

# Electromagnetic scattering from perfect electromagnetic conductor cylinders placed in magnetized plasma medium

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A generalized problem of electromagnetic scattering from perfect electromagnetic conductor (PEMC) circular cylinder hosted by magnetized plasma is investigated. The physical modeling of the scattering problem is done in the frame work of extended classical scattering theory. The cylindrical wave excitation is considered for both types of polarization i.e., TE and TM. The influence of plasma parameters and the effect of external magnetic field strength on the scattering amplitude is discussed and analyzed. The influence of external magnetic field strength on scattering amplitude is, in accordance with the literature. Further, the numerical results are compared with published literature and good agreement is found. It is concluded that scattering amplitude can be controlled with the increase of magnetic field strength, the radar cross section is decreased and vice versa. Moreover, stealth capability can be increased by the appropriate choice of perfect cylinder.

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## 1. Introduction

The electromagnetic interaction with plasma medium is the growing interest of many researchers and scientists due to its numerous practical applications in the field of communication, defense technology, aerospace sciences and optical sensing and many more [1-2]. Plasma can be modeled as a highly ionized state of a gas; mixture of electrons, ions and neutral particles. The ionosphere, shock wave and flames are some examples of plasma. Conventionally researchers and plasma physicists categorized the plasma in two classes one is the un-magnetized plasma and second is the magnetized plasma. The un-magnetized plasma is generally considered as an isotropic medium, under the influence of ambient (external) magnetic field the un-magnetized plasma converted into the magnetized plasma which is considered as an anisotropic medium and generally characterized by the permittivity tensor [3-4].

Recently, the plasma coated conducting objects have magnetized many researchers and engineers due to its scattering amplitude reducing capability. First time the electromagnetic interaction i.e., transmission, reflection and absorption with un-magnetized plasma was studied by Vidmar [5]. The propagation, reflection and absorption of electromagnetic waves through un-magnetized plasma

theoretically investigated and reported that plasma has ability to absorb maximum part of the incident electromagnetic energy and can be used for the protection of different targets. Further the electromagnetic interaction with magnetized plasma is carried out by using the numerical method and scattering matrix approach [6-7]. Furthermore, Cheng et al discussed the scattering of electromagnetic wave from anisotropic plasma coated perfect electric conductor (PEC), in which the PEC cylinder is concentrically coated [8]. Bou et al extended the Cheng's work, studied the scattering from eccentric plasma coated PEC cylinder and reported that the radar cross section (RCS) is sensitive to the plasma parameters i.e., plasma density, frequency and collision frequency as well as the position of cylinder from center [9].

Recently a general problem of scattering is studied i.e., electromagnetic scattering from anisotropic plasma coated PEMC cylinder and reported that PEMC cylinder has more stealth capability as compare to other perfect cylinders [10]. PEMC is the acronym of perfect electromagnetic conductor, which is the generalized form of perfect conductors i.e., perfect electric conductor (PEC) and perfect magnetic conductor (PMC). The PEMC is characterized by its scalar admittance parameter ( $M$ ), it has  $M = 0$  and  $M \rightarrow \pm\infty$  for PEC and PMC respectively [11]. Electromagnetic scattering from coated and un-

coated geometries (cylinder/ sphere) of PEMC has been studied extensively in literature [12-15].

In recent years, the emerging trend in researchers is, to place different objects in different complex host medium and analyze their stealth capability. Ghaffar et al studied the scattering of electromagnetic radiation from a PEMC sphere placed in chiral metamaterial and reported the influences of admittance and chirality parameters on scattered fields [16]. Recently a scattering problem is formulated in which the PEMC cylinder is placed in unmagnetized plasma medium and reported that stealth capability of plasma medium increases with the placement of PEMC cylinder as compared to other perfect cylinders [17] but the more complicated scattering problem i.e., scattering of electromagnetic radiation from PEMC cylinder placed in magnetized plasma has not been studied yet. In this research work, the theoretical formulation is carried out for the electromagnetic scattering from PEMC cylinder placed in magnetized plasma. The influence of magnetic field strength, plasma parameters and admittance parameters of PEMC cylinder on the radar cross section (RCS)/bistatic echo width is analyzed. The co and cross polarization scattering coefficients are obtained and compared under different conditions. In the whole study the time dependence of fields are taken as  $e^{-j\omega t}$ . The organization of this paper is as follows: In section 2 scattering theory for a perfect cylinders placed in magnetized plasma is formulated. Section 3 presents numerical results and discussion, whereas conclusion is presented in section 4.

