Evolution of anomalous vortex beam with different polarization states from conical diffraction to birefringence

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In order to discuss the evolution of anomalous vortex beams with different polarization states from conical diffraction to birefringence, the influences of the propagation distance and propagation direction of the incident beam on the propagation characteristics are analyzed based on the conical diffraction theory. The conical diffraction characteristics of elliptical anomalous vortex beams with different azimuth is also demonstrated. The results show the field intensity distribution of the circularly polarized anomalous vortex beam is more sensitive to the inclination factor than the circularly polarized Gaussian beam during the evolution from conical diffraction to birefringence. The light intensity distribution properties of elliptically polarized light is between circularly polarized and linearly polarized light. When an elliptically polarized beam passes through a biaxial crystal, the light field changes with the phase difference between the two perpendicular linearly polarized light of elliptically polarized light. It has application prospect in optical capture.

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1. Introduction

Conical diffraction occurs when the beam passes through a biaxial crystal at a certain angle. Conical diffraction can be divided into internal conical diffraction and external conical diffraction. It is called internal conical diffraction when the incident beam is parallel to an optical axis of biaxial crystal. The beam is a cone in the crystal and a hollow light column emitted from the crystal. External conical diffraction is that the incident beam is converging beam and the outgoing beam is conical.

The phenomenon of conical diffraction was predicted by Hamilton in 1832 [1]. In 1833, Lloyd reported experiments about the internal and external conical diffraction of sunlight. Thereafter, some interesting experimental phenomena of conical diffraction including Poggendorff ring and Raman spots were found [2]. In 1972, D. L. Portigal described the influence of chiral characteristics on conical diffraction [3]. In 1978, A. M. Belskii and A. P. Khapalyuk proposed the theory of internal and external conical diffraction diffraction, but the process was still relatively complicated [4]. In addition, biaxial crystals have effects on optical vortices. V. N. Belyi presents a new general method for generating first - order, second - order and high - order optical vortex Bessel beams using uniaxial and biaxial crystals [5]. With the Berry's improvement, the development of conical diffraction was promoted greatly [6]. In 2007, Y. P. Mikhailichenko demonstrated a high transmission optical scheme for conical diffraction with large dimensions [7]. In 2010, M. V. Berry discussed the cascaded problem of conical diffraction [8]. V. Peet reported the internal conical diffraction experiment of circularly polarized Laguerre-Gaussian vortex beam [9]. In 2012, D. P. Odwyer proposed a method to capture particles with conical diffraction of linearly polarized Gauss beams cascaded by biaxial crystals [10]. V. N. Belyi demonstrated the dynamics of the spin-to-orbit angular momentum conversion of a zero-order and high-order circularly polarized Bessel beam when such beams propagated along uniaxial and biaxial crystal optical axes [11]. In 2015, A. Peinado investigated the phase structure of the conical diffraction ring with an interferometer [12]. In the same year, R. T. Darcy observed the predicted low-intensity interference pattern [13]. In 2017, E. V. Kuznetsov reported the conical diffration in magneto-optic biaxial crystals [14]. C. N. Aelexeyev reported the unstability of fractional-order vortex beam under the condition of conical diffraction [15]. A. Brenier demonstrated the chiral characteristics and dichroism of conical diffraction [16]. In 2019, A. Brenier investigate the chiral properties, output light intensity and phase of conical diffraction of $Bi_2ZnOB_2O_6$ crystal [17].

The anomalous vortex beam carries the helical phase factor and has the application prospect in the fields of optical capture, biomedicine and micromechanics [18-19]. In 2013, Y. Yang demonstrated the generation method of anomalous vortex beam and the propagation characteristics of such beam in free space. It was found that the anomalous vortex beam will become an elegant Laguerre-Gaussian beam when such beam was propagation in the far field in free space [20]. The propagation properties of anomalous vortex beams through paraxial optical systems was investigated and an analytical expression of the M^2 factor of anomalous vortex beams was also given [21]. In 2015, D. Zhang reported the radiation force of anomalous vortex beams based on the diffraction theory and paraxial approximation [22]. In 2018, M. Zhang reported the tight focusing properties of anomalous vortex beams passing through an optical system with high numerical aperture [23]. However, the investigation on the conical diffraction of anomalous vortex beams was lacked, it is necessary to discuss the conical diffraction of anomalous vortex beams with different polarization states through biaxial crystals.

