also presented to illustrate the feasibility of theoretical system. By applying the same methodology for generating the proposed hyperchaotic system, an additional novel hyperchaotic example using memristor is also found.

2. New hyperchaotic system without equilibrium

Recently, Li et al. [13] have been developed a four-dimensional system with eight terms and three parameters (a, b, and c):

$$\begin{aligned} \dot{x} &= y - xz - yz + w \\ \dot{y} &= axz \\ \dot{z} &= y^2 - bz^2 \\ \dot{w} &= -cy. \end{aligned} \tag{1}$$

When a, b, $c \neq 0$, system (1) has the line equilibrium E(x, 0, 0, 0). It is noting that system (1) is hyperchaotic for various combinations of parameters. For example, the maximum hyperchaos is obtained for $a = 5$, $b = 0.28$, and $c = 0.05$ [13]. Furthermore, the infinite line of equilibrium points leads system (1) into a dynamical system with hidden attractors [4, 5, 14].

Obviously, it is easy to imagine that when applying a tiny perturbation to system (1), its hyperchaos may be preserved while the infinite equilibrium points are disappeared. Therefore a simple additional parameter d is added to system (1). As a result, a new system (denoted as the model HNE_1) is given as:

$$\begin{aligned} \dot{x} &= y - xz - yz + w \\ \dot{y} &= axz + d \\ \dot{z} &= y^2 - bz^2 \\ \dot{w} &= -cy, \end{aligned} \tag{2}$$

where $a = 5$, $c = 0.05$ and b, d are the real parameters.

System (2) is a four-dimensional autonomous flow with five quadratic nonlinearities. When $d = 0$, it becomes Li's system (1); when $d \neq 0$, however, the new system (2) possesses no any equilibrium points. It is interesting that, when $b = 0.28$, $d = -0.001$ and the initial conditions $(x_0, y_0, z_0, w_0) = (0, 0, 0.8, 0.02)$, the new system HNE_1 can display a hyperchaotic attractor with no equilibria, as shown in Fig. 1.

It has been known that the Lyapunov exponents measure the exponential rates of the divergence and convergence of nearby trajectories in the phase space of chaotic system. For a four-dimensional hyperchaotic system there are two positive Lyapunov exponents, one zero and one negative Lyapunov exponent [15]. Thus Lyapunov exponents of the system HNE_1 have been calculated using well-known algorithm in [16] to verify its hyperchaos when $b = 0.28$ and $d = -0.001$. In this work, Lyapunov exponents are denoted by λ_{L_i}, $i = 1, 2, 3, 4$ with $\lambda_{L_1} > \lambda_{L_2} > \lambda_{L_3} > \lambda_{L_4}$. Apparently, the system HNE_1 is hyperchaotic because it has more than one positive Lyapunov exponent $\lambda_{L_1} = 0.0756 > 0$, $\lambda_{L_2} = 0.0382 > 0$, $\lambda_{L_3} = 0$, and $\lambda_{L_4} = -1.6600$.

The Kaplan-Yorke fractional dimension, which presents the complexity of attractor, is defined by

$$D_{KY} = j + \frac{1}{\left|\lambda_{L_{j+1}}\right|} \sum_{i=1}^{j} \lambda_{L_i},$$

where j is the largest integer satisfying $\sum_{i=1}^{j} \lambda_{L_i} \geq 0$ and $\sum_{i=1}^{j+1} \lambda_{L_i} < 0.$ The calculated fractional dimension of system HNE_1 when $b = 0.28$ and $d = -0.001$ is $D_{KY} = 3.0686 > 3.$ Therefore, it indicates a strange attractor. In addition, the Poincaré map of the system HNE_1 also reflects properties of chaos (see Fig. 2).

