Four-scroll stellate new chaotic system

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This paper introduces a new chaotic system of three-dimensional autonomous ordinary differential equations, which can display strange four-scroll stellate chaotic attractors simultaneously. Some basic dynamical behaviors of the new system investigated via theoretical analysis by means of equilibria and lyapunov exponent spectrum. Finally, the chaos generator of the new chaotic system is experimentally confirmed via a novel analogue circuit design. It is convenient to use the system to purposefully generate chaos in chaos applications. A good qualitative agreement is illustrated between the simulation results.

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1. Introduction

As the first chaotic model, the Lorenz system has become a paradigm of chaos research [1]. Chen constructed another chaotic system [2], which nevertheless is not topologically equivalent to the Lorenz's [2, 3]. This system is the dual to the Lorenz system and similarly has a simple structure [3]. Lü and Chen found the critical new chaotic system [4], which represents the transition between the Lorenz and Chen attractors. For the investigation on generic 3D smooth quadratic autonomous systems, Sprott [5-7] found by exhaustive computer searching about 19 simple chaotic systems with no more than three equilibria. It is very important to note that some 3D autonomous chaotic systems have three particular fixed points: one saddle and two unstable saddle-foci (for example, Lorenz system [1], Chen system [2], Lu system [4]). The other 3D chaotic systems, such as the original Rossler system [8], DLS [9] and Burke-Show system [10], have two unstable saddle-foci. Yang and Chen found another 3D chaotic system with three fixed points: one saddle and two stable fixed points [11]. Recently, Yang et al. [12] and Pehlivan et al. [13] introduced and analyzed the new 3D chaotic systems with six terms including only two quadratic terms in a form very similar to the Lorenz, Chen, Lu and Yang–Chen systems, but they have two very different fixed points: two stable node-foci. Therefore, they are very interesting to further find out the new dynamics of the system.

This paper introduces one more interesting complex three-dimensional quadratic autonomous four-scroll stellate chaotic system, which can depict complex 4-scroll chaotic attractors simultaneously.

2. A new four-scroll stellate chaotic system and its analyses

Following nonlinear autonomous ordinary differential equations are the new chaotic system.

$$\begin{aligned}\dot{x} &= -a\cdot x + y + y\cdot z\\ \dot{y} &= x - a\cdot y + b\cdot x\cdot z\\ \dot{z} &= c\cdot z - b\cdot x\cdot y\end{aligned}$$

where $a, b, c \in R$ are parameters of the system. Typical parameters are a=4, b=0.5, c=0.6. Let us consider a volume in a certain domain of the state space. For the system, one has

$$\Delta V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} = -a - a + c = -2a + c < 0$$

As the divergence of vector field is negative, it can be concluded that the system is dissipative. It should be noticed that the system will always be dissipative if and only if $c < 2a$ with an exponential rate

$$\frac{dV}{dt} = e^{-2a+c}$$

In the dynamical system, a volume element V_0 is apparently contracted by the flow into a volume element $V_0\, e^{(-2a+c)\,t}$ in time t. It means that each volume containing the trajectory of this dynamical system shrinks to zero as $t\to\infty$ at an exponential rate (-2a+c). So, all this dynamical system orbits are eventually confined to a specific subset that have zero volume, the asymptotic motion settles onto an attractor of the system. This suggests that the dynamics may tend to an attractor as $t\to\infty$.

Phase portraits of the new chaotic system were achieved as performing the numerical simulation for initial conditions x_0=0.6, y_0=0, z_0=0 and parameters a=4, b=0.5, c=0.6, and shown in Fig. 1. As can be seen in the Fig. 2, the lyapunov exponents of the new system are

$$\lambda_1 = 0.2439,\ \lambda_2 = 0,\ \lambda_3 = -7.6254$$

for initial conditions x_0=0.6, y_0=0, z_0=0 and parameters a=4, b=0.5, c=0.6,

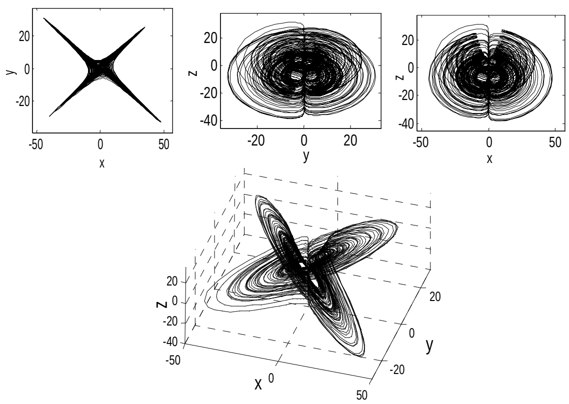


Fig. 1. 2D and 3D Phase portraits of the four-scroll stellate new chaotic system.