## 2. Analytical formulation

The whole space is divided into the two regions, region I represents the homogenous magnetized anisotropic plasma medium while the region II is taken as infinite PEMC circular cylinder, placed along the z-axis in the anisotropic plasma medium for the sake of convenience and symmetry. A uniform external magnetic field is also applied along the z-axis as depicted in Fig. 1.

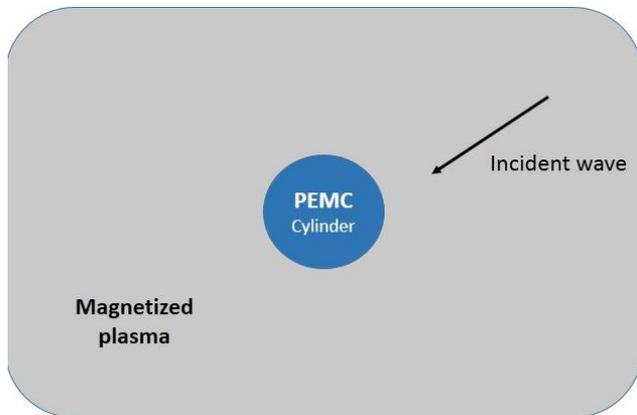


Fig. 1. PEMC cylinder placed in homogenous magnetized plasma.

The magnetized plasma medium is considered as anisotropic, cold, non-reciprocal, homogenous, incompressible and uniform. The density of plasma is kept constant. The magnetized cold plasma is characterized by the permittivity tensor [3-4, 8]

$$[\epsilon] = \begin{bmatrix} \epsilon_1 & j\epsilon_2 & 0 \\ -j\epsilon_2 & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_3 \end{bmatrix} \quad (1)$$

The elements of this tensor are the function of plasma parameters and their explicit expressions are given as

$$\epsilon_1 = 1 - \frac{\omega_p^2(\omega - j\nu)}{\omega[(\omega - j\nu)^2 - \omega_c^2]} \quad (2)$$

$$\epsilon_2 = \frac{\omega_c \omega_p}{\omega[(\omega - j\nu)^2 - \omega_c^2]} \quad (3)$$

$$\epsilon_3 = 1 - \frac{\omega_p^2}{\omega(\omega - j\nu)} \quad (4)$$

where the  $\omega_p$  is the plasma oscillation frequency,  $\omega_c$  is the cyclotron frequency,  $\nu$  is the effective collision frequency and  $\omega$  is the incident wave angular frequency. The  $\omega_p = \frac{ne^2}{m\epsilon_0}$  and the  $\omega_c = \frac{eB_0}{m}$ ,  $n$  is plasma density (electron density),  $m$  is the mass of electron,  $e$  is the magnitude of electron's charge and  $B_0$  is the external magnetic field strength. The wave numbers in anisotropic plasma for parallel (TM) polarization and perpendicular (TE) polarization are  $k_1 = \frac{k_0}{\sqrt{m}}$ , where  $m = \frac{\epsilon_1}{\epsilon_1^2 - \epsilon_2^2}$  and  $k_2 = \frac{k_0}{\sqrt{m_3}}$ , where  $m_3 = \frac{1}{\epsilon_3}$  respectively [8].

### 2.1 Cylindrical wave excitation:

The fields of rectilinear coordinates are expanded in terms of cylindrical coordinates  $(\rho, \varphi, z)$  and a plane wave is transformed into a cylindrical wave. Incident cylindrical wave is considered as a normal to the PEMC cylinder. There are the two possible state of polarization, (i) when the electric field is parallel to the cylinder axis called transverse magnetic (TM) polarization, (ii) when the magnetic field is parallel to the cylinder axis is called transverse electric (TE) polarization. In this present scattering problem, the both types of polarization of fields are discussed and their formulations are given below as

#### 2.2.1 Parallel polarization

Consider the normal incidence of parallel polarized electromagnetic waves on anisotropic plasma coated PEMC cylinder. The incident electromagnetic fields in terms of cylindrical coordinates  $(\rho, \varphi)$  is given as [12, 17]

$$E_z^i = E_0 \sum_{n=-\infty}^{\infty} j^n J_n(k_1 \rho) e^{jn(\varphi)} \quad (5)$$

$$H_{\varphi}^i = -\frac{E_0}{j\eta_0} \sum_{n=-\infty}^{\infty} j^n J_n'(k_1 \rho) e^{jn(\varphi)} \quad (6)$$