2. Conical diffraction theory

The electric field distribution of anomalous vortex beam is as follows [20]:

$$E_{n,m}(r,\theta)=E_0\left(\frac{r}{\omega}\right)^{2n+|m|}\exp\left(-\frac{r^2}{\omega^2}\right)\exp(im\theta) \tag{1}$$

Here, E_0 is the amplitude of light, n is the order of anomalous vortex beam, m is the topological charge of the beam. ω is the beam radius. r and θ are the parameters of the polar coordinate. If n=0 and m≠0 in Eq.(1), it become the field distribution of the ordinary vortex beam.

According to the conical diffraction theory [9][24], the light field of a parallel beam passing through a biaxial crystal with the paraxial approximation can be represented by **D** vector:

$$\mathbf{D}=\left[B_C\mathbf{I}+B_S+B_G\begin{bmatrix}0 & -i\\ i & 0\end{bmatrix}\right]\mathbf{d}_0 \tag{2}$$

where $\mathbf{d}_0$ is the Jones matrix, I is the identity matrix. The forms of B_C, B_S and B_G are as follows,

$$B_C(r,Z)=\int_0^{\infty}\int_0^{2\pi}Pa(P)e^{-\frac{1}{2}ikZP^2}\cos\left(\sqrt{k^2R_0^2P^2+k^2G^2}\right) i^m e^{im\phi_P}e^{-ik\Pr\cos(\phi_P-\varphi)}dPd\phi_P \tag{3}$$

$$B_S(r,Z)=\int_0^{\infty}\int_0^{2\pi}Pa(P)e^{-\frac{1}{2}ikZP^2}\frac{kR_0P}{\sqrt{k^2R_0^2P^2+k^2G^2}} \sin\left(\sqrt{k^2R_0^2P^2+k^2G^2}\right)i^m e^{im\phi_P}e^{ik\Pr\cos(\phi_P-\varphi)} \begin{bmatrix}\cos\phi_P & \sin\phi_P\\ \sin\phi_P & -\cos\phi_P\end{bmatrix}dPd\phi_P \tag{4}$$

$$B_G(r,Z)=-i\int_0^{\infty}\int_0^{2\pi}Pa(P)e^{-\frac{1}{2}ikZP^2}\frac{kG}{\sqrt{k^2R_0^2P^2+k^2G^2}} \sin\left(\sqrt{k^2R_0^2P^2+k^2G^2}\right)i^m e^{im\phi_P}e^{ik\Pr\cos(\phi_P-\varphi)}dPd\phi_P \tag{5}$$

Here, φ is the azimuth of θ after Fourier integration, ϕ_P is the variable corresponding to φ after another Fourier Bessel integral. The evolution from conical diffraction to birefringence can be analyzed according to reference [25]. The inclination degree of the beam can be described with the inclination factor u. $a(P)$ is the Fourier transform of the incident parallel beam and the P of this formula is a vector. The k is the wave number. G is related to the chirality of the crystal. R_0 is the radius of the ring of conical diffraction. Half cone angle of biaxial crystal and length of crystal are A and l, respectively. The product A and l is R_0.

3. Evolution of circular and linearly polarized anomalous vortex beams

In order to analyze the intensity distribution and variation law of the light field when the beam passes through the biaxial crystal at a certain angle relative to the optical axis, numerical simulation was carried out for different u and Z. Z is zero at the focal image plane. The parameters used in the simulation are as follows: λ = 632.8 nm, R_0 = 240 μm, ω = 25μm, n = 1, m = 1. The polarization states of the beam are circular polarization and linear polarization.

The Fig. 1 shows the field intensity distribution of the incident right circularly polarized light when Z is zero and u is 0, 10, 30 and 50, respectively. It was found that the light field distribution on the focal image plane is polycyclic structure with central symmetry when the incident anomalous vortex beam is right circularly polarized light. With the increase of the inclination factor u, the conical diffraction ring evolves to birefringence gradually. The intensity distribution of initial polycyclic structure collapsed into a symmetric arc with bicyclic structure gradually, or even a single short arc structure. Finally, the conical diffraction ring evolved into two bright spots. Fig. 2 shows the light field intensity distribution when the incident anomalous vortex beam is linearly polarized light with the direction of horizontal polarization. It can be found that the light field intensity in other directions are suppressed and the intensity distribution transforms to a single spot with the increase of inclination angle. When the polarized direction of the incident linear is horizontal, the brightest region of the ring on the focal image plane appears on the right and the corresponding dark region appears on the left. When the incident beam is vertically polarized, the brightest region of the ring appears on the left and the dark region appears on the right. For circularly polarized light, two bright spots are linearly polarized light which polarization directions are perpendicular to each other.

