Electromagnetic scattering from isotropic-plasma coated perfect electromagnetic conductor sphere

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The scattering of electromagnetic waves by a perfect electromagnetic conductor (PEMC) sphere coated with a plasma isotropic material is studied in this paper. The extended classical theory of scattering is used to physically model the scattering problem. All the fields are expanded in terms of spherical vector wave functions (SVWFs), and then scattering matrices are obtained by applying the boundary conditions at each interface i.e., free space/ plasma and plasma/ PEMC. The presented analysis and formulations are general for any perfect conductor cylinder (PEC, PMC, or PEMC) with general isotropic material coatings. After that, the Co and Cross-polarized scattering coefficients are numerically computed and used to calculate the radar cross section (RCS). The influence of plasma parameters i.e., plasma density (*n*) and effective collision frequency (*v*) on RCS is analyzed and reported that radar cross section can be controlled by choosing the appropriate plasma parameters. It is also concluded that plasma coating can be used for target protection.The comparisons of the computed results of the presented formulations with the published results of some special cases confirm the accuracy of the presented analysis.

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1. Introduction

Metamaterials with unprecedented abilities have potential applications in sensing, wave guiding, microwave controlling devices, electromagnetic invisibility cloak, perfect reflectors, lenses, and in phase shifters [1-4]. In addition, recently a new complex metamaterial is introduced in 2005 by Lindel and sihvolanamed as perfect electromagnetic conductor (PEMC), which behaves as perfect reflector [5-6]. It is a generalization of perfect electric conductor (PEC) and perfect magnetic conductor (PMC). The scalar admittance parameter M characterizes the PEMC.Unlike the PEC or PMC, PEMC contains additional cross polarization component which characterizes the PEMC as a bi-isotropic material.

Scattering of electromagnetic radiation from PEMC objects is main interest of many researchers and different geometries of PEMC has been studied in literature; Ruppin analyzed the scattering of electromagnetic radiation by coated and un-coated PEMC sphereand also by infinite length circular cylinder [7-9]. Scattering from coated and un-coated PEMC elliptical cylinder is also investigated in [10-11]. Moreover, the scattering of a radially oriented Hertz Dipole field by a PEMC sphere is also discussed by Ghaffar et al [12]. In recent years, Scattering of electromagnetic radiations from plasma coated perfect conducting objects has stimulate great interests of many people due to its stealth capability, which is a hot topic in defense technology. Plasma is a complex, quasi neutral material, can be considered as highly ionized state of a gas; having ions, electrons and neutral species. When the spacecraft having cylindrical and spherical structures i.e., missiles and satellitesenter into the atmosphere, they immersed in ionosphere plasma and plasma generated on their surfaces. This generated plasma coating can be modeled as layers of homogenous thicknesses of plasma coating on the perfect conducting missiles. When this plasma coated structures come under the influence of an external magnetic field like the earth's magnetic field, then it can be modeled as a conducting cylindrical or spherical structure coated with plasma in the presence of a magnetic field [13].

Keeping in view the practical applications of plasma coated structures in the field of aerospace sciences and defense technology a lot of work has been performed by many researchers with different geometries and analogies. K. Cheng. et al., discussed the 2-D electromagnetic scattering from anisotropic plasma coated perfect electric conductor cylinder [14]. The 3-D scattering of electromagnetic waves from anisotropic plasma sphere and anisotropic plasma coated conducting sphere have been reported [15]. In the recent year the analysis of scattered electromagnetic wave from eccentric plasma coated conducting cylinder is presented by Bo. Yina *et al*and the scattering amplitude is found sensitive to plasma parameters i.e., plasma density, effective collision frequency and plasma oscillation [16].

Recently, the electromagnetic scattering from anisotropic plasma coated PEMC cylinders has been investigated and reported that the PEMC helps to reduce the scattering amplitude. This work presents the more general formulation for any type of perfect conductor cylinder (PEC, PMC and PEMC) with plasma coating [17]. Further extended to the scattering of electromagnetic radiation from PEMC cylinder placed in un-magnetized plasma and reported that the PEMC cylinder has more stealth capability as compared to other perfect cylinders [18]. Moreover the electromagnetic scattering from plasma coated PEMC cylinder placed in the chiral metamaterial is also investigated [19-20].