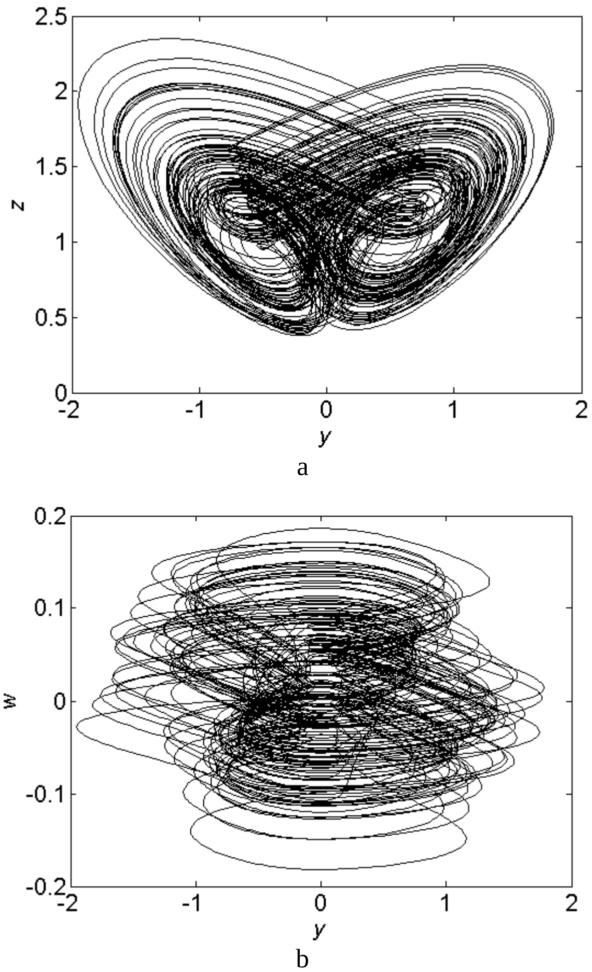


Fig. 1. Hyperchaotic attractor without equilibrium in the novel proposed system HNE_1 for b= 0.28, and d = –0.001 (a) in the y-z plane, (b) in the y-w plane.

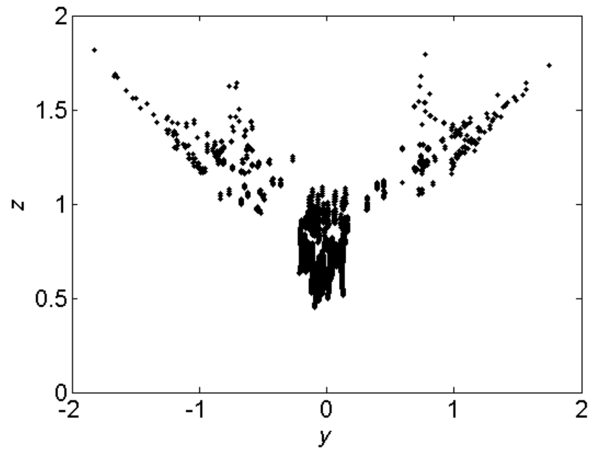


Fig. 2. Poincaré map in the y-z plane when x = 0.

In order to have detailed view of the novel system HNE_1 (2), its behavior with respect to the bifurcation parameter b is discovered. The bifurcation diagram (see Fig. 3) is obtained by plotting the local maxima of the state variable $z(t)$ when changing the value of b. Further, the numerical calculated result of Lyapunov exponents is shown in Fig. 4.

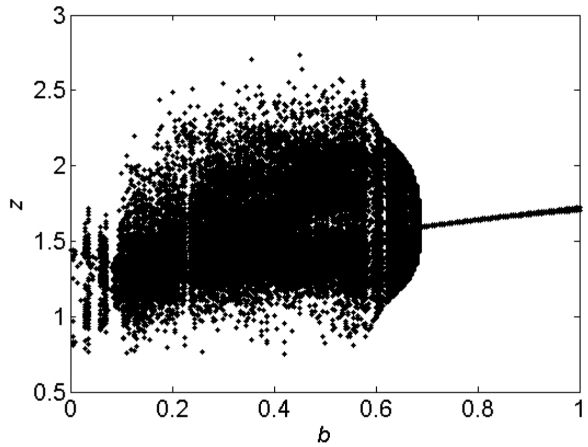


Fig. 3. Bifurcation diagram of z_{max} with d = –0.001 and b as varying parameter.

