Effect of plasma-vacuum boundary for coherent transition radiation

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During the propagation of an electron bunch through the discontinuity of the (Coulomb) self-fields for the sharp boundary with different dielectric constant, a surface current of background electrons is generated, which in return emits transition radiation. In this study a comparison for temporal field profile of THZ pulse has been investigated for different quasi-monoenergetic (momentum) distribution. Our calculation shows a rather well THZ signal for higher pick quasi-monoenergetic distribution but according to experimental results, most electrons bunches follow the Boltzmann energy distribution. In our comparison, for distributions consisting quasi- monoenergetic electron, we have shown that the optimum THZ coherent radiation will occur; furthermore in smaller emission (or observation) angle it would be considerable.

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1. Introduction

Over the last decade, the THz radiation is being the one of the interested research areas of scientific studies. There are various schemes to generate THz radiation, such as optical rectification, Quantum cascade intersubband, varactor Frequency doublers and photoconductive antenna. A direct way to generate the THz emission is using the femtosecond laser pulses passing through the electro-optic crystals, semiconductors and optical rectification [1, 2]. Recently, the THz emission is being generated by bending electron bunches in a magnetic field, traversing a refractive index [3]. The interaction of high intense femtosecond laser with plasma, leads to a well collimated ultra short *MeV* electron bunch due to the transverse wave breaking [4-7]. The plasma-vacuum boundary already acts as the dielectric discontinuity. The generation of electromagnetic pulses from plasma channels induced by femtosecond light strings, are studied by several authors [7, 8]. The sub-THz pulses emitted by the filamentary structure from an intense femtosecond laser pulse were also detected [9-13].

1.1. Transition radiation

In this work, we consider the plasma dielectric discontinuity for THz emission. Transition radiation is emitted by passing the electrons bunch particle through the boundary of two different dielectric materials. The electrons bunch could be considered at three different time steps: a) Through a medium with unity dielectric constant, b) Passing a boundary, c) Going into a second medium with a dielectric constant greater than unity. The relativistic electrons velocities cause the (Coulomb) self-field to have a more transverse orientation. Electrons bunch Propagation in the second medium would experience a screened bunch partial self-field due to the presence of background electrons. By considering the Maxwell equations and writing as the wave equation [14] we will have:

$$(c^2\nabla^2 - \frac{\partial^2}{\partial t^2})E = 4\pi \frac{\partial}{\partial t}(J_p + J_b) + 4\pi c^2 \nabla(\rho_\rho + \rho_b) \tag{1}$$

That in this equation $J_p, J_b, \rho_\rho, \rho_b$ and E are the plasma current, electron bunch current density, plasma charge density, electron bunch charge density and the electric field, respectively.

The effectiveness of the screening is proportional to the dielectric constant. It means that a discontinuity in electric field is occurred when electron passes the sharp interface. In order to cancel this discontinuity the background electrons at the surface are moved transversely so this will be a confirmation of the Maxwell's equations. The bunch-driven surface current is the main source for driving force and emission of electro-magnetic radiation that is referred to transition radiation. Using the continuity equation and Poisson's equation, we will have:

$$(c^2\nabla^2 - \varepsilon\omega^2)E = \frac{4\pi}{i\omega\varepsilon}[c^2\nabla(\nabla \cdot J_b) + \omega^2\varepsilon\ J_b] \tag{2}$$

that $\varepsilon = 1 - \frac{\omega_p^2}{\omega^2}$, $E = E_p + E_h$, E_p is particle field and E_h is radiation field. The complete solution to Eq. (2)

will yield both the particular solution (particle field E_p) as well as the homogeneous solution (radiation field E_h) that could lead us to Coherent transition radiation. Coherent transition radiation (CTR) is discussed in this paper. It is based on the emission of the LWFA- produced electron bunch as it propagates through the plasma-vacuum boundary. This was first observed experimentally by Leemans et al. [15].

1.2. Diffraction effects

The effect of a boundary with a finite transverse size causes a diffraction radiation (DR) that is added to TR. Therefore the total radiation field amplitude will be reduced by the DR effect.

The diffraction function varies by changing the boundary sizes, as the lower frequencies, experiences a suppression due to the diffraction effect.

1.3. Energy distributions

The CTR emission amplitude does not have a strong dependence on momentum *u* for relativistic electrons. The results show that the emission of higher energetic electrons bunch marginally is larger than the lower energetic electrons bunch. For the bunch produced by the laser wake field accelerator, the accelerated electrons are characterized by a large energy spread distribution, but the CTR expression can be simplified for the mono-energetic distribution.

The single and two picks of quasi-monoenergetic electron accelerated by the laser wake field accelerator, presented in the work of mangles, et al. [16].