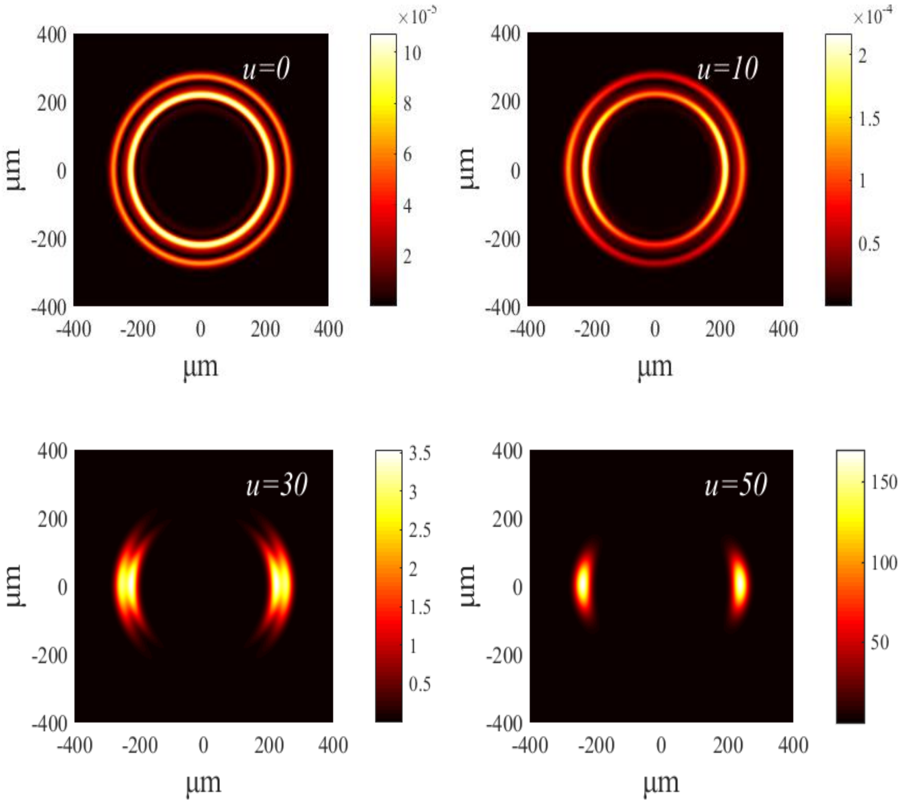

Fig. 1. The field intensity distribution of right circularly polarized light on the focal image plane (color online)

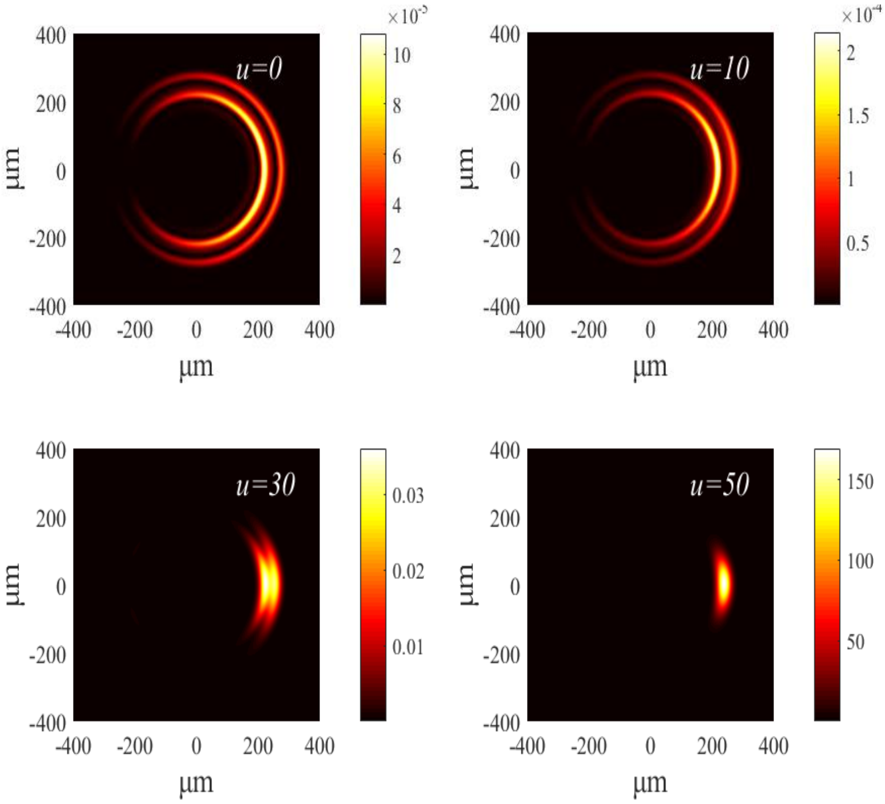

Fig. 2. The field intensity distribution of linearly polarized light on the focal image plane (color online)