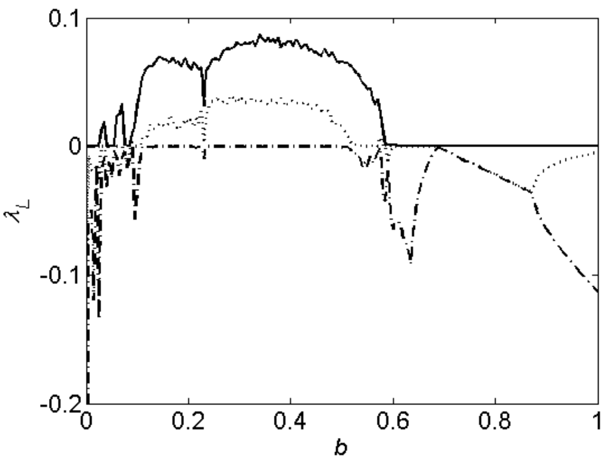


Fig. 4. Three largest Lyapunov exponents λ_{L_1}, λ_{L_2}, λ_{L_3} (solid line, dot line, and dash-dot line, respectively) of system (2) versus b for d = –0.001.

Both the bifurcation diagram and the corresponding Lyapunov spectrum evidently indicate that there are some windows of limit cycles, of chaotic behavior and of hyperchaotic behavior. For example, when the parameter b decreases from 1.0, the system HNE_1 goes through a limit cycle, torus and chaos before getting hyperchaos. Fig. 5 illustrates the periodic orbit of the system HNE_1 for the parameter b = 0.8.

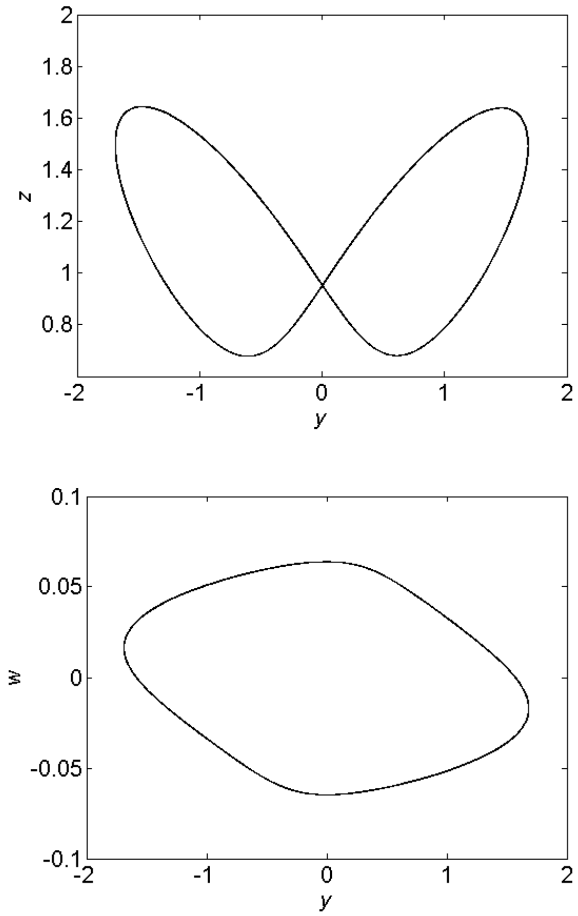


Fig. 5. The periodic orbit of system (2) for b = 0.8, and d = –0.001.