In the literature, the scattering of electromagnetic radiations from plasma coated PEC sphere has been studiedwhile the more general and complex problem of electromagnetic scattering from anisotropic plasma-coated PEMC sphere has not been studied yet. Here we carried out analytical scattering study for the isotropic plasma coated PEMC sphere due to its practical importance as narrated in the above discussion. This study is a generalization of the classical scattering from PEC coated spheres and it reveals an extra field term which is the cross polarized component in the scattered field. In the whole study the time dependence of fields are taken as $e^{-j\omega t}$. The organization of this paper is as follows: Section 2 contains the analytical formulation of scattering theory for isotropic plasma coated perfect electromagnetic conductor sphere. Section 3 presents numerical results and discussion, whereas conclusion is presented in section 4.

2. Analytical formulation

2.1. Isotropic plasma coated PEMC sphere geometry

The whole space is divided into three regions i.e., region I is free space, region II is isotropic plasma coating and region III represents the PEMC sphere as depicted in the Fig. 1. Where the radius of the PEMC sphere is a and b is the radius of the isotropic plasma coated PEMC sphere. The thickness of the isotropic, homogeneous plasma coating/layer is $b-a$. The wavenumber in region I is $k_0 = \omega\sqrt{\varepsilon_0\mu_0}$and,in region II i.e., isotropic plasma region is k_1 which is equal to $k = k_o(\varepsilon_p)^{1/2}$ as given in [19], where k_o is the wave number in free space and $\varepsilon_p = 1 - \frac{i\omega_o{}^2}{(v+i\omega)\omega}$is the relative permittivity of plasma medium, which is the function of (electron) plasma frequency$\omega_o = \sqrt{\frac{ne^2}{m\varepsilon_o}}$, effective collision frequency v and the incident field frequency ω.

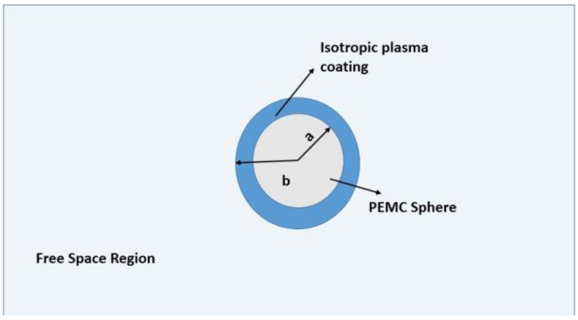

Fig. 1. Schematic view of the electromagnetic scattering from isotropic plasma-coated PEMC sphere.

2.2 Spherical Wave Excitation from isotropic plasma coated PEMC sphere

The incident electromagnetic fields are expanded in terms of spherical coordinate system (r,θ,φ) and the spherical vector wave functions are as given [7-8].

$$\vec{M}^{(l)}_{\sigma mn\gamma} = \vec{\nabla} \times \left[\vec{r}Y_{\sigma mn}(\theta,\varphi)R^{(l)}_n(k_\gamma r)\right] \tag{1}$$

$$\vec{N}^{(l)}_{\sigma mn\gamma} = \frac{1}{k_\gamma}\left[\vec{\nabla} \times \vec{M}^{(l)}_{\sigma mn\gamma}\right] \tag{2}$$

The spherical harmonic$Y_{\sigma mn}(\theta,\varphi)$ is characterized by the subscript $\sigma = e$ or o. For$\sigma = e$ (even) and $\sigma = o$ (odd) the spherical harmonic$Y_{\sigma mn}(\theta,\varphi)$ will be even and odd respectively. Where radial function $R^{(l)}_n(k_\gamma r)$ presents appropriate kind of spherical Bessel function $J_n(k_\gamma r)$, spherical Neumann function $n_n(k_\gamma r)$ and spherical Hankel function $h_n(k_\gamma r)$ for l =1, 2 and 3 respectively. The subscript γ in k_γ is to characterize the wavenumber in different regions i.e., $\gamma = 0$ for free space region, and$\gamma = 1$ for isotropic plasma region respectively.

