

Enlargement of omni-directional reflection range by cascading photonic crystals

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In the present communication, we study one-dimensional photonic crystal in which alternate layers are made up of materials like Na_3AlF_6 and Ge. For our three different structures, namely PC1, PC2 and PC3 in which the filling factors are 0.333, 1.083 and 2.125, respectively, are considered first. Another structure PC4 which is a combination of PC1/PC2/PC3 has also been considered to study the effect of cascading. Using transfer matrix method, the reflectivity of each photonic crystal is plotted as a function of normalized frequency and omni directional photonic band gaps (PBGs) are identified. We found that large frequency range of omni-directional PBG can be obtained from $\text{Na}_3\text{AlF}_6/\text{Ge}$ system by cascading photonic band gap structures.

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1. Introduction

Recently, photonic crystals with forbidden photonic bands, which are known as photonic band gap (PBG) materials, have drawn considerable attention of many investigators for the study of their fundamental physical properties as well as for their potential applications in photonic devices [1-3]. Photonic band gap (PBG) materials may be designed in one-, two- and three-dimensions. But one-dimensional PBG materials are given more emphasis because such structures can be fabricated more easily at any wavelength scale and their analytical study and numerical computations are simpler. Also, their fabrication is compatible with the modern integrated circuit technology, particularly in the optical region of electromagnetic spectrum. Ideally, omni-directional reflector is a mirror having cent percent reflectivity at any angle of incidence for both TE and TM polarized electromagnetic waves for a particular wavelength or a range of wavelength. Thus, periodic structures have become important structures in photonics [4-8].

In 1998, Fink et al. [9, 10] reported for the first time that one-dimensional dielectric lattice displays total omni-directional reflection for incident light under certain conditions. They fabricated a stack of nine alternate polystyrene/tellurium layers having a thickness of a few micro-metres and demonstrated omni directional reflection over the wavelength range from $10\mu\text{m}$ to $15\mu\text{m}$. Further works by various researchers found many interesting results. Gallas et al. [11] reported the annealing effect in the Si/SiO_2 PBG based omni-directional reflectors. Chen et al. [12] fabricated six bi-layers of SiO_2 and TiO_2 quarter

wave films using the sol gel method and found an omni-directional photonic band gap of about 70nm in near infrared region. Chigrin et al. [13, 14] fabricated a lattice consisting of 19 layers of $\text{Na}_3\text{AlF}_6/\text{ZnSe}$ and found that an omni-directional photonic band gap exists in the spectral range 604.3nm to 638.4nm. Ojha et al. [15,16] theoretically studied omni-directional high reflector for infrared wavelengths, large omni-directional reflection using combination of periodic and Fibonacci structures respectively.

In the present communication, we study the omni-directional photonic band gaps in one-dimensional photonic crystals made up of $\text{Na}_3\text{AlF}_6/\text{Ge}$ periodic multi-layers and a cascaded structure of three such structures having different filling factors.

2. Theoretical analysis

Consider electromagnetic wave propagation in one-dimensional (1D) system that consists of alternate layers of dielectric materials with different refractive indices. We consider the system to be isotropic and nonmagnetic. The refractive index profile of the PBG is given by

$$n(x) = \begin{cases} n_L, & 0 < x < d_L \\ n_H, & d_L < x < d_H \end{cases} \quad \text{with } n(x) = n(x+d) \quad (1)$$

where d_L and d_H are the thicknesses of the layers, and $d = d_L + d_H$ is the lattice period. Here the x-axis is taken along the direction perpendicular to the layers. To calculate the

reflectivity of this PBG structure, we have used transfer matrix method (TMM) [17].

For a plane wave propagating in the z-direction, the electric field can be written as

$$E = E(x) \cdot \exp[i(\omega t - \beta x)] \quad (2)$$

where, β is the x-component of the wave vector and ω is angular frequency.