The Fig. 3 shows the light field intensity distribution with different *u* when the observed plane is 10 mm from the focal image plane. The distribution also changes from the central symmetric structure when the incident anomalous vortex beam is right circularly polarized light. With the increase of *u*, the intensity evolves towards birefringence. Compared with the two spots with the same intensity on the focal image plane, the intensity of the two spots is not the same when *u* is 50. The low intensity region diffuses to the edge, while the high intensity light field moves to the center. Compared with Gauss beam, the intensity of anomalous vortex beam is more sensitive with the increase of *u*. By taking the maximum intensity 1/*e* as the reference standard, the ratio of the light field intensity including the 1/e of maximum intensity to the total intensity is quantitatively analyzed. For example, the intensity ratio of the right circularly polarized anomalous vortex beam is 21.68%, 6.23%, 0.46% and 0.32% when u is 0, 10, 20 and 30, respectively. Under the same conditions, the corresponding intensity ratio of Gauss beam is 18.33%, 11.75%, 4.48% and 2.85%. It can be seen that the conical diffraction experiment of anomalous vortex beam requires higher precision to adjust the crystal position than the Gauss beam. Fig. 4 shows the intensity distribution of the linearly polarized anomalous vortex beam with horizontal polarization direction when observed plane is also 10 mm from the focal image plane. As the beam is far away from the focal image plane and has a slight inclination angle, the horizontal polarized beam is concentrated in the right region of light field. With the inclination angle increasing, the intensity distribution becomes more and more concentrated, and it becomes a bright spot when *u is* 30.

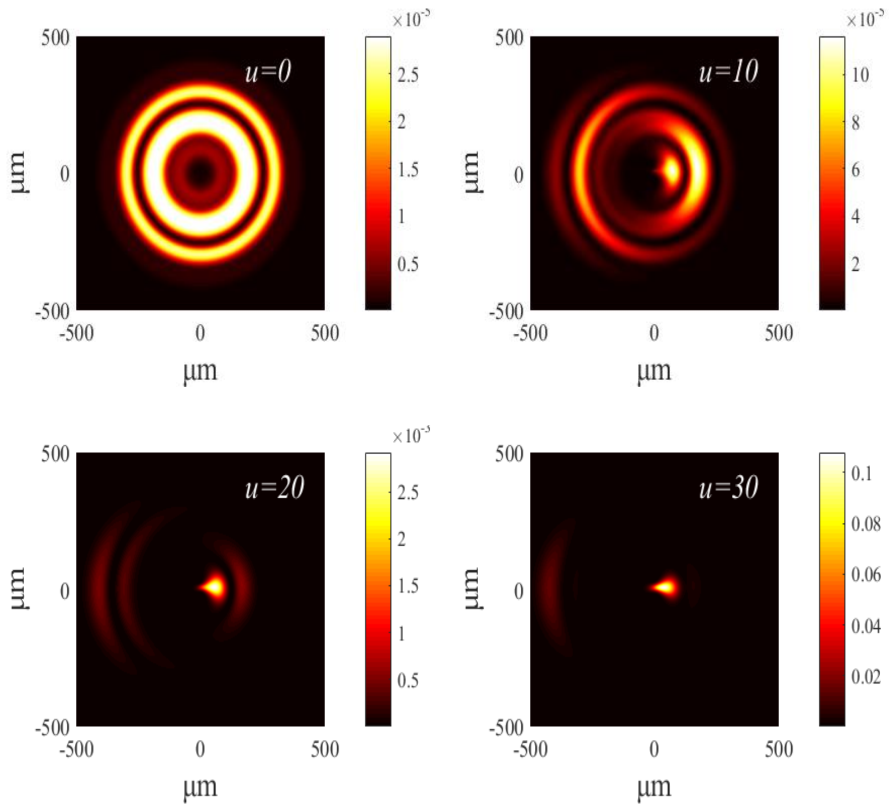

Fig. 3. The intensity distribution of right circularly polarized light when Z is10 mm (color online)