$$\vec{E}^i = E_0\sum_{n=1}^{\infty} i^n \frac{2n+1}{n(n+1)}(\vec{M}^{(1)}_{o1n0} - i\vec{N}^{(1)}_{e1n0}) \tag{3}$$

$$\vec{H}^i = -\frac{E_0}{\eta_0}\sum_{n=1}^{\infty} i^n \frac{2n+1}{n(n+1)}(\vec{M}^{(1)}_{e1n0} + i\vec{N}^{(1)}_{1n0}) \tag{4}$$

where $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$ is the impedance of free space and scattered fields in region I are given as

$$\vec{E}^s = E_0\sum_{n=1}^{\infty} i^n \frac{2n+1}{n(n+1)}\left(a^s_n\vec{M}^{(3)}_{o1n0} + c^s_n\vec{M}^{(3)}_{e1n0} - ib^s_n\vec{N}^{(1)}_{e1n0} - id^s_n\vec{N}^{(1)}_{o1n0}\right) \tag{5}$$

$$\vec{H}^s = -\frac{E_0}{\eta_0}\sum_{n=1}^{\infty} i^n \frac{2n+1}{n(n+1)}\left(b^s_n\vec{M}^{(3)}_{e1n0} + d^s_n\vec{M}^{(3)}_{o1n0} + ia^s_n\vec{N}^{(1)}_{o1n0} + ic^s_n\vec{N}^{(1)}_{e1n0}\right) \tag{6}$$

where $a_n{}^s$and $b_n{}^s$ are co-polarized scattering coefficients and $c_n{}^s$and $d_n{}^s$are cross polarized field scattering coefficient. The transmitted fields in region II i.e., isotropic plasma coating are

$$\vec{E}^t = E_0 \sum_{n=1}^{\infty} i^n \frac{2n+1}{n(n+1)} \big(a_n^{t1} \vec{M}_{o1n1}^{(1)} + a_n^{t2} \vec{M}_{o1n1}^{(2)} + c_n^{t1} \vec{M}_{e1n2}^{(1)} + c_n^{t2} \vec{M}_{e1n2}^{(2)} - i b_n^{t1} \vec{N}_{e1n1}^{(1)} - i b_n^{t2} \vec{N}_{e1n1}^{(2)} - i d_n^{t1} \vec{N}_{o1n2}^{(1)} - i d_n^{t2} \vec{N}_{o1n2}^{(1)} \big) \qquad (7)$$

$$\vec{H}^t = -\frac{E_0}{\eta} \sum_{n=1}^{\infty} i^n \frac{2n+1}{n(n+1)} \big(b_n^{t1} \vec{M}_{e1n1}^{(1)} + b_n^{t2} \vec{M}_{e1n1}^{(2)} + d_n^{t1} \vec{M}_{o1n2}^{(1)} + d_n^{t2} \vec{M}_{o1n2}^{(2)} + i a_n^{t1} \vec{N}_{o1n1}^{(1)} + i a_n^{t2} \vec{N}_{o1n1}^{(2)} + i c_n^{t1} \vec{N}_{e1n2}^{(1)} + i c_n^{t2} \vec{N}_{e1n2}^{(1)} \big) \qquad (8)$$

where $\eta = \sqrt{\mu_0/\varepsilon_p \varepsilon_0}$ is the intrinsic impedance of isotropic plasma coating.

2.3 Boundary conditions and Scattering matrices

The boundary condition at the surface of PEMC sphere ($r = a$) is given as [7-8, 10]

$$\left.\begin{aligned} \boldsymbol{n} \times (\vec{\boldsymbol{H}} + M\vec{\boldsymbol{E}}) = 0 \\ \boldsymbol{n} \cdot (\vec{\boldsymbol{D}} - M\vec{\boldsymbol{B}}) = 0 \end{aligned}\right\} \qquad (9)$$