The electric field distribution within each layer can be expressed as the sum of an incident plane wave and a reflected plane wave. The incident light can be either transverse electric (TE) (electric field is perpendicular to the plane of incidence) or transverse magnetic (TM) mode (magnetic field is perpendicular to plane of incidence). The amplitudes of the fields in the first layer of N^{th} unit cell and $(N-1)^{\text{th}}$ unit cell are related by a matrix ,

$$\begin{pmatrix} A_{N-1} \\ B_{N-1} \end{pmatrix} = M_N \begin{pmatrix} A_N \\ B_N \end{pmatrix} \quad (3)$$

where, $M_N = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}$ is the transfer matrix, A_N and B_N

respectively are the coefficients of the right and left hand side propagating waves in the first layer of the N^{th} unit cell. Considering the structure of N period of layers with alternate refractive indices n_L and n_H having thicknesses d_L and d_H , the coefficients of propagating waves in right and left side of the multilayer structure are related to each other. Thus multiplying transfer matrices of each cell we get e relation

$$\begin{pmatrix} A_0 \\ B_0 \end{pmatrix} = M_1 M_2 M_3 \dots M_N \begin{pmatrix} A_N \\ B_N \end{pmatrix} \quad (4)$$

The coefficient of reflection is given by

$$r_N = \left(\frac{B_0}{A_0} \right)_{B_N=0} \quad (5)$$

The reflectance of the structure (for TE and TM modes) is related to the coefficient of reflectivity as [18],

$$R_N = |r_N|^2 \quad (6)$$

For an infinitely extended Photonic crystal, according to Bloch's theorem, the Bloch wave vector K is obtained by solving the eigen-value problem

$$M_N \begin{pmatrix} A_N \\ B_N \end{pmatrix} = e^{iKd} \begin{pmatrix} A_N \\ B_N \end{pmatrix} \quad (7)$$

The dispersion relation between K and ω for Bloch waves, can be expressed as

$$K(\omega) = \frac{1}{d} \cos^{-1} \left[\frac{1}{2} (M_{11} + M_{22}) \right] \quad (8)$$

In the next section, using equation (6), we will compute reflectivity of the structure considered here for both TE and TM modes of polarization.

The criterion for the existence of omnidirectional reflection (ODR) is that there are no propagating modes that can couple the incident wave. From Snell's law $n_i \sin \theta_i = n_j \sin \theta_j$, where $j=1,2$; we can see that the refracted angle θ_j is restricted to a certain range, where n_i and n_j ($j=1,2$) are the refractive indices of incident medium, first dielectric layer ($j=1$) and second dielectric layer ($j=2$). If the maximum refracted angle is smaller than the internal Brewster's angle $\theta_j^B = \tan^{-1}(n_2/n_1)$, the incident wave from the outside can not couple to the Brewster window, leading to the total reflection for all incident angles.

3. Results and discussion

For the sake of numerical computation of the reflectivity of one-dimensional photonic crystals (PCs) considered in the present work, we choose Na_3AlF_6 ($n_1=1.34$) as low refractive index material and Ge ($n_2=4.2$) as high refractive index material.

We consider the four PCs having 10 lattice periods each, as follows

PC1: $n_1=1.34$, $n_2=4.2$; $d_1=0.75d$, $d_2=0.25d$ (filling factor=0.333)

PC2: $n_1=1.34$, $n_2=4.2$; $d_1=0.52d$, $d_2=0.48d$ (filling factor=1.083)

PC3: $n_1=1.34$, $n_2=4.2$; $d_1=0.32d$, $d_2=0.68d$ (filling factor=2.125)

PC4: cascaded structure of PC1, PC2 and PC3 [PC1(10 pairs)/PC2 (10 pairs)/PC3 (10 pairs)]