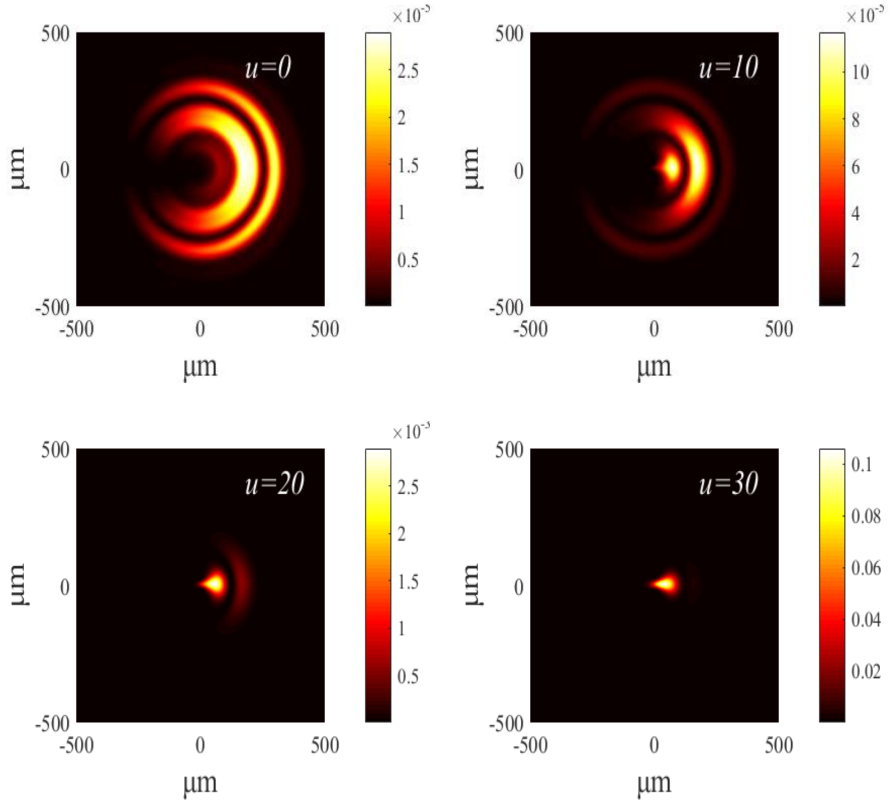

Fig. 4. The intensity distribution of linearly polarized light when Z is10 mm (color online)

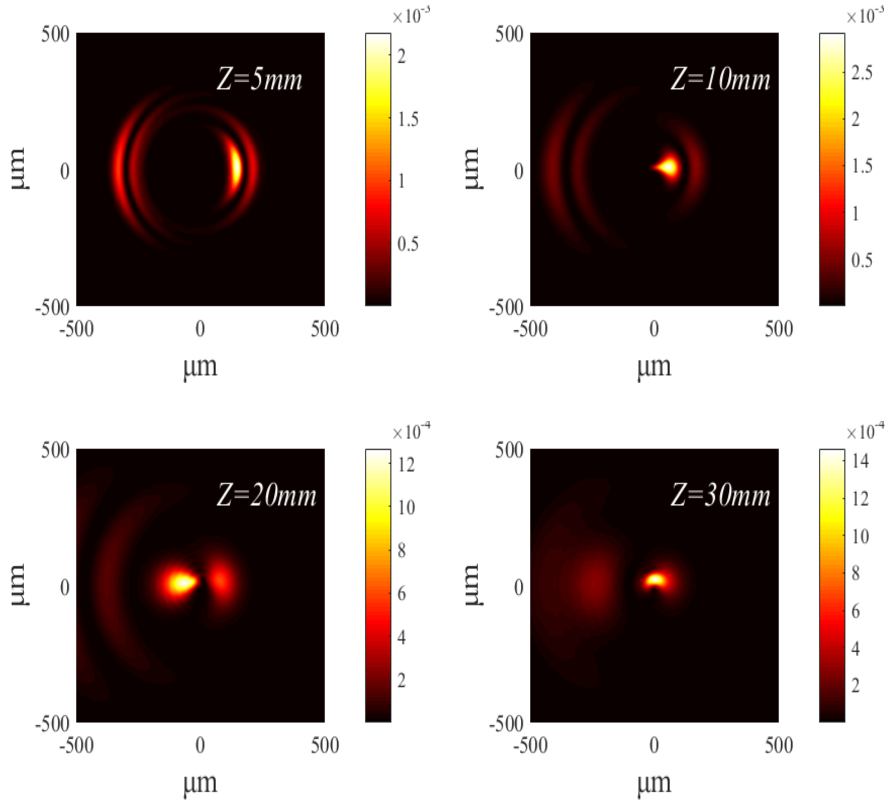

Fig. 5. The intensity distribution of the circularly polarized light when u is 20 (color online)