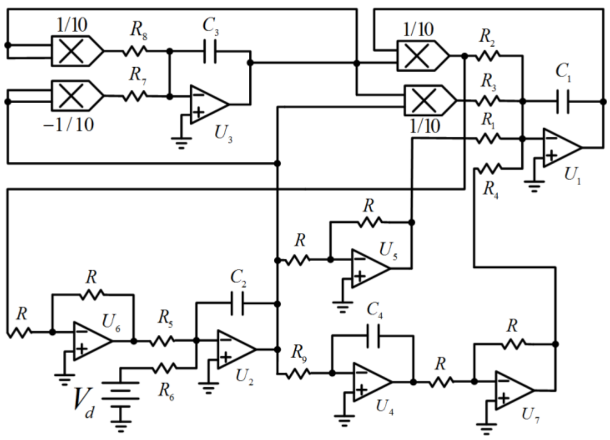
3. Circuit implementation

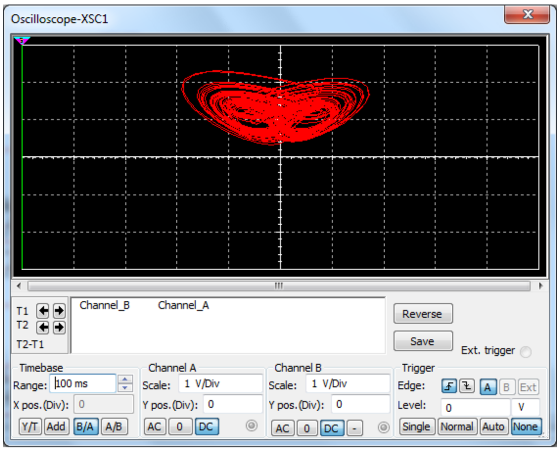
An electronic circuit is designed to realize the new proposed hyperchaotic system (2). As shown in Fig. 6, common off-the-shelf components such as resistors, capacitors, operational amplifiers, and multipliers are used. The state variables x, y, z, and w of system (2) are the voltages across the capacitors C_1, C_2, C_3, and C_4, respectively. As a result, the corresponding equations of circuit can be written as below

$$\begin{aligned}
\dot{x} &= \frac{1}{R_1C_1}y - \frac{1}{10R_2C_1}xz - \frac{1}{10R_3C_1}yz + \frac{1}{R_4C_1}w \\
\dot{y} &= \frac{1}{10R_5C_2}xz - \frac{1}{R_6C_2}V_d \\
\dot{z} &= \frac{1}{10R_7C_3}y^2 - \frac{1}{10R_8C_3}z^2 \\
\dot{w} &= -\frac{1}{R_9C_4}y.
\end{aligned} \qquad (3)$$

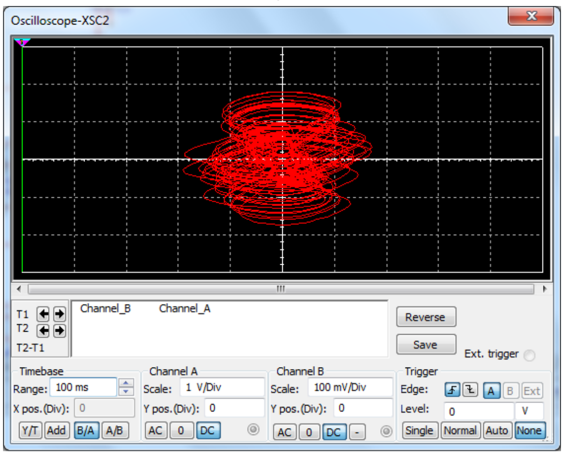
The power supplies are ±15 volts while the values of circuit elements in Fig. 6 are chosen as follows: $C_1 = C_2 = C_3 = C_4 = 1\text{nF}$, $R_1 = R_4 = R_6 = R = 100\text{k}\Omega$,

$R_2 = R_3 = R_7 = 10\text{k}\Omega$, $R_5 = 2\text{k}\Omega$, $R_8 = 35.714\text{k}\Omega$, $R_9 = 2\text{M}\Omega$, and $V_d = 1\text{mV}_{DC}$. The designed circuit is implemented in the electronic simulation package NI Multisim and the results are presented in Fig. 7. A good agreement between the theoretical and circuital attractors (see Fig. 1 and Fig. 7) confirms the feasibility of the hyperchaotic system without equilibrium (2).





a



b

Fig. 7. Hyperchaotic attractor of the designed electronic circuit obtained from Multisim (a) in the y-z plane, (b) in the y-w plane.