In above, $\boldsymbol{n}$ is the unit normal and M is the scalar admittance parameter, which characterizes the PEMC. For tangential field components the boundary condition is $H_t^i + M E_t^i + H_t^s + M E_t^s = 0$ (10)

and the boundary condition for the radial field components is

$$\varepsilon_0 E_r^i - M \mu_0 H_r^i + \varepsilon_0 E_r^s - M \mu_0 H_r^s = 0 \qquad (11)$$

The boundary conditions at the interface of plasma and free space at $r = b$ are given below

$$E_t^i + E_t^s = E_t^t \qquad r = b, \qquad 0 \le \varphi \le 2\pi \quad (12)$$
$$H_t^i + H_t^s = H_t^t \qquad r = b, \qquad 0 \le \varphi \le 2\pi \quad (13)$$

Implementing the above boundary conditions to fields from (3)-(8), the two sets of six linear equations are obtained, which are written in the following matrices form. By numerical computation the transmitted coefficients ($a_n^{t1}, a_n^{t2}, b_n^{t1}, b_n^{t2}, c_n^{t1}, c_n^{t2}, d_n^{t1}, d_n^{t2}$) and scattered coefficients ($a_n^s, b_n^s, c_n^s, d_n^s$) are extracted from these matrices.

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ J_n(k_0 b) \\ \frac{\eta}{k_0 b}[k_0 b\, J_n(k_0 b)]' \\ 0 \end{bmatrix} = \begin{bmatrix} M\eta\, J_n(k_1 a) & M\eta\, n_n(k_1 a) & -J_n(k_1 a) & -n_n(k_1 a) & 0 & 0 \\ [k_1 a\, J_n(k_1 a)]' & [k_1 a\, n_n(k_1 a)]' & M\eta[k_1 a\, J_n(k_1 a)]' & M\eta[k_1 a\, n_n(k_1 a)]' & 0 & 0 \\ 0 & 0 & \eta_0\, J_n(k_1 b) & \eta_0\, n_n(k_1 b) & 0 & -\eta h_n(k_0 b) \\ J_n(k_1 b) & n_n(k_1 b) & 0 & 0 & -h_n(k_0 b) & 0 \\ \frac{\eta_0}{k_1 b}[k_1 b\, J_n(k_1 b)]' & \frac{\eta_0}{k_1 b}[k_1 b\, n_n(k_1 b)]' & 0 & 0 & \frac{\eta}{k_1 b}[k_1 b\, h_n(k_1 b)]' & 0 \\ 0 & 0 & \frac{1}{k_1 b}[k_1 b\, J_n(k_1 b)]' & \frac{1}{k_1 b}[k_1 b\, n_n(k_1 b)]' & 0 & \frac{-1}{k_0 b}[k_0 b\, h_n(k_0 b)]' \end{bmatrix} \begin{bmatrix} a_n^{t1} \\ a_n^{t2} \\ d_n^{t1} \\ d_n^{t2} \\ a_n^s \\ d_n^s \end{bmatrix}$$

and

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ J_n(k_0 b) \\ \frac{\eta}{k_0 b}[k_0 b\, J_n(k_0 b)]' \\ 0 \end{bmatrix} = \begin{bmatrix} J_n(k_1 a) & n_n(k_1 a) & -M\eta\, J_n(k_1 a) & -M\eta\, n_n(k_1 a) & 0 & 0 \\ M\eta[k_1 a\, J_n(k_1 a)]' & M\eta[k_1 a\, n_n(k_1 a)]' & [k_1 a\, J_n(k_1 a)]' & [k_1 a\, n_n(k_1 a)]' & 0 & 0 \\ 0 & 0 & J_n(k_1 b) & n_n(k_1 b) & 0 & -h_n(k_0 b) \\ \frac{1}{k_1 b}[k_1 b\, J_n(k_1 b)]' & \frac{1}{k_1 b}[k_1 b\, n_n(k_1 b)]' & 0 & 0 & \frac{-1}{k_0 b}[k_0 b\, h_n(k_0 b)]' & 0 \\ \eta_0\, J_n(k_1 b) & \eta_0\, J_n(k_1 b) & 0 & 0 & -\eta h_n(k_0 b) & 0 \\ 0 & 0 & \frac{\eta_0}{k_1 b}[k_1 b\, J_n(k_1 b)]' & \frac{\eta_0}{k_1 b}[k_1 b\, n_n(k_1 b)]' & 0 & \frac{-\eta}{k_0 b}[k_0 b\, h_n(k_0 b)]' \end{bmatrix} \begin{bmatrix} b_n^{t1} \\ b_n^{t2} \\ c_n^{t1} \\ c_n^{t2} \\ b_n^s \\ c_n^s \end{bmatrix}$$