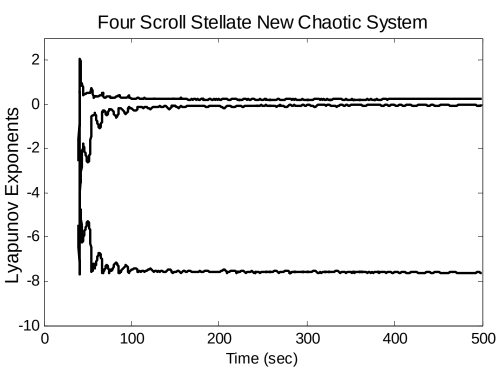


Fig. 2. Lyapunov Exponents of the new chaotic system for initial conditions x_0=0.6, y_0 = 0, z_0 = 0 and parameters a=4, b=0.5, c=0.6.

The new system equations have five equilibrum points as $(0,0,0)$,

$$(\pm 3.648,\ \mp 2.361,\ -7.179),$$
$$(\pm 2.548,\ \pm 1.968,\ 4.179)$$

for a=4, b=0.5, c=0.6.

For the fixed point $E_1(\text{x}^*, \text{y}^*, \text{z}^*) = (\ 0\ ,\ 0\ ,\ 0\ )$ the Jacobian matrix of the system is given as follows:

$$J(E_1) = \begin{pmatrix} -4 & 1 & 0 \\ 1 & -4 & 0 \\ 0 & 0 & 0.6 \end{pmatrix}$$

Obviously, the characteristic equation about the equilibria E_1 is:

$$\det(\lambda I - J(E_1)) = \lambda^3 + 7.4 \cdot \lambda^2 + 10.2 \cdot \lambda - 9 = 0,$$

by solving the characteristic equation, the eigenvalues are found as

$$\lambda_1 = -5, \quad \lambda_2 = -3, \quad \lambda_3 = 0.6\ .$$

For the fixed point $E_2(\text{x}^*, \text{y}^*, \text{z}^*) = (\ 3.648,\ -2.361,\ -7.179)$ the Jacobian matrix of the system is given as follows:

$$J(E_2) = \begin{pmatrix} -4 & -6.179 & -2.361 \\ \text{-2.590} & -4 & 1.824 \\ 1.181 & -1.824 & 0.6 \end{pmatrix}$$

Obviously, the characteristic equation about the equilibria E_2 is:

$\det(\lambda I - J(E_2)) = \lambda^3 + 7.4 \cdot \lambda^2 + 1.312 \cdot \lambda + 48.928 = 0$, by solving the characteristic equation, the eigenvalues are found as

$$\lambda_1 = -8, \quad \lambda_2 = 0.3 + 2.455 \cdot \mathbf{i},$$
$$\lambda_3 = 0.3 - 2.455 \cdot \mathbf{i}$$

For the fixed point $E_3(\text{x}^*, \text{y}^*, \text{z}^*) = (\ -3.648,\ +2.361,\ -7.179)$ the Jacobian matrix of the system is given as follows:

$$J(E_3) = \begin{pmatrix} -4 & -6.179 & 2.361 \\ \text{-2.590} & -4 & -1.824 \\ -1.181 & 1.824 & 0.6 \end{pmatrix}$$

Obviously, the characteristic equation about the equilibria E_3 is:

$\det(\lambda I - J(E_3)) = \lambda^3 + 7.4 \cdot \lambda^2 + 1.312 \cdot \lambda + 48.928 = 0$, by solving the characteristic equation, the eigenvalues are found as

$$\lambda_1 = -8,\ \lambda_2 = 0.3 + 2.455 \cdot \mathbf{i},\ \lambda_3 = 0.3 - 2.455 \cdot \mathbf{i}\ .$$

For the fixed point $E_4(\text{x}^*, \text{y}^*, \text{z}^*) = (\ 2.548,\ 1.968\ ,\ 4.179\ )$ the Jacobian matrix of the system is given as follows:

$$J(E_4) = \begin{pmatrix} -4 & 5.179 & 1.968 \\ 3.090 & -4 & 1.274 \\ -0.984 & -1.274 & 0.6 \end{pmatrix}$$