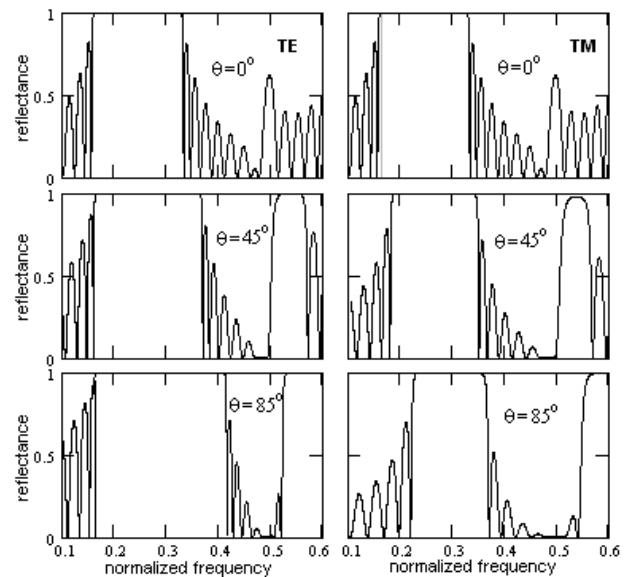


Fig. 1. Reflectance vs normalized frequency for PC1.

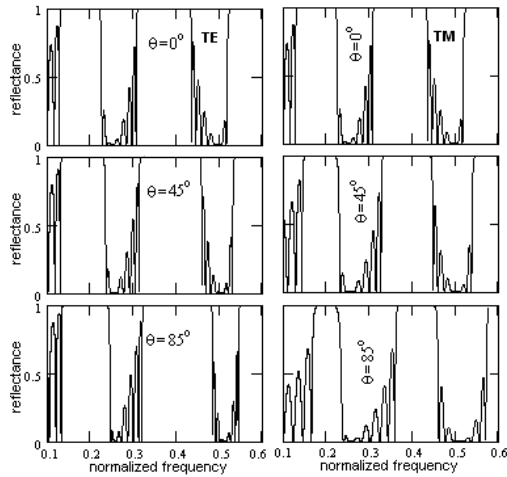


Fig. 2. Reflectance vs normalized frequency for PC2.

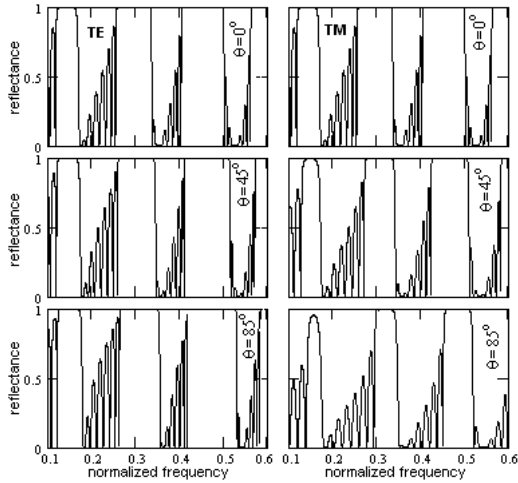


Fig. 3. Reflectance vs normalized frequency for PC3.

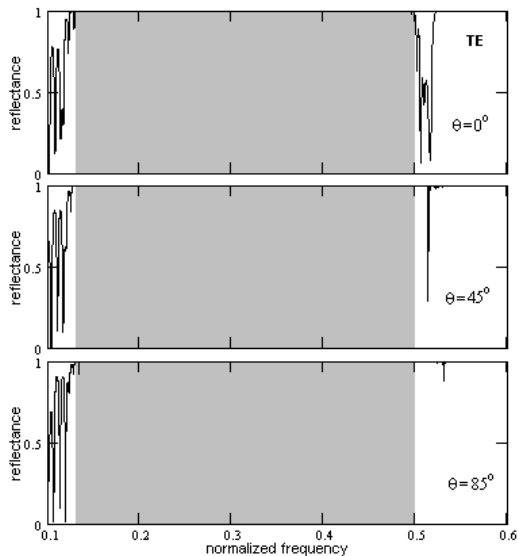


Fig. 4. Omni directional gap of PC4 for TE polarization.

The reflectance spectra (reflectance vs normalized frequency $\omega d/2\pi c$) of PC1, PC2 and PC3 are shown in Figs. 1, 2 and 3 respectively. It can be seen from these figures that the edges of each of the photonic band gaps (PBGs) for both polarizations increase with increasing angle of incidence. Hence the omni-directional PBGs for both TE and TM polarizations are defined by the region from the upper edge of a photonic band gap at the incident angle 85° and the lower edge of that photonic band gap at incident angle 0° . It is noticeable that for each photonic band gap, omni-directional PBG for the TM polarization is completely located within that for the TE polarization. Therefore, for each of the photonic band gap, the overall omni-directional PBG for PC1, PC2 and PC3 is no more than the omni-directional PBG for the TM polarization.