After the numerical computation of scattering coefficients, the normalized forward and backward radar cross sections (RCS) are calculated by the following formulas as

$$\sigma(180°) = \left(\frac{2}{\rho}\right)^2 \left[\left|\sum_{n=1}^{\infty}(-1)^n\left(n+\frac{1}{2}\right)(b_n{}^s - a_n{}^s)\right|^2 + \left|\sum_{n=1}^{\infty}(-1)^n\left(n+\frac{1}{2}\right)(c_n{}^s + d_n{}^s)\right|^2\right] \quad (14)$$

$$\sigma(0°) = \left(\frac{2}{\rho}\right)^2 \left[\left|\sum_{n=1}^{\infty}\left(n+\frac{1}{2}\right)(b_n{}^s + a_n{}^s)\right|^2 + \left|\sum_{n=1}^{\infty}\left(n+\frac{1}{2}\right)(c_n{}^s - d_n{}^s)\right|^2\right] \quad (15)$$

3. Results and discussion

In this section, numerical results of above formulated analytical solutions are presented. The influence of plasma parameters on radar cross section (RCS) is discussed and analyzed graphically. Throughout all the numerical results and graphs, the incident frequency is kept $f = 2\ GHz$, the radius of the sphere is taken as $a = 0.1\lambda_o$ and the Co and Cross polarized scattering coefficients are plotted against the alpha (α), where the relation between alpha (α) and admittance parameters is $M\eta = \tan\alpha$ [7-8]. The scattering problem i.e., electromagnetic scattering from plasma coated PEMC sphere is verified and checked under special conditions. First the plasma coating is replaced by the dielectric coating and problem transformed into the scattering of electromagnetic radiations from dielectric coated PEMC sphere and good agreement is found [8], as shown in Fig. 2. Further the present scattering theory is compared with the scattering problem of un-coated PEMC sphere [8], by taking the thickness of the coating$b - a = 0$ as given in Fig. 3.

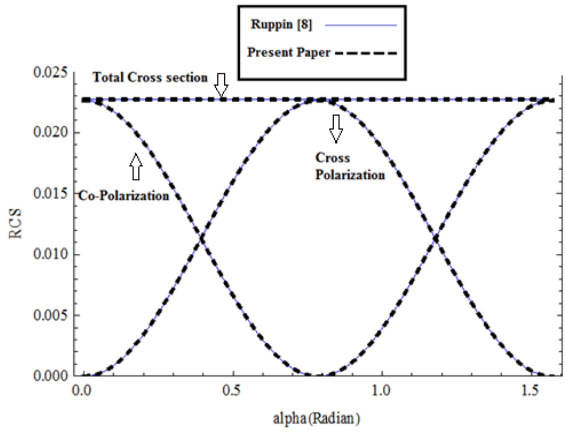

Fig. 2. Comparison between total RCS, Co polarized and Cross polarized contribution to RCS of dielectric coated PEMC sphere ($b/a = 1.1$, $\rho = 3, \epsilon_r = \mu_r = 0.01$).

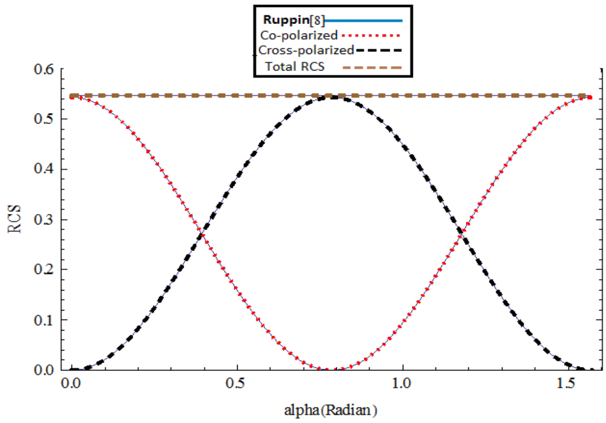

Fig. 3. Comparison between total RCS, Co polarized and Cross polarized contribution to RCS of un-coated PEMC sphere.