Where  $\eta = \sqrt{\frac{\mu_0}{\epsilon_0}}$  is the free space impedance,  $J_n(\cdot)$  is the Bessel function of first kind and  $H_n^{(2)}(\cdot)$  is the hankel function of second kind, where prime denotes the derivative with respect to whole argument. The scattered fields in region I are

$$E_z^s = E_0 \sum_{n=-\infty}^{\infty} j^n a_n H_n^1(k_1 \rho) e^{jn(\varphi)} \quad (7)$$

$$H_{0\varphi}^s = -\frac{E_0}{j\eta} \sum_{n=-\infty}^{\infty} j^n a_n H_n^{1'}(k_1 \rho) e^{jn(\varphi)} \quad (8)$$

$$H_{0z}^s = -\frac{jE_0}{\eta} \sum_{n=-\infty}^{\infty} j^n b_n H_n^1(k_2 \rho) e^{jn(\varphi)} \quad (9)$$

$$E_{0\varphi}^s = -E_0 \sum_{n=-\infty}^{\infty} j^n b_n H_n^{1'}(k_2 \rho) e^{jn(\varphi)} \quad (10)$$

**2.2.2 Perpendicular polarization**

Now, Consider the normal incidence of perpendicular polarized electromagnetic radiation on anisotropic plasma coated PEMC cylinder. By applying the duality transformation on the parallel polarization, the perpendicular polarization can be derived as

$$E_{\varphi}^i = -E_0 \sum_{n=-\infty}^{\infty} j^n J_n'(k_2 \rho) e^{jn(\varphi)} \quad (11)$$

$$H_{0z}^i = \frac{E_0}{j\eta} \sum_{n=-\infty}^{\infty} j^n J_n(k_2 \rho) e^{jn(\varphi)} \quad (12)$$

The scattered fields in the region I are given as

$$H_{0z}^s = \frac{jE_0}{\eta} \sum_{n=-\infty}^{\infty} j^n a_n' H_n^1(k_2 \rho) e^{jn(\varphi)} \quad (13)$$

$$E_{0\varphi}^s = -E_0 \sum_{n=-\infty}^{\infty} j^n a_n' H_n^{1'}(k_2 \rho) e^{jn(\varphi)} \quad (14)$$

$$H_{0\varphi}^s = -\frac{E_0}{j\eta} \sum_{n=-\infty}^{\infty} j^n b_n' H_n^{1'}(k_1 \rho) e^{jn(\varphi)} \quad (15)$$

$$E_{0z}^s = E_0 \sum_{n=-\infty}^{\infty} j^n b_n' H_n^1(k_1 \rho) e^{jn(\varphi)} \quad (16)$$

**Boundary conditions and scattering matrices:**

The incident fields and the scattered fields should be continuous at the interface of PEMC cylinder and magnetized plasma. The fields have to satisfy the following boundary condition at

$$\left. \begin{aligned} n \times (H + ME) &= 0 \\ n \cdot (D - MB) &= 0 \end{aligned} \right\} \quad (17)$$

In above,  $n$  is the unit normal and  $M$  is the scalar admittance parameter, which characterizes the PEMC. The tangential form of the above boundary condition is

$$\left. \begin{aligned} H_{\varphi}^i + ME_{\varphi}^i + H_{\varphi}^s + ME_{\varphi}^s &= 0 \\ H_z^i + ME_z^i + H_z^s + ME_z^s &= 0 \end{aligned} \right\} \quad (18)$$

By implementing the above boundary conditions in (18), the following 2 X 2 scattering matrices are obtained. For the case of transverse electric (TM) or parallel polarization

$$\begin{bmatrix} jM\eta H_n^1(k_1 \rho) & H_n^1(k_2 \rho) \\ H_n^{1'}(k_1 \rho) & jM\eta H_n^{1'}(k_2 \rho) \end{bmatrix} \begin{bmatrix} a_n \\ b_n \end{bmatrix} = \begin{bmatrix} -jM\eta J_n(k_1 \rho) \\ -J_n'(k_1 \rho) \end{bmatrix} \quad (19)$$

When the incident fields have transverse electric (TE) or perpendicular polarization then scattering matrix is

$$\begin{bmatrix} H_n^1(k_2 \rho) & jM\eta H_n^1(k_1 \rho) \\ H_n^{1'}(k_2 \rho) & (1/jM\eta) H_n^{1'}(k_1 \rho) \end{bmatrix} \begin{bmatrix} a_n' \\ b_n' \end{bmatrix} = \begin{bmatrix} -J_n(k_2 \rho) \\ -J_n'(k_2 \rho) \end{bmatrix} \quad (20)$$

From these scattering matrices, the Co and Cross polarized coefficients can be calculated for both type of polarization either TE or TM. After the computation of Co and Cross polarized coefficients, the bistatic echo width is calculated which is equal to the ratio of total scattered power to the incident power [10].