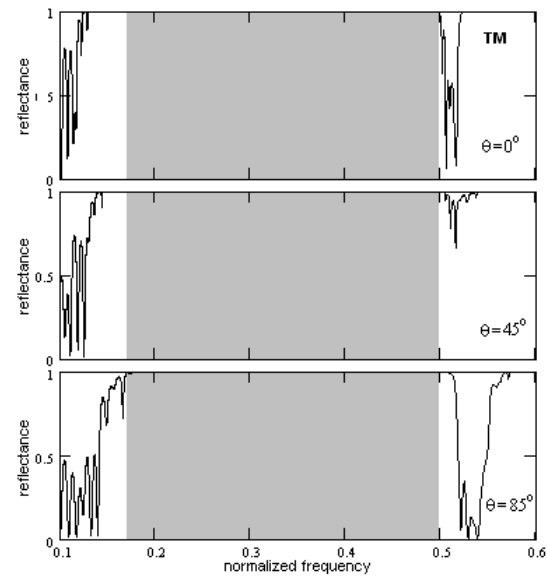


Fig. 5. Omni directional gap of PC4 for TM polarization.

The structure PC4 is a cascaded structure of PC1, PC2 and PC3. Figs. 4 and 5 show the variation of reflectance versus normalized frequency showing photonic band gaps for the TE and the TM modes for different angles of incidence. It is observed that the omni-directional (ODR) photonic band gaps are very large. Also, from these figures it is clear that, for normal incidence TE and TM polarizations degenerate. Consequently, the overall ODR photonic band gap covers a wide range in the normalized frequency scale. Thus, the frequency range of total reflection in some sense is enhanced to a great extent for all incident angles.

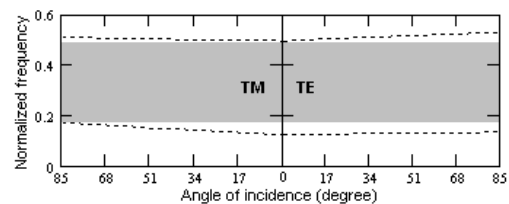


Fig. 6. Photonic band structure of PC4.

Photonic band structure in terms of frequency and incident angle for the PC4, is shown in Fig. 6. The ODR photonic band gap for the TM polarization is no more than the overall omni-directional PBG for any polarization. The shaded region between two dotted lines shows the total omnidirectional photonic band gap for both polarizations. From the analysis of Figs. 5, 6 and 7 and the results summarized in Table 1, it is clear that, the total omnidirectional reflection range for the PC4 is from $0.170\omega d/2\pi c$ to $0.496\omega d/2\pi c$ with total ODR bandwidth of $0.326\omega d/2\pi c$. The bandwidth is 1.5 times more from the bandwidth of overlapped structure of two photonic crystals [19].

Table 1. 100% reflection ranges for PC4.

Incident angle	TE Polarization		TM Polarization	
	Ranges of 100% reflection ($\omega d/2\pi c$)	Band width	Ranges of 100% reflection ($\omega d/2\pi c$)	Band width
0°	0.122-0.496	0.374	0.122-0.496	0.374
45°	0.130-0.510	0.380	0.136-0.504	0.368
85°	0.133-0.529	0.396	0.170-0.509	0.339

4. Conclusions

From the aforementioned discussions, the total reflection frequency range is substantially enlarged for all incident angles and for both TM and TE polarizations. Subsequently, the omnidirectional total reflection frequency range for any polarization is enlarged. It is obvious that one can use more 1D PCs to form multiple photonic heterostructures to get a very wide omni-directional total reflection frequency range as desired provided that the directional PBGs of the adjacent 1D PCs at any incident angle cascade each other in tandem.

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