Fig. 5 shows the light field intensity distribution of the incident anomalous vortex beam with circularly polarized light when *u* is 20 and *Z* is 5 mm, 10 mm, 20 mm and 30 mm, respectively. When z is 10 mm, the bright spot on the right side deviates to the left. When z is 20 mm and 30 mm, two bright spots in the center changes in opposite directions. The light field at one end of the edge spreads further and the intensity becomes weaker. Fig. 6 shows the evolution of light field intensity when the incident anomalous vortex beam is linearly polarized. When z is from 10 mm to 20 mm, two bright spots move from right to left. Here, a conical diffraction interference pattern can be observed. The investigation of the influence of inclination angle and propagation distance on the intensity distribution of conical diffraction is very helpful for the observation in conical diffraction experiments.

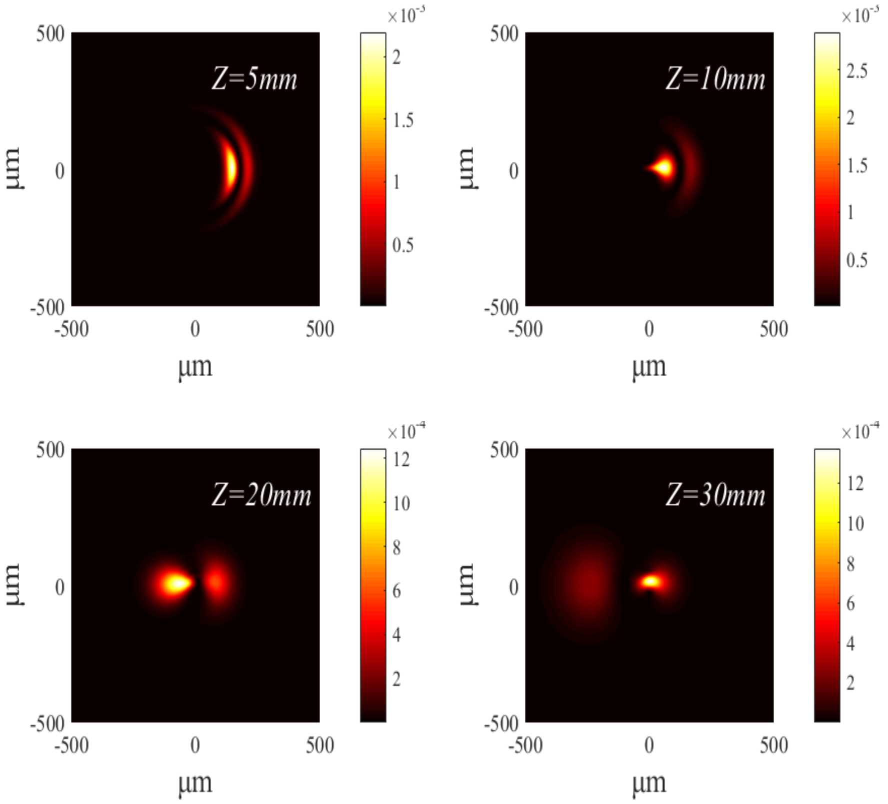

Fig. 6. The intensity distribution of the linearly polarized light when u is 20 (color online)

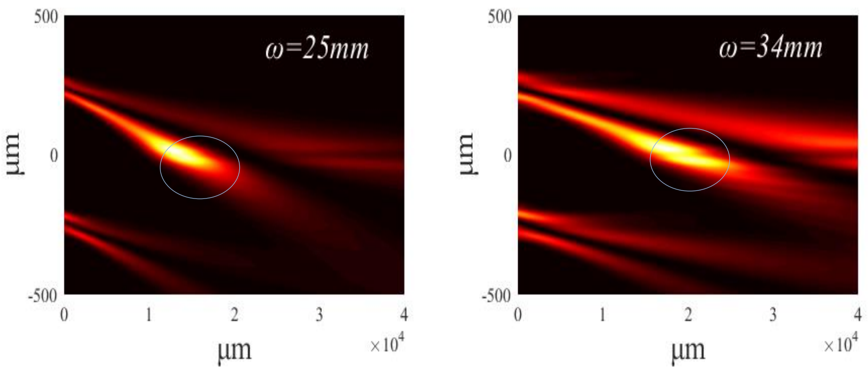

Fig. 7. The intensity distribution on the side when u is 20 (color online)

Fig. 7 shows the field intensity distribution on the side of the crystal when the incident anomalous vortex beam is circularly polarized. It is easy to observe that the tilted beam has a turning point where the propagation direction is changed. Near this turning point showed with an elliptical ring in Fig. 7, the beam no longer propagates in the same direction. The interference pattern of conical diffraction can be observed. The inclination angle of the beam is also related to the waist radius of the beam. It was found that clear interference pattern can be observed by selecting the appropriate beam waist radius and adjusting the inclination angle.