2. CTR waveform in the temporal domain

This study is established on three main parts: **a)** Investigation of the transverse boundary size effect on the diffraction function and coherent transition radiation, **b)** monoenergetic electron bunches with different momentums and **c)** calculation of E_{CTR} in deferent emission angle (or angle of observation) .

a) **Investigation of the transverse boundary size effect on the diffraction function and Coherent transition radiation:** in this part, we calculated numerically the diffraction function $D(f,\rho)$ and $E_{CTR}(t)$ for different values of the transverse boundary size ρ. To do so we have used the Parseval's theorem in the expression of total energy (W) radiated through the $Z=Z_0$ plane in the far-field. [17]

By considering the Electrical field (E_{TR}) as the diffraction- limited transition radiation, we would have:

$$w = \frac{c}{4\pi}\int dt \int dxdy(\vec{E}_{TR}\times\vec{B}_{TR})\cdot e_z$$
$$= \frac{c}{2\pi}\int_0^\infty \frac{d\omega}{2\pi}\int\frac{d^2q}{(2\pi)^2}(k\cdot e_z)E^*(\omega,q,z)E(\omega,q,z) \quad (3)$$

Considering the relation

$$dk_x dk_y = 2qdq = k^*\cos\theta\, d\Omega \quad (4)$$

the spectral and angular differential energy distribution become as follow:

$$\frac{d^2w}{d\omega d\Omega} = \frac{\omega^2\cos^2\theta}{(2\pi)^4 c}E^*_{TR}(\omega,q,z).E_{TR}(\omega,q,z)_\perp \quad (5)$$

which can be written as:

$$\frac{d^2w}{d\omega d\Omega} = N\int du\ g(u)\left|\varepsilon D\right|^2 + N(N-1)\left|\int d^3rduf(r,u)\varepsilon De^{i\psi}\right|^2 \quad (6)$$

In above equation, $f(\mathbf{r}, u)$, and D are the momentum distribution function and diffraction function of the electron bunch, respectively. $f(\mathbf{r}, u)$ is normalized bythe relation $\int d^3rduf(r,u)=1$, also $g(u)$ is defined as a function which satisfies the condition $\int d^3rf(r,u)=g(u)$,and . On the right side of the Eq.(6), the first term is incoherent part of contribution which is neglected because of $N(N-1)>>N$,.

We calculated numerically the diffraction profile $D(f,\rho)$ as a function of the transverse boundary size and frequency domain, assuming a mono-energetic momentum distribution $g(u)=\delta(u-u_0)$, with longitudinal length $\sigma_z=15$ $_{\mu m}$, momentum u=10 and emission angle or angle of observation $\theta=0.3_{rad}$ .The result is shown in Fig. 1. The high stability of low terahertz frequency whit small size of transverse boundary can clearly be seen in this result.

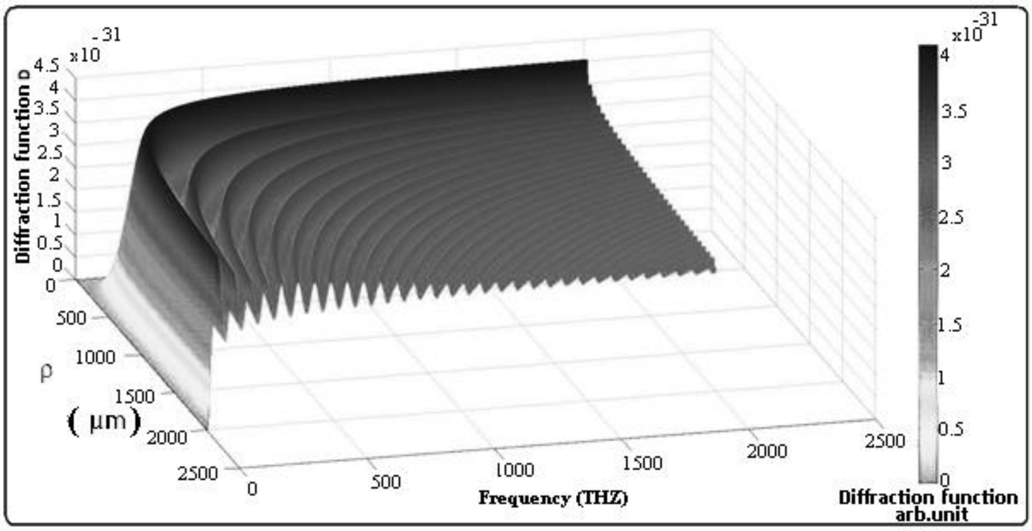

Fig. 1. The diffraction function $D(f,\rho)$ calculated for different transverse boundary sizes, up to 2000 $(\mu\, m)$.

