Exact Thirring optical solitons in birefringent fibers with cubic-quintic nonlinearity by G'/G-expansion method

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This work deals with Thirring optical solitons in birefringent fibers with cubic-quintic nonlinearity. The vector coupled nonlinear Schrödinger equation for the propagation of Thirring optical solitons is studied analytically by the G'/G-expansion scheme. As a consequence, exact traveling wave solutions, along with the existence conditions, are reported.

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1. Introduction

Birefringence is a natural phenomenon that occurs in optical fibers [1-3], and the nonlinear dynamical model for the propagation of optical solitons in birefringent fibers is given by the vector coupled nonlinear Schrödinger equation (NLSE) [1-6]. When the cross-phase modulation (XPM) much more than the self phase modulation (SPM), i.e. SPM is negligible, the soliton in birefringent fibers is known as Thirring optical soliton, which is the product of perfect balance between XPM and group velocity dispersion (GVD) [7-10].

Very recently, we investigated the dynamics of Thirring optical solitons in birefringent fibers with Kerr law and parabolic law nonlinearities, and obtained explicit Thirring bright, dark and singular solitons based on the sub-equations expansion approach, traveling wave hypothesis and the P^6 model scheme [7-10]. However, the study of Thirring solitons is not comprehensive. In this paper, we will focus on the nonlinear dynamics of Thirring optical solitons with cubic-quintic (parabolic law) nonlinearity, and a new integration tool that is the G'/G-expansion scheme [11-15] is applied to extract the traveling wave solutions. As a result, analytical Thirring dark and singular soliton solutions are derived. Hence, this work is an extension of our previous results.

2. Governing equation

In the presence of parabolic law nonlinearity and spatio-temporal dispersion (STD), the mathematical model for the propagating of Thirring optical solitons through birefringent fibers is given by the following vector coupled NLSE [8]:

$$\begin{aligned} &iq_t + a_1 q_{xx} + b_1 q_{xt} + c_1 |r|^2 q \\ &+ (d_1 |q|^2 |r|^2 + e_1 |r|^4) q = 0 \end{aligned} \tag{1}$$

$$\begin{aligned} &ir_t + a_2 r_{xx} + b_2 r_{xt} + c_2 |q|^2 r \\ &+ (d_2 |r|^2 |q|^2 + e_2 |q|^4) r = 0 \end{aligned} \tag{2}$$

where $q(x,t)$ and $r(x,t)$ represent the optical wave profiles of two split pulses, while x and t are the spatial and temporal variables.

In Equations (1) and (2), the first, second and third terms represent the linear temporal evolution, GVD and STD, respectively. STD should be taken into account in order to make the dynamical model well-posed. In addition, the fourth and fifth terms are due to the parabolic law nonlinearity [16-30]. Here, a_l , b_l and c_l for $l = 1, 2$ give the coefficients of GVD, STD and XPM, and finally d_l and e_l terms account for the quintic nonlinearity.

In our recent work [8], Equations (1) and (2) were solved analytically by the traveling wave hypothesis and the P^6 model approach. This paper will employ a different way to integrate Equations (1) and (2). It is the G'/G-expansion scheme.

3. Exact soliton solutions

Firstly, we make the following traveling wave hypotheses [8]:

$$q(x,t) = P_1[\eta(x,t)] \exp[i\varphi_1(x,t)] \tag{3}$$

$$r(x,t) = P_2[\eta(x,t)] \exp[i\varphi_2(x,t)] \tag{4}$$

where $\eta(x,t) = B(x - vt)$ and $\varphi_l(x,t) = -\kappa_l x + \omega_l t + \delta_l$.

In Equations (3) and (4), B gives the inverse width, and v is the velocity of two polarized solitons. For

$l=1,2$, κ_l, ω_l and δ_l represent frequencies, wave numbers and phase constants, respectively.

Substituting the above hypotheses into Equations (1) and (2) yields

$$\begin{aligned}&(a_l-b_l v)B^2P_l''-(a_l\kappa_l^2+\omega_l-b_l\omega_l\kappa_l)P_l\\&+c_lP_{\bar{l}}^2P_l+d_lP_l^3P_{\bar{l}}^2+e_lP_{\bar{l}}^4P_l=0\end{aligned} \tag{5}$$

$$v=\frac{2a_l\kappa_l-b_l\omega_l}{b_l\kappa_l-1} \tag{6}$$

for $l=1,2$ and $\bar{l}=3-l$.