**3. Numerical results and discussion**

In this section, some numerical results are presented, to gain the better understanding and more physical aspects of above formulated scattering problem i.e., electromagnetic scattering from perfect electromagnetic conductor (PEMC) cylinder placed in magnetized cold plasma. First of all, to check the accuracy of our work and functionality of used Software package i.e., MATHEMATICA, some results are compare with already published literature under special conditions and good agreement is found. After the insurance of correctness of the analytical formulation, further results for transverse electric (TE) and transverse magnetic (TM) polarization are presented by varying different plasma and physical parameters i.e., plasma density ( $n$ ), effective collision frequency ( $\nu$ ) and external magnetic field strength ( $B_0$ ) respectively, as according to the literature.

Fig. 2.a shows the comparison between the Co and Cross polarized bistatic echo widths. In above formulated theory, when external magnetic field is removed i.e.,  $B_0 = 0$ , the cyclotron frequency becomes zero i.e.,  $\omega_c = 0$  and the anisotropic relative permittivity tensor transforms

into the isotropic plasma, relative permittivity values become  $\epsilon_2 = 0$  &  $\epsilon_1 = \epsilon_3 = 1 - \frac{\omega_p^2}{\omega[(\omega - j\nu)]}$ , and our scattering formulation converts into the electromagnetic scattering from PEMC cylinder placed in un-magnetized plasma medium [17].

Fig. 2b shows the comparison of radar cross section (RCS) between present paper and literature, when the hosting medium i.e., plasma is replaced by the free space then the present scattering problem converts into the scattering of electromagnetic radiation from PEMC cylinder [12] and good agreement is found.

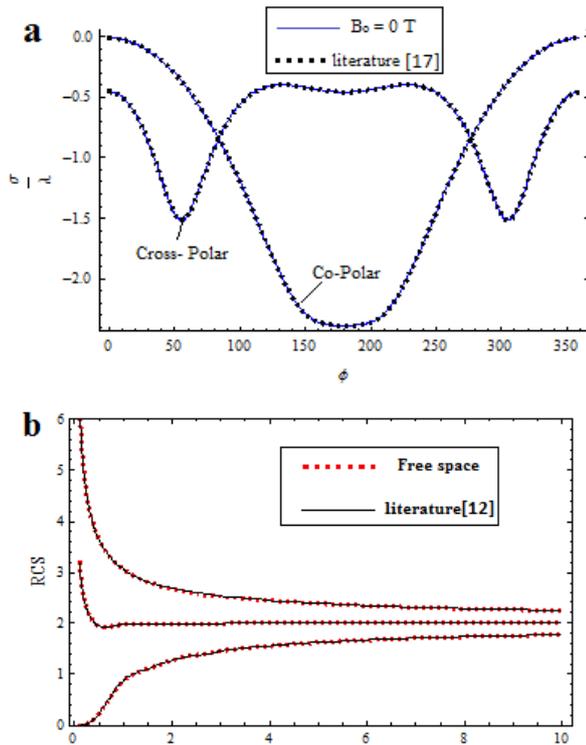


Fig. 2. (a) Comparison between Co and Cross-Polarized bistatic echo widths of PEMC cylinder placed in un-magnetized plasma ( $B_0 = 0.0$  T,  $a = 0.2\lambda_0$ ,  $f = 1$  GHz,  $n = 1.0 \times 10^{15} \text{m}^{-3}$  and  $\nu = 1.0 \times 10^{10} \text{Hz}$ ) (b) Comparison between scattering cross section and size of PEMC cylinder “a” placed in free space. ( $M\eta = 0, 1$  and  $\infty$ ).

Fig. 3 shows the comparison between the normalized Co and Cross polarized bistatic echo widths of a PEMC cylinder placed in magnetized cold plasma medium. The Fig. 3a and Fig. 3b shows the comparison between Co and Cross polarized bistatic echo widths for both transverse electric (TE) and transverse magnetic (TM) polarization respectively and also depicts that Co and Cross polarized fields have opposite behavior, furthermore, both type of polarizations TE/TM the scattered fields have same results because due to mixing of TE and TM fields at the PEMC cylinder interface.

