Fatigue behaviour of crumb rubber concrete under flexural load

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This paper presents the results of a study on fatigue behaviour of crumb rubber concrete (CRC) under flexural load. An experimental program was carried out to investigate the fatigue lives of CRC at various stress levels. The four-point flexural tests on CRC beam specimens ($100 \times 100 \times 400$ mm) were conducted. CRC beam specimens were also tested under static flexural loading to obtain the flexural strength of CRC. The test results indicate that the flexural fatigue lives of CRC follow the double-parameter Weibull distribution. The regression parameters of the fatigue equation corresponding to different survival probabilities are obtained. The flexural fatigue strength of CRC for the desired level of survival probability can be evaluated by using the fatigue equation.

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1. Introduction

Crumb rubber concrete (CRC) is known as a promising material because of its good performance and the utilization of crumb rubber from discarded scrap tyres. The properties of fresh and hardened CRC were investigated in previous papers. CRC showed higher toughness and impact resistance, lower density and better sound absorption and insulation compared with conventional concrete [1-4].

In this paper, the flexural fatigue behavior of CRC is experimentally studied. The test results indicate that the flexural fatigue lives of CRC follow the double-parameter Weibull distribution. The regression parameters of the fatigue equation corresponding to different survival probabilities are obtained through the regression analysis.

2. Experimental

2.1 Materials and mixture proportions

Ordinary Portland cement was used in this study. The coarse aggregate was crushed stone with diameter of 5-20 mm. The fine aggregate was natural sand with a maximum size of 5 mm. Fly ash was used as mineral addition. Crumb rubber content used was 90 kg/m³. The details of the mixture proportions are shown in Table 1. The CRC beam specimens ($100 \times 100 \times 400$ mm) were cast and cured for 90 days.

Materials	(kg/m ³)
Portland cement	367
Water	165
Coarse aggregate	1055
Fine aggregate	372
Crumb rubber	90
Fly ash	156
Superplasticizer	5.2

Table 1. Mixture proportions of CRC.

2.2 Testing procedure

The four-point flexural tests were conducted on beam specimens $(100 \times 100 \times 400 \text{ mm})$ with a closed-loop universal testing machine. The support span is 300 mm and the loading span is 100 mm. The statical flexural tests were carried out to determine the flexural strength of CRC prior to the fatigue tests. The stress level *S* is defined as: $S = P_{\text{max}}/P_u$, where P_{max} is the maximum load and P_u is the static ultimate load. The stress levels selected in the fatigue test are 0.90, 0.85, 0.80 and 0.75. The load cycle characteristic value *R* ($R = P_{\text{min}}/P_{\text{max}}$, P_{min} is the minimum load) applied in the fatigue test is 0.1. The flexural fatigue test is carried out in load control with a sinusoidal waveform at a frequency of 5Hz (when stress level *S* is equal to 0.90 and 0.85) or 10 Hz (when stress level *S* is equal to 0.80 and 0.75). The number of beam specimens used in statical flexural test and flexural fatigue test is given in Table 2.

Table 2. The number of beam specimens used in test.

Flexural fatigue test			Statical flexural	
S=0.9	S=0.85	S=0.80	<i>S</i> =0.75	test
4	4	4	4	3

3. Weibull distribution

The fatigue data of concrete usually exhibit larger discreteness because of material heterogeneity. Weibull distribution has been widely adopted to process the data of fatigue test. The Weibull probability density function $f_N(n)$ and cumulative distribution function $F_N(n)$ can be expressed by

$$f_N(n) = \frac{\alpha}{u - n_0} \left(\frac{n - n_0}{u - n_0}\right)^{\alpha - 1} \exp\left[-\left(\frac{n - n_0}{u - n_0}\right)^{\alpha}\right]$$
(1)

$$F_{N}(n) = 1 - \exp\left[-\left(\frac{n - n_{0}}{u - n_{0}}\right)^{\alpha}\right]$$
(2)

Where, α is the shape parameter or the Weibull slope at stress level *S*; *u* is the scale parameter or the characteristic fatigue life at stress level *S*; n_0 is the position parameter or the minimum fatigue life at stress level *S*.

The value of n_0 may be considered as zero because of the discreteness of fatigue data and safer reliability. This will yield double-parameter Weibull distribution. From Eq. (2), the survivorship function P(n) can be given

$$P(n) = 1 - F_N(n) = \exp\left[-\left(\frac{n}{u}\right)^{\alpha}\right]$$
(3)

Taking twice natural logarithm for both sides of Eq. (3) gives

$$\ln\left[\ln\left(\frac{1}{P}\right)\right] = \alpha \ln n - \alpha \ln u \tag{4}$$

Setting $Y = \ln \left[\ln \left(1/P \right) \right]$, $X = \ln N$, $\beta = \alpha \ln u$, then

$$Y = \alpha X - \beta \tag{5}$$

It can be seen from Eq. (5) that, if a linear trend is observed between Y and X, the fatigue data follow the Weibull distribution.

4. Results and discussion

4.1 Weibull distribution verification

According to the probability theory of Weibull distribution, the empirical survivorship function P can be expressed by

$$P = 1 - \frac{i}{k+1} \tag{6}$$

where, k is the total number of the fatigue data at certain stress level, i is the sequence number of failure specimens at certain stress level.

The linear regression is carried out for the fatigue data according to Eq. (5). Table 3 shows the results of Weibull regression analysis when the stress level *S* is equal to 0.90, 0.85, 0.80 and 0.75. The corresponding determination coefficients R^2 are 0.9385, 0.9671, 0.9492 and 0.9512. Graphs are plotted between $\ln[\ln(1/P)]$ and $\ln N$ in Fig. 1. From Fig. 1, a linear trend is observed for the fatigue data, which indicates that the relationship between $\ln[\ln(1/P)]$ and $\ln N$ is linear for various stress levels. This indicates that the fatigue data follow the double-parameter Weibull distribution. Consequently, the double-parameter Weibull distribution is a reasonable assumption for the fatigue data of CRC.

Table 3. The results of Weibull regression analysis.

S	β	α	R^2
0.90	7.4845	1.1447	0.9385
0.85	10.0373	1.1816	0.9671
0.80	10.31	0.9526	0.9492
0.75	26.228	2.1415	0.9512



Fig. 1. Weibull regression analysis of fatigue data of CRC.

4.2 Regression parameters of the fatigue equation of CRC

The double-logarithm fatigue equation is used to investigate the fatigue behavior of CRC in this study. The double-logarithm fatigue equation is defined as:

$$\lg S = \lg a - b \lg N \tag{7}$$

Where, S is the stress level, N is the fatigue life.

The parameters a and b of the Eq. (7) can be obtained through the regression analysis. Table 4 shows the regression parameters a and b of the fatigue equation corresponding to different survival probabilities. The Eq. (7) can be used to evaluate the flexural fatigue strength of CRC for the desired level of survival probability [5].

Table 4. Regression parameters	a	and	b
of the fatigue equation	ı.		

Р	а	b	R^2
0.95	1.001	0.025	0.993
0.90	1.028	0.026	0.996
0.80	1.056	0.027	0.987
0.70	1.073	0.030	0.982
0.60	1.089	0.031	0.975
0.50	1.101	0.031	0.971

5. Conclusion

The fatigue lives of CRC follow the double-parameter Weibull distribution. The regression parameters of the Weibull distribution for the fatigue lives of CRC are obtained. The regression parameters of the fatigue equation corresponding to different survival probabilities are obtained through the regression analysis. The flexural fatigue strength of CRC for the desired level of survival probability can be predicted by using the fatigue equation.

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