Equation (6) gives the velocity of two polarized solitons that poses a constraint condition

$$\begin{aligned}&(2a_1\kappa_1-b_1\omega_1)(b_2\kappa_2-1)\\&=(2a_2\kappa_2-b_2\omega_2)(b_1\kappa_1-1)\end{aligned} \tag{7}$$

Below, Equation (5) will be integrated to obtain explicit traveling wave solutions by G'/G-expansion method.

Based on the balancing principle, we assume that Equation (5) admits the traveling wave solution in the form

$$P_l(\eta)=A_l\left(\frac{G'(\eta)}{G(\eta)}\right)^{\frac{1}{2}} \tag{8}$$

where A_l is a real constants to be determined later, and $G(\eta)$ satisfies

$$G''(\eta)+\lambda G'(\eta)+\mu G(\eta)=0 \tag{9}$$

where λ and μ are real constants.

Substituting Equations (8) and (9) into Equations (5), and setting the coefficients of $\left(\frac{G'(\eta)}{G(\eta)}\right)^m$ ($m=-\frac{3}{2},\frac{1}{2},\frac{3}{2},\frac{5}{2}$) to zero, we obtain a set of nonlinear algebraic equations

$$\begin{aligned}&m=\frac{5}{2}:\\&\frac{3}{4}(a_l-b_l v)B^2+d_lA_l^2A_{\bar{l}}^2+e_lA_{\bar{l}}^4=0\end{aligned} \tag{10}$$

$$m=\frac{3}{2}:(a_l-b_l v)B^2\lambda+c_lA_{\bar{l}}^2=0 \tag{11}$$

$$\begin{aligned}&m=\frac{1}{2}:(\frac{1}{2}\mu+\frac{1}{4}\lambda^2)(a_l-b_l v)B^2\\&-(a_l\kappa_l^2+\omega_l-b_l\omega_l\kappa_l)=0\end{aligned} \tag{12}$$

$$m=-\frac{3}{2}:-\frac{1}{4}\mu^2(a_l-b_l v)B^2=0 \tag{13}$$

Solving the above nonlinear system yields

$$A_l=2\sqrt{\frac{b_{\bar{l}}\omega_{\bar{l}}\kappa_{\bar{l}}-a_{\bar{l}}\kappa_{\bar{l}}^2-\omega_{\bar{l}}}{\lambda c_{\bar{l}}}} \tag{14}$$

$$B=\frac{2}{\lambda}\sqrt{\frac{a_l\kappa_l^2+\omega_l-b_l\omega_l\kappa_l}{a_l-b_l v}} \tag{15}$$

$$\mu=0 \tag{16}$$

$$\begin{aligned}&16c_ld_l(b_{\bar{l}}\omega_{\bar{l}}\kappa_{\bar{l}}-a_{\bar{l}}\kappa_{\bar{l}}^2-\omega_{\bar{l}})\\&+16c_{\bar{l}}e_l(b_l\omega_l\kappa_l-a_l\kappa_l^2-\omega_l)\\&-3c_l^2c_{\bar{l}}=0\end{aligned} \tag{17}$$

where λ, κ_l, ω_l and δ_l are real arbitrary constants.

Equation (14) gives the amplitude of the two polarized solitons, which naturally introduces the restriction

$$\lambda c_{\bar{l}}(b_{\bar{l}}\omega_{\bar{l}}\kappa_{\bar{l}}-a_{\bar{l}}\kappa_{\bar{l}}^2-\omega_{\bar{l}})>0 \tag{18}$$

Equation (15) gives the inverse width of the two polarized solitons that poses a constraint condition

$$(a_l-b_l v)(a_l\kappa_l^2+\omega_l-b_l\omega_l\kappa_l)>0 \tag{19}$$

Equation (17) gives other existence condition for the Thirring solitons.

From Equation (16), exact solution to Equation (6) can be obtained, which is given by

$$G(\eta)=C_0+C_1\exp(-\lambda\eta) \tag{20}$$

where C_0 and C_1 are the integration constants.