4. Discussion

Memristor [17] was introduced as the fourth basic circuit element besides the three classical elements (the resistor, the inductor, and the capacitor). Recently, researchers at Hewlett-Packard (HP) Labs have implemented successfully nano-scale memristors [18]. That motivates a new attractive area in which memristor-based systems can be realized with special unknown features [19-21]. Up to now, the intrinsic nonlinear characteristic of memristor could be exploited in constructing novel chaotic systems with complex dynamics [22-25]. It has been found that some of such memristor-based systems can display chaotic attractor with an infinite number of equilibrium due to the presence of memristor [24, 25]. Especially, a hyperchaotic system using memristor has been presented in [26]. The dimensionless equations of the system have been given as

$$\begin{aligned}
\dot{x} &= -a(2x + y + yz)\\
\dot{y} &= -b(x - xz) + y(0.1 + 0.6w^2)\\
\dot{z} &= -z - cxy\\
\dot{w} &= y.
\end{aligned} \tag{4}$$

Obviously, the equilibria of memristor-base system (4) is E(0, 0, 0, w). In other words, system (4) has a real line equilibrium and belongs to a new category of hyperchaotic systems with hidden attractors [4, 5].

In order to illustrate the effectiveness of the mentioned approach in Section 2, a new no-equilibrium memristor-based system is also constructed in this Section. By adding a tiny control parameter d into system (4), a novel memristor-based system without equilibrium (called HNE_2) has been obtained in the following form

$$\begin{aligned}
\dot{x} &= -a(2x + y + yz)\\
\dot{y} &= -b(x - xz) + y(0.1 + 0.6w^2) + d\\
\dot{z} &= -z - cxy\\
\dot{w} &= y.
\end{aligned} \tag{5}$$

The selected parameters, Lyapunov exponents, Kaplan-Yorke dimension, and initial conditions of new system (5) are reported in Table 1. Two positive Lyapunov exponents indicate hyperchaos in new proposed memristor-based system HNE_2.

To the best of our knowledge, there are few works relating to the conversion of a chaotic system with hidden attractor, which is rarely reported in the literature, into a new chaotic system with hidden attractor [27]. In particular, there is no report about the hyperchaotic memristor-based system without equilibrium. Therefore this work has enlarged the list of hidden hyperchaotic attractors.

Table 1. Two proposed novel systems without equilibrium which can exhibit hyperchaos

Model	Parameters	LEs	D_{KY}	(x_0, y_0, z_0, w_0)
HNE_1	a = 5 b = 0.28 c = 0.05 d = –0.001	0.0756 0.0382 0 –1.6600	3.0686	0.0 0.0 0.8 0.02
HNE_2	a = 5 b = 6 c = 6 d = –0.001	0.1244 0.0136 0 –10.8161	3.0128	0.0 0.01 0.01 0.0

5. Conclusions

This paper presents a novel hyperchaotic system without equilibrium which is generated from a known system with an infinite line of equilibrium points. The transformation from a hidden hyperchaotic attractor into another new hidden hyperchaotic attractor contributes towards little knowledge about the special characteristic of such systems.

Furthermore, a new memristor-based hyperchaotic system without equilibrium is also constructed by using the introduced methodology. Because hyperchaos is better than conventional chaos in a variety of areas, for instance, hyperchaos increases the security of chaotic-based communication systems significantly [28, 29], designing hyperchaotic circuits based on memristor will be investigated in the future works.

Acknowledgements

This research is funded by Vietnam National Foundation for Science and Technology Development (NAFOSTED) under grant number 102.99-2013.06.