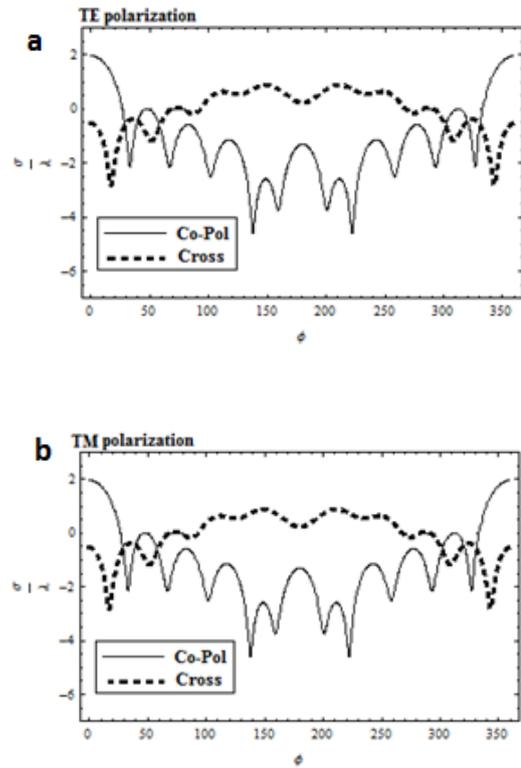


Fig. 3. (a) Comparison between Co and Cross-polarized bistatic echo widths of PEMC cylinder for TE polarization (b) Comparison between Co and Cross-polarized bistatic echo widths of PEMC cylinder for TM polarization ( $B_0 = 0.1$  T,  $a = \lambda_0$ ,  $f = 10$  GHz,  $n = 1.0 \times 10^{17} \text{m}^{-3}$  and  $\nu = 1.0 \times 10^{10} \text{Hz}$ ).

Fig. 4 shows the influence of external magnetic field strength on the forward and backward scattering amplitudes for TE and TM polarization. Fig. 4a shows the comparison between Co polarized bistatic echo width while the Fig. 4b shows the comparison between Cross polarized echo width. It is clear from these results, with the increase in external magnetic field strength, the scattering amplitude decreases because by increasing the external magnetic field strength, the anisotropy increase, which causes the increase in the stealth capability of plasma medium. The same effect of magnetic field strength on plasma scattering amplitude reducing ability is discussed in [18]. This comparison also shows that the influence of magnetic field strength has different response to TE and TM polarization, which is same in above comparison.