All these comparisons show the correctness and generalization of our recently developed scattering problem. This check on analytical theory, allow us to proceedfurther and get more results concerning the plasma coated PEMC sphere i.e., the influence of plasma parameters on RCS. Fig.4 shows the comparison between Co and Cross polarized RCS of plasma coated PEMC sphere, which clearly shows that the Co and Cross polarized coefficients have opposite behavior to each other. Further, it is also shown that the cross polarized component vanishes at extreme values of the alpha (α) i.e., $\alpha = 0.0$ radian and $\alpha = 1.5$ radian which shows that the cross polarization component is only associated with the PEMC neither PEC nor PMC.

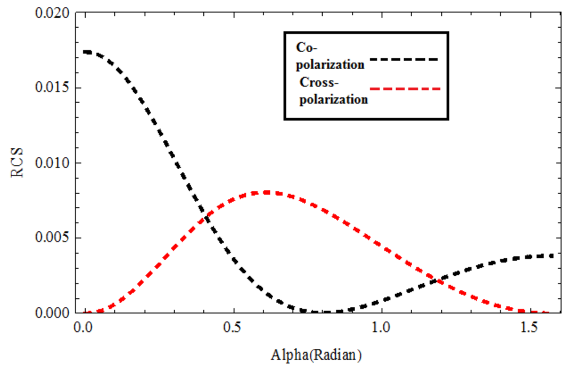

Fig. 4. Comparison between Co and Cross polarized RCS

($a = 0.1\lambda_0, b = 1.5\ a\ , f = 1\ GHz, n = 1.0 \times 10^{15} m^{-3}\ \&\ v = 1.0 \times 10^{10} Hz$).

The Fig. 5 shows the comparison between Co-polarized RCS at different values of plasma density (n), which clear from this comparison that with the increase of plasma density the RCS is decreasing.Similar behavior in the comparison of Cross-polarized RCS is given in Fig.6. This density dependent behavior of Co and Cross polarized RCS allows us to take plasma coating as a target protector for PEMC spherical geometries. The effect of effective collision frequency (v) on Co and Cross polarized contribution in RCS is depicted in Fig. 7 and 8 respectively. In which clearly shown that with the increase in the effective collision frequency the RCS in also increasing because of the permittivity of the plasma medium which consists of two parts i.e., real (transmission) and imaginary (absorption) part, the imaginary part is inversely proportional to the effective collision frequency (v). With the increase in effective collision frequency the absorption decreases and transmission increases, which is responsible for the increase in the RCS in these comparisons.

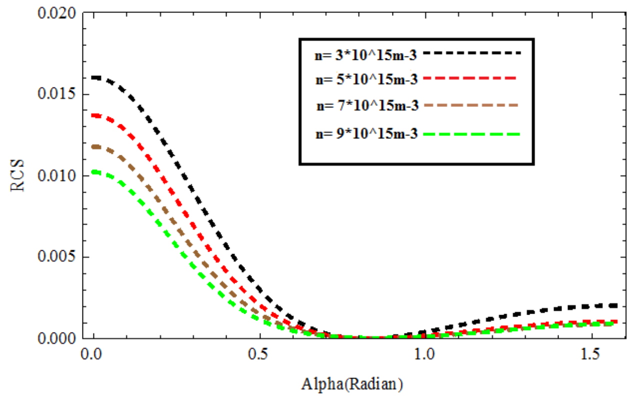

Fig. 5. Comparison between C0-polarized RCS components of plasma-coated PEMC sphere at different values of plasma density ($a = 0.1\lambda_0, b = 1.5\,a\,, f = 2\ GHz, \&\ v = 1.0 \times 10^{10} Hz$).