References

[1] J. C. Sprott, Phys. Rev. E **50**, 397 (1994).
[2] L. P. Silnikov, Sov. Math. Docklady **6**, 163 (1965).
[3] L. P. Silnikov, Sov. Math. U.S.S.R.-Shornik **10**, 91 (1970).
[4] G. A. Leonov, N. V. Kuznetsov, O. A. Kuznetsova, S. M. Seledzhi, V. I. Vagaitsev, Trans. Syst. Control **6**, 54 (2011).
[5] G. A. Leonov, N. V. Kuznetsov, Int. J. Bifur. Chaos **23**, 1330002 (2013).
[6] X. Wang, G. R. Chen, Commun. Nonlinear Sci. Numer.Simulat. **17**, 1264 (2012).
[7] Z. C. Wei, I. Pehlivan, Optoelectron. Adv. Mater.–Rapid Comm. **6**,742(2012).
[8] Z. C. Wei, Phys. Lett. A **376**, 102 (2011).
[9] S. Jafari, J. C. Sprott, S. M. R. H. Golpayegani, Phys. Lett. A **377**, 699 (2013).
[10] Z. Wang, S. Cang, E. O. Ochola, Y. Sun, Nonlinear Dyn. **69**, 531 (2012).
[11] Z. C. Wei, Y. Tang, H. Chen, I. Pehlivan, Optoelectron. Adv. Mater.–Rapid Comm. **7**, 984 (2013).
[12] C. Li, J. C. Sprott, Int. J. Bifur. Chaos **24**, 1450034 (2014).
[13] C. Li, J. C. Sprott, W. Thio J. Exp. Theo. Phys. **118**, 494 (2014).
[14] S. Jafari, J. C. Sprott, Chaos Solitons Fractals **57**, 79 (2013).
[15] O. E. Rölsser, Phys. Lett. A **71**, 155 (1979).
[16] A. Wolf, J. B. Swift, H. L. Swinney, J. A. Vastano, Physica D **16**, 285 (1985).
[17] L. O. Chua, IEEE Trans. Circ. Theory **18**, 507 (1971).
[18] D. B. Strukov, G. S. Steeart, R. S. Williams Nature **453**, 80 (2008).
[19] X. Wang, Y. Chen, H. Xi, D. Dimitrov, Electr. Dev. Lett. **30**, 294 (2009).
[20] S. P. Kosta, Y. P. Kosta, M. Bhatele, Y. M. Dubey, A. Gaur, S. Kosta, J. Gupta, A. Patel, B. Patel, Int. J. Med. Eng. Inf. **3**, 16 (2011).
[21] T. Driscoll, J. Quinn, S. Klein, H. T. Kim, B. J. Kim, Y. V. Pershin, M. Di Ventra, D. N. Basov, Appl. Phys. Lett. **97**, 093502 (2012).
[22] B. Muthuswamy, P. P Kokate, IETE Tech. Rev. **26**, 415 (2009).
[23] B. Muthuswamy, Int. J. Bifur. Chaos **20**, 1335 (2010).
[24] M. Itoh, L. O. Chua, Int. J. Bifur. Chaos **18**, 3183 (2008).
[25] B. C. Bao, J. P. Xu, G. H. Zhou, Z. H. Ma, L. Zou, Chin. Phys. B **20**, 120502 (2011).
[26] Q. Li, S. Hu, S. Tang, G. Zeng, Int J. Circ. Theory Appl. DOI: 10.1002/cta.1912 (2013).
[27] V.-T. Pham, C. Volos, S. Jafari, Z. Wei, X. Wang, Int. J. Bifur. Chaos **24**, in press (2014).
[28] V. Udaltsol, J. Goedgebuer, L. Larger, J. Cuenot, P. Levy, W. Rhodes, Optics Spectroscopy **95**, 114 (2003).
[29] S. Sadoudi, C. Tanougast, M. S. Azzaz, A. Dandache, EURASIP J. Image Video Processing **43**, 1 (2013).

*Corresponding author: pvt3010@gmail.com