4. Evolution of elliptically polarized anomalous vortex beams

When an elliptically polarized anomalous vortex beam passes through a crystal without chiral characteristics, the field intensity of the conical diffraction on the focal image plane changes with the phase difference between the two perpendicular linearly polarized light of elliptically polarized light. When the long and short axes of elliptically polarized beams are exchanged, the light field intensity will also change. The parameters of simulation are follows: λ = 632.8 nm, R_0 = 240 μm, ω = 25 μm. The amplitude ratio of horizontally polarized light to vertically polarized light is two. Fig. 8 shows the field intensity distribution of conical diffraction of elliptically polarized anomalous vortex beam with phase differences. The elliptically polarized beam was synthesized by linearly polarized light with two polarization directions perpendicular to each other at different phase differences.

It is linearly polarized light if the phase difference is 0 or π. When the phase difference is $\pi/2$ and $3\pi/2$, positive elliptically polarized light is generated. It is oblique elliptically polarized light with the other phase differences. The weak intensity region of the conical diffracted light field is also related to the phase difference. When the incident beam is positive elliptically polarized light, the intensity of the weak region in the light field intensity reaches maximum value. When the phase difference is zero and π, the light field intensity of weak intensity region is zero. Numerical simulation shows that the high region of field intensity is always distributed on the right side and the light field intensity also changes with the phase difference when the amplitude of horizontal polarization is higher. When the vertical polarization coefficient is higher, the high region of field intensity is always on the left side.

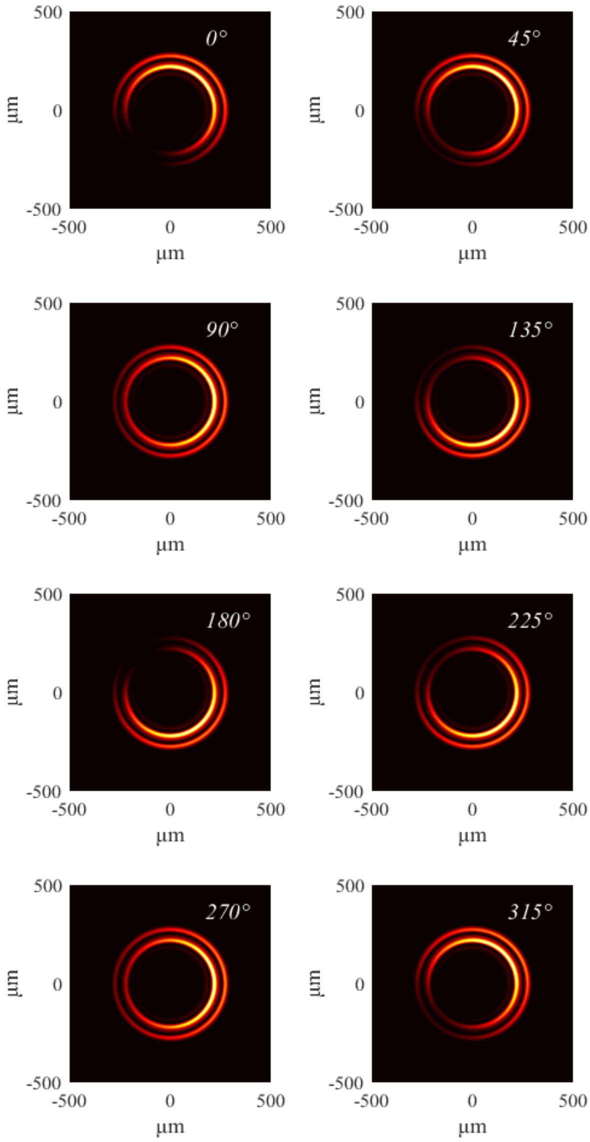

Fig. 8. Intensity distribution of elliptic polarized light on the focal image plane with different phase differences (color online)

Fig. 9 shows the intensity distribution of the conical diffraction when the incident beam is positive elliptic polarization. Because the length of the ellipse axis is different, the intensity distribution is obviously different on the focal image plane. In particular, when the tilt angle increases, the intensity is concentrated at both ends and the distribution of intensity is also directly related to the difference in the amplitude of horizontal polarization and vertical polarization. In addition, it is found that the intensity distribution of elliptically polarized light is between circular polarization and linear polarization at different propagation distances and inclination angles. From the intensity distribution of optical field, it can be seen that conical diffraction of elliptically polarized light may play a role of filtering selection in optical capture.