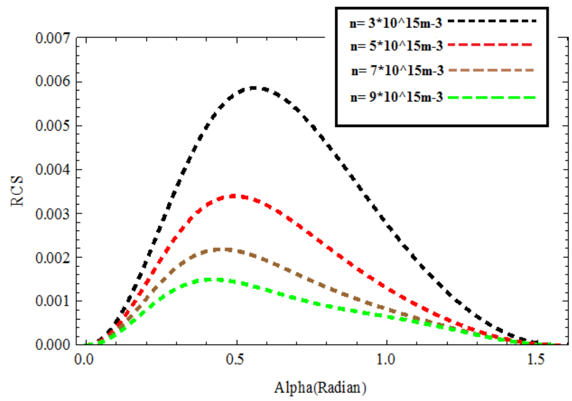

Fig. 6. Comparison between Cross-polarized RCS components of plasma-coated PEMC sphere at different values of plasma density ($a = 0.1\lambda_0, b = 1.5\,a\,, f = 2\ GHz, \&\ v = 1.0 \times 10^{10} Hz$).

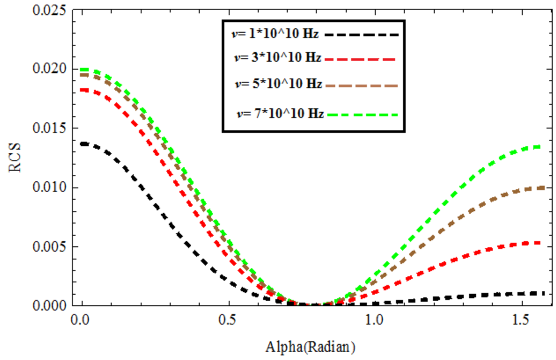

Fig. 7.Comparison between Co-polarized RCS components of plasma-coated PEMC sphere at different values of effective collision frequency ($a = 0.1\lambda_0,\ b = 1.5\,a,\ f = 2\ GHz\ \&n = 5.0 \times 10^{15} m^{-3}$).

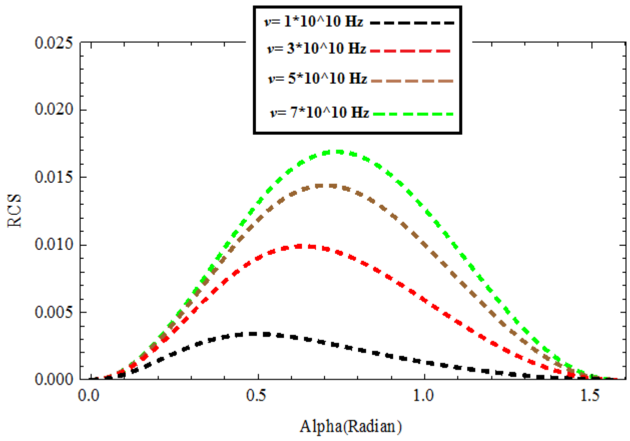

Fig. 8. Comparison between Cross-polarized RCS components of plasma-coated PEMC sphere at different values of effective collision frequency ($a = 0.1\lambda_0,\ b = 1.5\,a,\ f = 2\ GHz\ \&n = 5.0 \times 10^{15} m^{-3}$).

4. Concluding remarks

Electromagnetic scattering from plasma coated perfect electromagnetic conductor sphere is formulated, in the frame work of extended classical scattering theory. The spherical wave excitation is considered. Scattering coefficients are computed from the scattering matrices. To get more insight physics and studies, the Co and Cross polarized contributions in RCS are plotted against the different values of alpha (α)as well as the effect of different values of plasma parameters is also presented. The following conclusions can be drawn from the previously demonstrated numerical results;

- The Co and Cross polarized scattering coefficients are only associated with the PEMC and independent of type of the coating either dielectric or plasma.
- The presently formulated scattering theory for plasma coated PEMC sphere, is applicable on any type of coating i.e., plasma, plasmonics and metamaterial.

- The RCS can be controlled and tuned by choosing the appropriate plasma parameters i.e., plasma density (*n*) and effective collision frequency (*v*).
- The dense isotropic, homogenous plasma coatings can be used for the protection of PEMC spherical objects.

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