By applying the method of stationary phase [14], the electric field profile of the coherent transition radiation pulse can be evaluated by:

$$E_{CTR}(x,\omega) = -\frac{2eN}{cR}\langle \varepsilon(\theta,u)D(\omega,u,\theta,\rho)F(\omega,u,\theta)\rangle e^{ikR} e_{\perp} \tag{7}$$

That in this equation, the emission angle, or angle of observation, is defined as the angle θ between the wave vector **k** and the z axis, $k=\omega/c$ and $\varepsilon(\theta,u)=\frac{u\sqrt{1+u^2}\sin\theta}{1+u^2\sin^2\theta}$. The unit vector $e_{\perp}$ lies in the $k-e_z$ plane, and is perpendicular to the vector of observation **k**. The normalized electron momentum u is related to the electron velocity through the formula $u=\gamma\beta$,where $\gamma=1/\sqrt{1-\beta^2}=\sqrt{1+u^2}$ and $\beta=v/c$. N, R, ρ, F and D are the number of electrons which incident at the interface between the plasma and vacuum , the distance from the emitting source to the observer, the transverse boundary size, the form factor and the diffraction function, respectively. Using the inverse fourier-transform integral, the temporal electric field profile $E_{CTR}(x,t)$ can be written :

$$E_{CTR}(x,t) = -\frac{eN}{\pi R}e_{\perp}\int dk\langle \varepsilon(\theta,u)D(k,u,\theta,\rho)F(k,u,\theta)\rangle e^{-ik(ct-R)} \tag{8}$$

Then by Considering a Gaussian form factor the electrical field profile of the coherent transition radiation can be calculated numerically. Fig. 2 depicts a single-cycle waveforms, with the field profile approaching a half-cycle profile by increasing the amount of transverse boundary sizeρ. One can also observe that as transverse boundary size decreases i) the amplitude of the field strength is reduced, ii) the negative side-wings in E_{CTR} (t) become more pronounced, iii) these side-wings move closer to the center of the pulse.

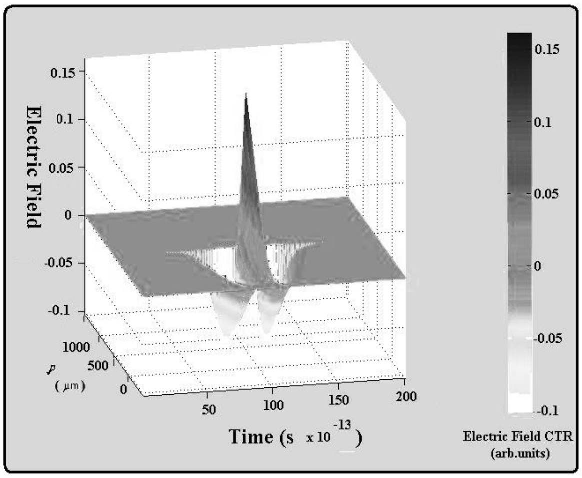

Fig. 2. Terahertz electrical field (E_{CTR}) calculated for different transverse boundary sizes, up to 1000 $(\mu\, m)$.

b) Coherent transition radiation electric field (E_{CTR}) generated by different electron momentum (u).

Fig. 3 shows the electrical field profile obtained in our study as a function of time (E_{CTR}) that generated by the different monoenergetic electron bunches. It can be concluded from this figure that by electrons momentum (u) increment, the amplitude of the electric field is increased. Although the electron bunch with $u = 40$ has 4 times more momentum than the bunch with the normalized electron momentum u = 10, the increase in emission amplitude is marginal. Furthermore, this figure demonstrates the weak momentum dependence of the electric field amplitude.

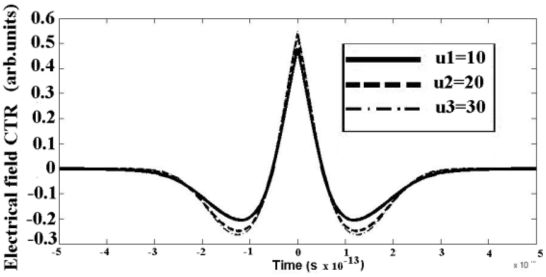

Fig. 3. The electric field profile E_{CTR} (t) for electron bunches with different momentum.