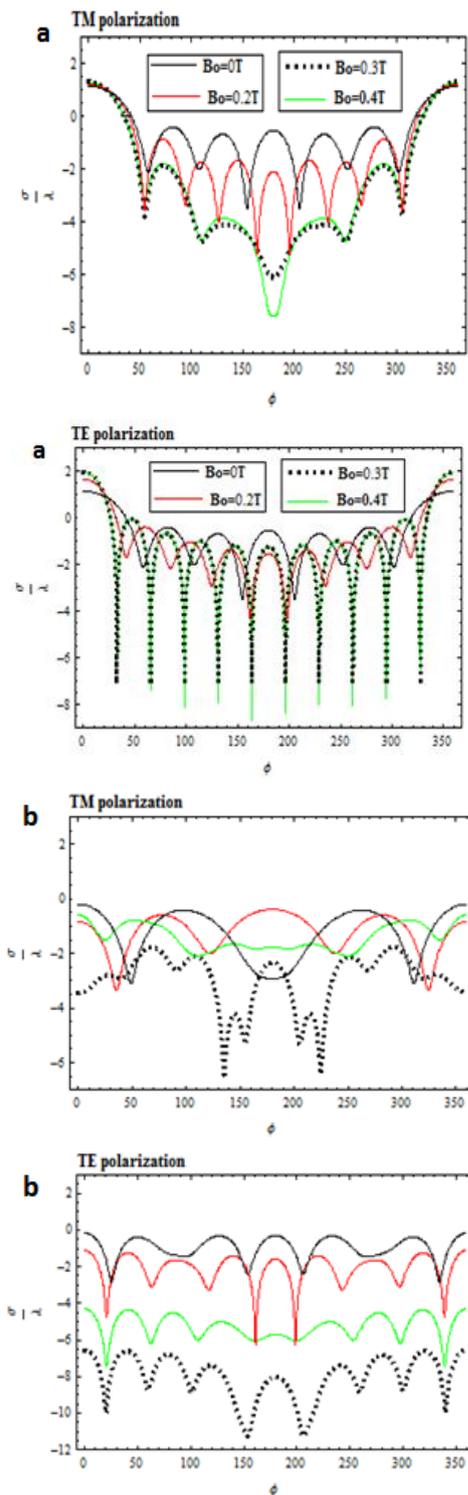


Fig. 4. (a) Co-polarized bistatic echo widths of PEMC cylinder placed in magnetized plasma medium for TE and TM polarization. (b) Cross-polarized bistatic echo widths of PEMC cylinder placed in magnetized plasma medium for TE and TM polarization. ( $a = \lambda_0, f = 10 \text{ GHz}, n = 1.0 \times 10^{18} \text{ m}^{-3}$  and  $v = 1.0 \times 10^{10} \text{ Hz}$  ).

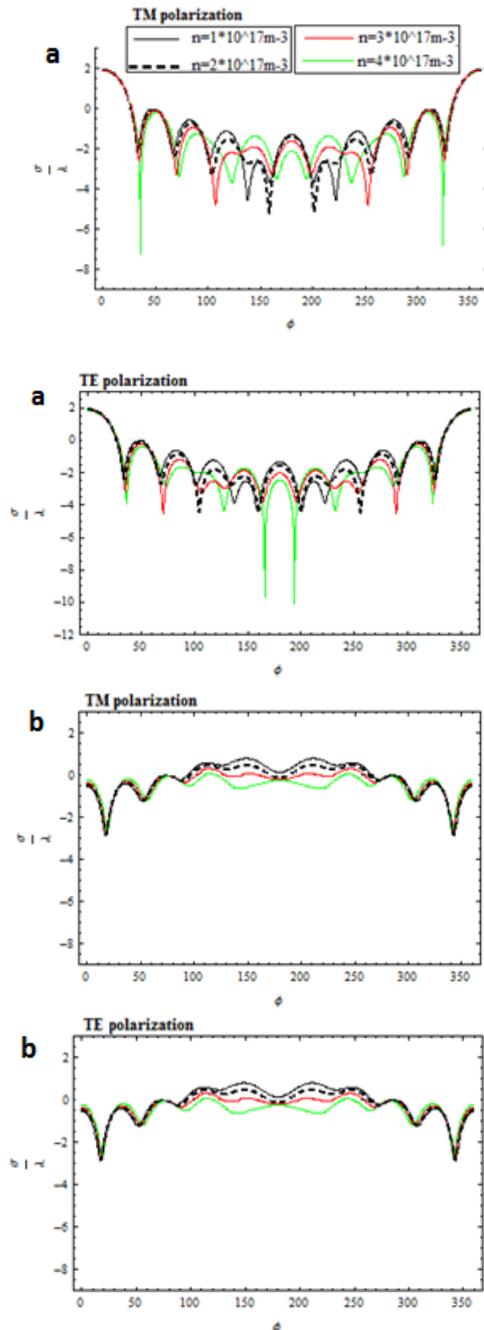


Fig. 5. (a) Influence of density on Co-polarized bistatic echo widths for TE and TM polarization. (b) Influence of density on Cross-polarized bistatic echo widths for TE and TM polarization. ( $B_0 = 0.2 \text{ T}, a = \lambda_0, f = 10 \text{ GHz}$  and  $v = 1.0 \times 10^{10} \text{ Hz}$  ).

Fig. 5 and 6 shows the comparison between Co and Cross polarized bistatic echo widths for parallel (TM) and perpendicular (TE) polarization. The influence of plasma density and the effective collision frequency is analyzed in Fig. 5 and 6 respectively. It is clear from the Fig. 5, that with the increase of plasma density the scattering amplitude decreases and further it is analyzed that the Co-polarized bistatic echo width has different response for different states polarization i.e., TE and TM, but has same

response for the cross polarized bistatic echo width as Fig. 6 depicts.