Finally, substituting Equations (8), (14), (15) and (20) into Equations (3) and (4), we get analytical traveling wave solutions to Equations (1) and (2) as follows:

$$q(x,t)=2\left\{-\frac{(b_2\omega_2\kappa_2-a_2\kappa_2^2-\omega_2)}{c_2}\times\frac{C_1\exp\left[-2\sqrt{\frac{a_l\kappa_l^2+\omega_l-b_l\omega_l\kappa_l}{a_l-b_lv}}(x-vt)\right]}{C_0+C_1\exp\left[-2\sqrt{\frac{a_l\kappa_l^2+\omega_l-b_l\omega_l\kappa_l}{a_l-b_lv}}(x-vt)\right]}\right\}^{\frac{1}{2}}\times\exp[i(-\kappa_1x+\omega_1t+\delta_1)] \tag{21}$$

$$r(x,t)=2\left\{-\frac{(b_1\omega_1\kappa_1-a_1\kappa_1^2-\omega_1)}{c_1}\times\frac{C_1\exp\left[-2\sqrt{\frac{a_l\kappa_l^2+\omega_l-b_l\omega_l\kappa_l}{a_l-b_lv}}(x-vt)\right]}{C_0+C_1\exp\left[-2\sqrt{\frac{a_l\kappa_l^2+\omega_l-b_l\omega_l\kappa_l}{a_l-b_lv}}(x-vt)\right]}\right\}^{\frac{1}{2}}\times\exp[i(-\kappa_2x+\omega_2t+\delta_2)] \tag{22}$$

where the speed of traveling wave solutions is given by Equation (6). The constraint conditions for the existence of those traveling waves (21) and (22) are given by Equations (7) and (17)-(19).

Remark 1: If we take $C_0=C_1$, traveling wave solutions (21) and (22) will become to the Thirring dark solitons

$$q(x,t)=\left\{-\frac{2(b_2\omega_2\kappa_2-a_2\kappa_2^2-\omega_2)}{c_2}\times\left(1-\tanh\left[\sqrt{\frac{a_l\kappa_l^2+\omega_l-b_l\omega_l\kappa_l}{a_l-b_lv}}(x-vt)\right]\right)\right\}^{\frac{1}{2}}\times\exp[i(-\kappa_1x+\omega_1t+\delta_1)] \tag{23}$$

$$r(x,t)=\left\{-\frac{2(b_1\omega_1\kappa_1-a_1\kappa_1^2-\omega_1)}{c_1}\times\left(1-\tanh\left[\sqrt{\frac{a_l\kappa_l^2+\omega_l-b_l\omega_l\kappa_l}{a_l-b_lv}}(x-vt)\right]\right)\right\}^{\frac{1}{2}}\times\exp[i(-\kappa_2x+\omega_2t+\delta_2)] \tag{24}$$

Remark 2: If we take $C_0=-C_1$, traveling wave solutions (21) and (22) will reduce to the Thirring singular solitons

$$q(x,t)=\left\{-\frac{2(b_2\omega_2\kappa_2-a_2\kappa_2^2-\omega_2)}{c_2}\times\left(1-\coth\left[\sqrt{\frac{a_l\kappa_l^2+\omega_l-b_l\omega_l\kappa_l}{a_l-b_lv}}(x-vt)\right]\right)\right\}^{\frac{1}{2}}\times\exp[i(-\kappa_1x+\omega_1t+\delta_1)] \tag{25}$$

$$r(x,t)=\left\{-\frac{2(b_1\omega_1\kappa_1-a_1\kappa_1^2-\omega_1)}{c_1}\times\left(1-\coth\left[\sqrt{\frac{a_l\kappa_l^2+\omega_l-b_l\omega_l\kappa_l}{a_l-b_lv}}(x-vt)\right]\right)\right\}^{\frac{1}{2}}\times\exp[i(-\kappa_2x+\omega_2t+\delta_2)] \tag{26}$$

4. Conclusion

The vector coupled NLSE (1) and (2) that describes the propagation of Thirring optical solitons through birefringent fibers is investigated analytically. The cubic-quintic nonlinearity and STD are taken into account. Via the G'/G-expansion approach, exact traveling wave solutions are obtained, which are given by Equations (21) and (22). Finally, we find that those traveling wave solutions will degenerate to Thirring dark and singular solitons by choosing appropriate parameter values of C_1 and C_2. It should be noted that these results were not reported in our recent work.

In future studies, we will use the complex envelope function ansatz to study the dynamics of Thirring combo-solitons in birefringent fibers. The Kerr law and parabolic law will be discussed. These results will be reported later.

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