Obviously, the characteristic equation about the equilibria E_4 is:

$$\det(\lambda I - J(E_4)) = \lambda^3 + 7.4 \cdot \lambda^2 - 1.244 \cdot \lambda + 28.48 = 0,$$

by solving the characteristic equation, the eigenvalues are found as

$$\lambda_1 = -8,\ \lambda_2 = 0.3 + 1.863 \cdot \mathbf{i},\ \lambda_3 = 0.3 - 1.863 \cdot \mathbf{i}\ .$$

For the fixed point $E_5(x^*, y^*, z^*) = (\ -2.548,\ -1.968\ , 4.179\ )$ the Jacobian matrix of the system is given as follows:

$$J(E_5) = \begin{pmatrix} -4 & 5.179 & -1.968 \\ 3.090 & -4 & -1.274 \\ 0.984 & 1.274 & 0.6 \end{pmatrix}$$

Obviously, the characteristic equation about the equilibria E_5 is :

$$\det(\lambda I - J(E_5)) = \lambda^3 + 7.4 \cdot \lambda^2 - 1.244 \cdot \lambda + 28.48 = 0$$

, by solving the characteristic equation, the eigenvalues are found as

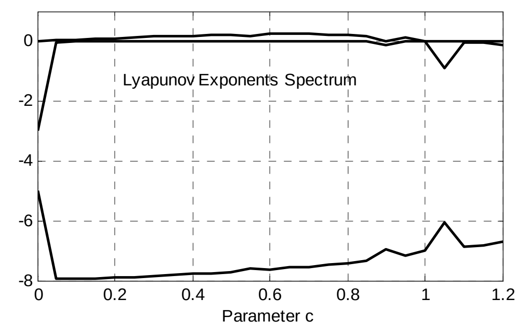
$$\lambda_1 = -8,\ \lambda_2 = 0.3 + 1.863 \cdot \mathbf{i},\ \lambda_3 = 0.3 - 1.863 \cdot \mathbf{i}\ .$$

The equilibria and eigenvalues for certain systems are tabulated in Table 1.

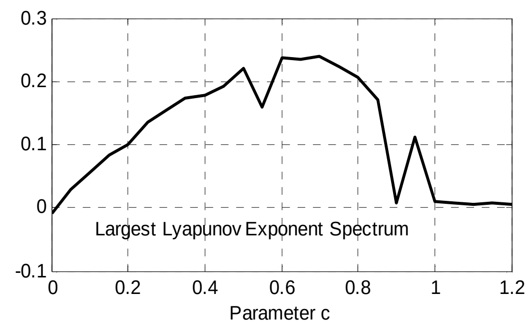
Table 1. Equilibria and eigenvalues for certain chaotic systems.

System	Parameters	Equilibria	Eigenvalues
Lorenz system	a=10, b=8/3, c=28	$(0,0,0)$	-22.8277, -2.6667, 11.8277
		$(\pm 6\sqrt{2},\ \pm 6\sqrt{2},\ 27)$	$-13.8546,\ 0.0940 \pm 0.1945 \cdot i$
Chen system	a=35, b=3, c=28	$(0,0,0)$	-30.8359, -3, 23.8359
		$(\pm 3\sqrt{7},\ \pm 3\sqrt{7},\ 21)$	$-18.4288,\ 4.2140 \pm 14.8846 \cdot i$
Lü system	a=36, b=3, c=20	$(0,0,0)$	-36, -3, 20
		$(\pm 2\sqrt{15}, \pm 2\sqrt{15}, 20)$	$-22.6516,\ 1.8258 \pm 13.6887 \cdot i$
The new system(1)	a=4, b=0.5, c=0.6	$(0,\ 0,\ 0)$	-5, -3, 0.6
		$(\pm 3.648,\ \mp 2.361,\ -7.179)$	$-8,\ 0.3 \pm 2.455 \cdot i$
		$(\pm 2.548,\ \pm 1.968,\ 4.179)$	$-8,\ 0.3 \pm 2.455 \cdot i$
			$-8,\ 0.3 \pm 1.863 \cdot i$
			$-8,\ 0.3 \pm 1.863 \cdot i$

Fig. 3 shows the Lyapunov Exponents Spectrum of the new system for varying parameter c, and constant parameters a=4, b=0.5. As can be seen from the Lyapunov exponents spectrum, the new system is chaotic when a positive Lyapunov exponent.