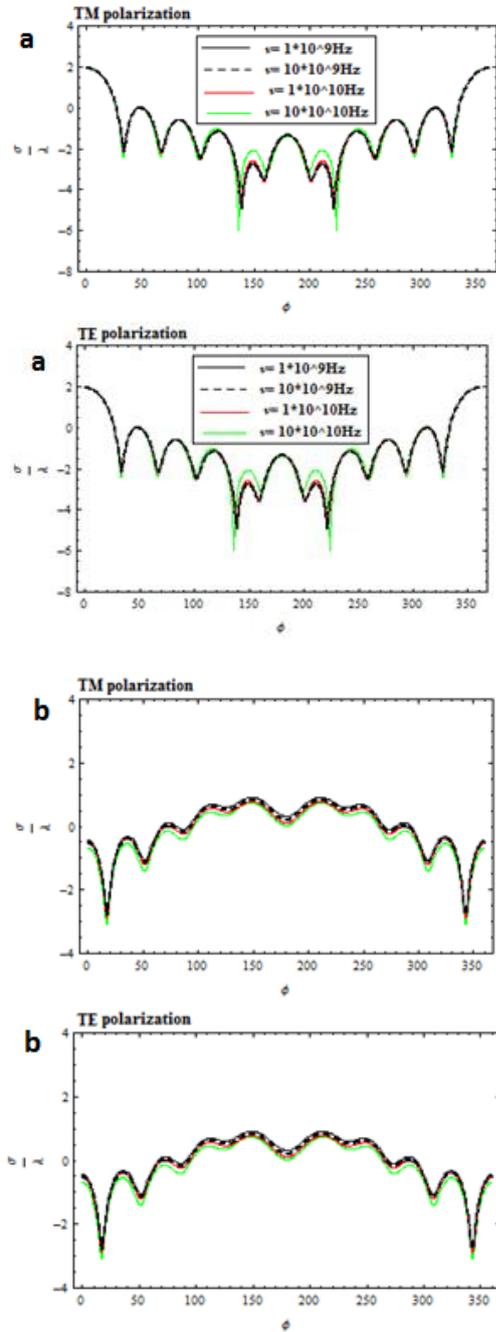


Fig.6 (a.) Influence of effective collision frequency on Co-polarized bistatic echo widths for TE and TM polarization. (b)Influence of effective collision frequency on Cross-polarized bistatic echo widths for TE and TM polarization. ( $B_0 = 0.2 \text{ T}$ ,  $a = \lambda_0$ ,  $f = 10 \text{ GHz}$  and  $n = 1.0 \times 10^{18} \text{ m}^{-3}$ ).

Fig. 7 shows the comparison between scattered fields of Co and Cross polarized coefficients from different cylinders i.e., PMC, PEC and PEMC placed in the magnetized plasma medium. It is clear from the Fig. 7.a that for the case of TM polarization, PEMC cylinder helps

to reduce the scattering cross section in the presence of plasma medium while for the TE polarization case the PMC cylinder is preferable for the sake of reduction in the scattering amplitude. Where the Fig. 7.b depict that the cross polarization coefficient exists only for the PEMC cylinder regardless the type of hosting medium either the magnetized plasma or un-magnetized plasma and the type of polarization either TE or TM.