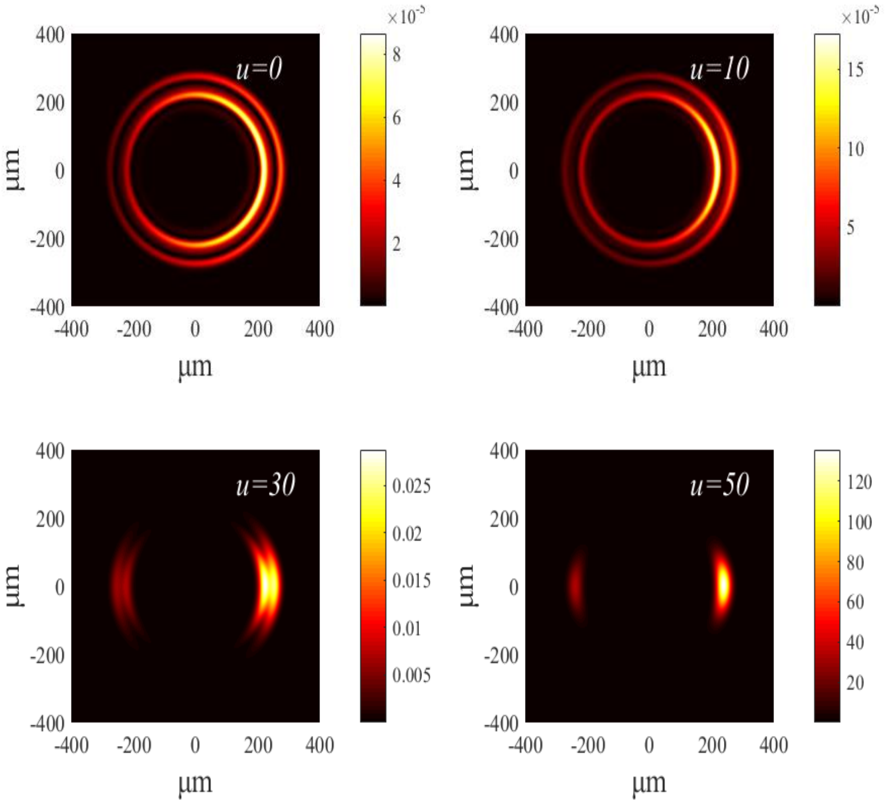

Fig. 9. Intensity distribution of elliptically polarized light field on the focal image plane (color online)

5. Conclusion

Based on the theory of conical diffraction, the evolution from conical diffraction to birefringence of circularly polarized, elliptically polarized and linearly polarized anomalous vortex beams is analyzed in this paper. The influence of propagation distance and inclination degree relative to optical axis on intensity distribution are discussed. The conical diffraction characteristics of elliptically polarized anomalous vortex beams which ratio of long axis to short axis is two and the change of azimuth of long axis are discussed. The results show the intensity of circularly polarized anomalous vortex beams is more sensitive to the inclination angle than the circularly Gauss beams with the same inclination. For observing the conical diffraction of anomalous vortex beams, it is necessary to have higher precision to adjust the crystal position. When the inclination angle between the incident anomalous vortex beam and the optical axis increases, the circularly polarized light on the focal image plane becomes two bright spots and the linearly polarized light becomes one bright spot. The elliptically polarized light falls between the two. In the process of oblique propagation, the field intensity changes more complicated with the increase of distance. When the amplitude of horizontally polarized light of elliptically polarized light is larger, the strong intensity region on the focal image plane is always on the right. When the vertical polarization amplitude is larger, the strong intensity region is always distributed on the left. With the change of the phase difference angle, the strong intensity region will also move. It is obvious that the field intensity distribution of conical diffraction was effected by the amplitude difference of two linearly polarized light with perpendiculars directions. In addition, the conical diffraction of elliptic polarization may have the prospect in optical capture for filtering selected particles.

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