(a)



(b)

Fig. 3. (a) Lyapunov Exponent Spectrum of the new system for varying parameter c, and constant parameters a=4, b=0.5. (b) Largest Lyapunov Exponent Spectrum of the new system for varying parameter c, and constant parameters a=4, b=0.5.

3. Circuit realization of the four-scroll chaotic system

The designed electronic circuit schematic and the Orcad-PSpice simulation results of the new chaotic circuit for parameters a=4, b=0.5, c=0.6, are seen in Fig. 4 and Fig. 5-6 respectively.

Chaotic differential equations of the new circuit are given below.

$$\dot{x} = \frac{1}{R_1\ C_1} x - \frac{1}{R_2\ C_1} y - \frac{1}{R_3\ C_1} y \cdot z$$

$$\dot{y} = \frac{1}{R_4\ C_2} x - \frac{1}{R_5\ C_2} y + \frac{1}{R_6\ C_2} x \cdot z$$

$$\dot{z} = \frac{1}{R_7\ C_3} z - \frac{1}{R_8\ C_3} x \cdot y$$

4. Conclusions

This article introduces one three-dimensional autonomous interesting new chaotic system which can display strange four-scroll stellate chaotic attractors simultaneously. Our investigation was completed using a combination of theoretical analysis and simulations.The chaos generator of the new chaotic system are confirmed via a novel electronic circuit design. Electronic circuitry of the new chaotic system is simple. It is convenient to use the new system to purposefully generate chaos in chaos applications. We believe that the unknown dynamical behaviors of this chaotic attractor deserve further investigation and are desirable for engineering applications in the near future.

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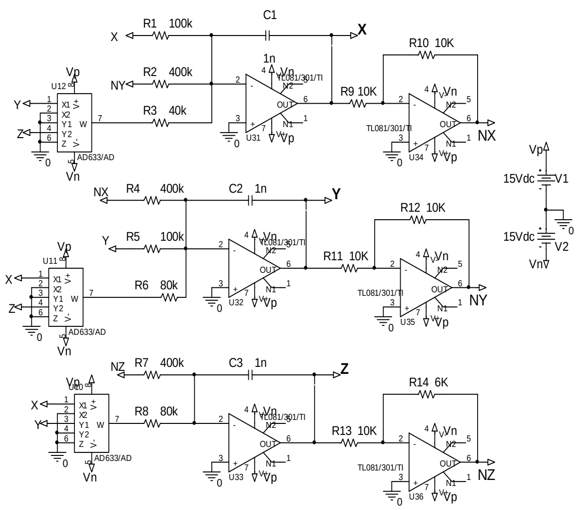

Fig. 4. The electronic circuit schematic of the new chaotic system.

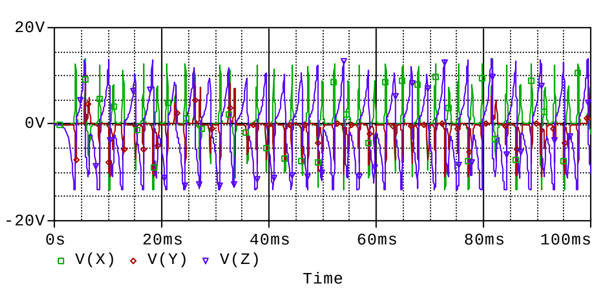

Fig. 5. x, y, z chaotic signals of the new chaotic system against to time.

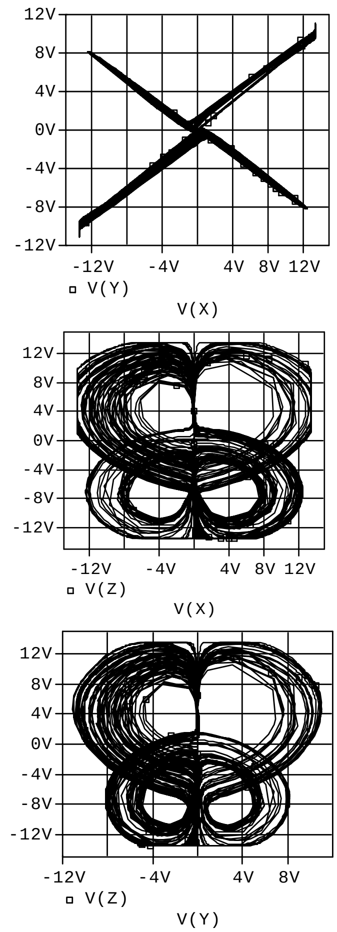

Fig. 6. Phase portraits of the new chaotic system.

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