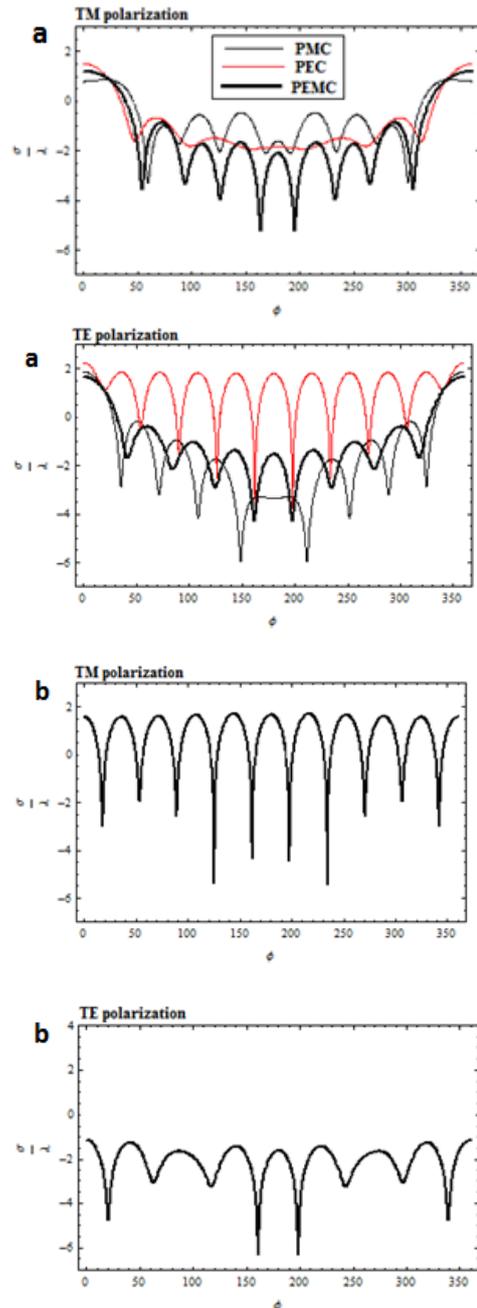


Fig. 7. (a) Co-polarized bistatic echo widths of PEC, PMC and PEMC cylinder placed in magnetized plasma. (b) Cross-polarized bistatic echo widths of PEC, PMC and PEMC cylinder placed in magnetized plasma. ( $B_0 = 0.2 \text{ T}$ ,  $a = \lambda_0$ ,  $f = 10 \text{ GHz}$ ,  $n = 1.0 \times 10^{18} \text{ m}^{-3}$  and  $\nu = 1.0 \times 10^{10} \text{ Hz}$ ).

### Concluding Remarks:

The following conclusions are extracted from the above numerical results, which increase the novelty and importance of work

- The electromagnetic response to PEMC cylinder placed in magnetized plasma medium is almost same for both type of polarizations i.e., transverse electric (TE) and transverse magnetic (TM).
- The scattering amplitude can be increased or decreased by increasing or decreasing the magnetic field strength ( $B_0$ ) respectively.
- The Co and Cross polarized bistatic echo widths are sensitive to the plasma parameters, the scattering amplitude can be controlled by the choosing the appropriate plasma parameters i.e., plasma density ( $n$ ) and effective collision frequency ( $\nu$ ).
- Furthermore, the scattering amplitude can also be controlled by the appropriate type of the cylinder. PEMC and PMC are good candidates to increase the stealth capability of magnetized plasma for TM and TE polarization respectively.
- The cross-polarized scattering component is only associated with the PEMC cylinder, neither PEC nor PMC cylinder. It is independent of plasma parameters as well as the physical parameters of hosting medium.

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### References

- [1] J. R. Roth, CRC Press **2** (2001).
- [2] F. F. Chen, M. D. Smith, Plasma, John Wiley & Sons, Inc.. (1984).
- [3] K. Miyamoto, Plasma physics for nuclear fusion. Cambridge, Mass., MIT Press, 1980. 625 p. Translation, 1. (1980).
- [4] Y. Geng, X. Wu, L. W. Li, Radio Science, **38**(6) (2003).
- [5] R. J. Vidmar, IEEE Trans. on Plasma Science, **18**(4), 733 (1990).
- [6] M. Laroussi, IEEE Trans. on Plasma Science, **21**(4), 366 (1993).
- [7] B. J. Hu, G. Wei, S. L. Lai, IEEE Trans. on Plasma Science, **27**(4), 1131 (1999).
- [8] H. C. Chen, D. K. Cheng, IEEE Transactions on Antennas and Propagation, **12**(3), 348 (1964).
- [9] B. Yin, F. Yang, H. Hao, C. Li, Radiation Effects & Defects in solids, <http://dx.doi.org/10.1080/10420150.2012.723003>.
- [10] A. Ghaffar, M. Z. Yaqoob, M. A. Alkanhal, M. Sharif, Q. A. Naqvi, AEU-International Journal of Electronics and Communications, **68**(8), 767 (2014).
- [11] I. V. Lindell, A. H. Sihvola, Journal of Electromagnetic Waves and Applications, **19**(7), 861 (2005).
- [12] R. Ruppın, Journal of Electromagnetic Waves and Applications, **20**(13), 1853 (2006).
- [13] R. Ruppın, Progress in Electromagnetics Research Letters, **8**, 53 (2009).
- [14] S. Ahmed, Q. A. Naqvi, Optics Communications, **281**(23), 5664 (2008).
- [15] R. Ruppın, Progress In Electromagnetics Research Letters, **8**, 53 (2009).
- [16] A. Ghaffar, S. I. Ahmad, R. Fazal, S. Shukrullah, Q. A. Naqvi, Optik-International Journal for Light and Electron Optics, **124**(21), 4947 (2013).
- [17] A. Ghaffar, M. Z. Yaqoob, M. A. Alkanhal, S. Ahmed, Q. A. Naqvi, M. A. Kalyar, Optik-International Journal for Light and Electron Optics (2014).
- [18] F. Jin, H. Tong, Z. Shi, D. Tang, P. K. Chu, Computer physics communications, **175**(8), 545 (2006).

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