C-calculation of E_{CTR} in deferent emission angle or angle of observation

By using the Eq. (6), considering a full coherence with no diffraction limitation (*D* = *F* = 1) and

$$\varepsilon(\theta, u) = \frac{u\sqrt{1+u^2}\sin\theta}{1+u^2\sin^2\theta}$$, we calculated numerically

E_{CTR} for the bunch length $\sigma_z = 15_{\mu m}$ with the normalized electron momentum u = 10. Fig. 4 shows our calculated Coherent transition radiation electric field (E_{CTR}) versus different observation angles θ. The result shows a periodic emission (or observation) angle domain that indicates the low quality of terahertz predominantly is emitted at larger emission angles. In contrast at the smaller observation angle, the maximum amplitude of radiation will obtain.

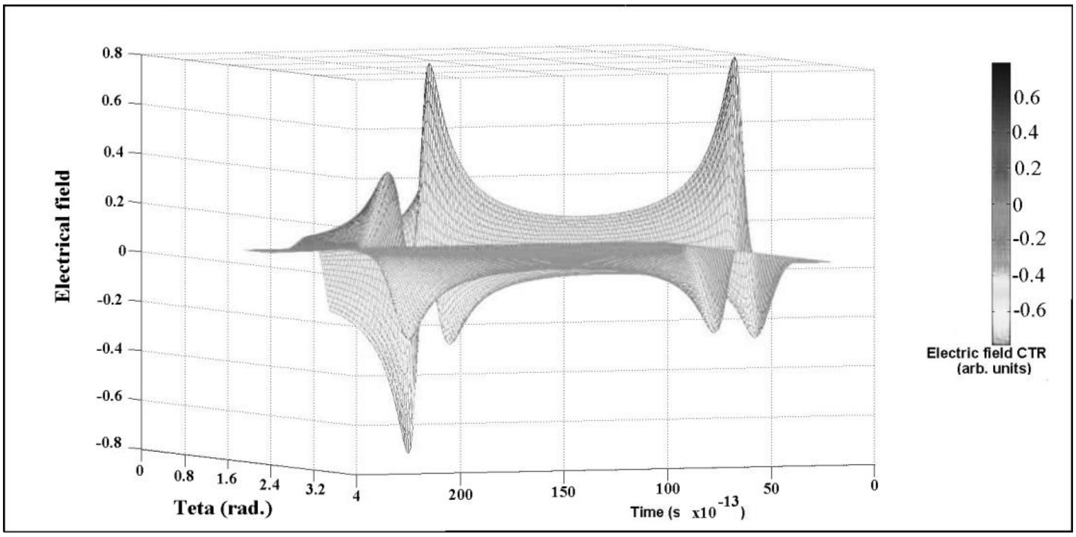

Fig. 4. Calculated Coherent transition radiation electric field (E_{CTR}) versus different observation angles θ. The bunch length $\sigma_z = 15_{\mu m}$ with the electron momentum u = 10. Full coherence with no diffraction was considered.

3. E_{CTR}(t) for various electron momentum distribution

In the Fig. 5 we have compared the CTR waveforms of a mono-energetice momentum $g(u) = \delta(u - u_0)$ with the electron momentum value u= 10 to the Boltzmann momentum distribution $g(u) = \frac{1}{u_t}\exp(\frac{-u}{u_t})$ with $u_t = 10$. It can be seen that the amplitude of the coherent transition radiation electric field for monoenergetic electron distribution is nearly %20 greater than the one for Boltzman distribution at $u_0 = u_t = 10$.

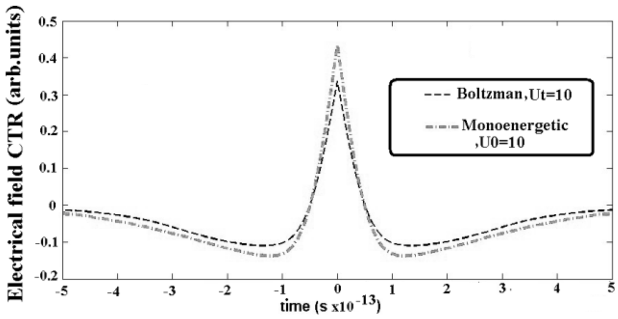

Fig. 5. The electric field profile E_{CTR}(t) at different momentum distributions.

4. Conclusion

This work presents the numerical results of coherent transition radiation from a plasma-vacuum boundary. We have shown that the electrons bunch with femtosecend (fs) duration that are produced by LWFA, can generate coherent transition radiation when the radiation wavelength is longer than the bunch length. In this study, our new investigation also compares the pulse shape of CTR emission amplitude for different number of electron bunch in monoenergetic energy distribution and Boltzman distributions. Our results show that there is a more effective CTR signal emission for the monoenergetic electron bunches contrary to the Boltzmann distribution. Consequently CTR waveform depends on the transverse boundary size ρ and also the value of electron momentum distribution (u) and emission angle.

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