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Eccentric connectivity index of bridge graphs

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The eccentric connectivity index $\xi(G)$ of the graph $G = G(V, E)$ is defined as $\xi(G) = \sum_{u \in V} \deg_G(u)\varepsilon(u)$ where $\deg_G(u)$ denotes the degree of vertex *u* in the graph G and $\varepsilon(u)$ is the largest distance between *u* and any other vertex *v* of *G*. In this paper, we calculate the eccentric connectivity index of bridge graph of the given graphs and distinguished vertices of them.

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1. Introduction

The graph theory has successfully provided chemists with a variety of very useful tools, namely, the topological index. A topological index is a numeric quantity of the structural graph. In this paper, all of the graphs are assumed as connected simple graphs that are undirected.

Suppose that $G = G(V, E)$ be a graph. The eccentric connectivity index ξ(*G*) of the graph *G* is defined as $\xi(G) = \sum_{u \in V} \deg(u)\varepsilon(u)$, where for a given vertex *u* of *V*, its eccentricity ε(*u*) is the largest distance between *u* and any other vertex v of G that their distance *d*(*u*, *v*) as the length of the shortest path connecting *u* and *v* in *G*. The maximum eccentricity over all vertices of *G* is called the diameter of *G* and denoted by *D*(*G*) and the minimum eccentricity among the vertices of *G* is called the radius of *G* and denoted by *R*(*G*). The set of vertices whose eccentricity is equal to the radius of *G* is called the center of *G*. It is well known that each tree has either one or two vertices in its center. In some research papers [1-15], the authors have computed the topological index eccentric connectivity index ξ(*G*) of some graphs. The aim of this article is to continue this problem and compute the eccentric connectivity index of a bridge graph of the given graphs and distinguished vertices of them.

Suppose that $G_1 = G(V_1, E_1)$ and $G_2 = G(V_2, E_2)$ are two graphs that the vertices sets V_1 and V_2 are disjoint. Let $u_1 \in V_1$ and $u_2 \in V_2$ are given. The bridge graph of these two graphs with respect to u_1 and u_2 that is $B = B(G_1, G_2, u_1, u_2)$, is defined a graph that its vertices set $V(B) = V_1 \bigcup V_2$ and its edges set $E(B) = E_1 \bigcup E_2 \bigcup \{u_1u_2\}$ where u_1u_2 is a new edge. In the same way, we can define the bridge graph of the n graphs. Suppose that $G_1 = G(V_1, E_1)$, $G_2 = G(V_2, E_2)$, … and $G_n = G(V_n, E_n)$ for $n > 2$ be graphs for which the vertices sets V_1, V_2, … and V_n are disjoint. Let $u_1 \in V_1$; $u_{2,1}, u_{2,2} \in V_2$; $u_{3,1}, u_{3,2} \in V_3$; …; $u_{n-1,1}, u_{n-1,2} \in V_{n-1}$ and $u_n \in V_n$ are given. The bridge graph of these n graphs with respect to these given vertices that is

$$B = B(G_1, G_2, ..., G_n, u_1, u_{2,1}u_{2,2}, u_{3,1}, u_{3,2}, ..., u_{n-1,1}, u_{n-1,2}, u_n)$$

is defined as a graph that its vertices set $V(B) = V_1 \bigcup V_2 \bigcup ... \bigcup V_n$ and its edges set $E(B) = E_1 \bigcup E_2 \bigcup ... \bigcup E_n \bigcup \{u_1u_{2,1}, u_{2,2}u_{3,1}, ..., u_{n-1,2}u_n\}$ where $u_1u_{2,1}, u_{2,2}u_{3,1}, ...,$ $u_{n-1,2}u_n$ are new edges.

2. Main results

In this section, we first compute the eccentric connectivity index of the bridge graph of two given graphs, then the eccentric connectivity index of the bridge graph of n given graphs for $n > 2$ will be computed.

Theorem 1. Suppose that $G_1 = G(V_1, E_1)$ and $G_2 = G(V_2, E_2)$ are two graphs that the vertices sets V_1 and V_2 are disjoint. Let $u_1 \in V_1$; $u_2 \in V_2$ are given and $B = B(G_1, G_2, u_1, u_2)$ is the bridge graph of these two graphs with respect to u_1 and u_2. For a given vertex $u \in V(B)$, if $\varepsilon_1(u)$ be the eccentricity of u, as a vertex of G_1 and $\varepsilon_2(u)$ be the eccentricity of u, as a vertex of G_2, then the eccentric connectivity index of the bridge graph B, $\xi(B)$, is given by

$$\xi(B)=\sum_{u\in V_1}\deg_B(u).Max\{d(u,u_1)+\varepsilon_2(u_2)+1;\varepsilon_1(u)\}$$
$$+\sum_{u\in V_2}\deg_B(u).Max\{d(u,u_2)+\varepsilon_1(u_1)+1;\varepsilon_2(u)\}.$$

Proof. By definition of the eccentricity of a vertex in a graph, we conclude that for any vertex $u\in V(B)$, $\varepsilon(u)$ the eccentricity of u, as a vertex of B is given by $\varepsilon(u)=Max\{d(u,u_1)+\varepsilon_2(u_2)+1;\varepsilon_1(u)\}$, if $u\in V_1$ and similarly $\varepsilon(u)=Max\{d(u,u_2)+\varepsilon_1(u_1)+1;\varepsilon_2(u)\}$, if $u\in V_2$. This completes our proof.

Theorem 2. Suppose that $G_1=G(V_1,E_1)$, $G_2=G(V_2,E_2)$, …, $G_n=G(V_n,E_n)$, $n>2$, and the vertex sets V_1, V_2, …and V_n are disjoint. Let $u_1\in V_1$; $u_{2,1},u_{2,2}\in V_2$; $u_{3,1},u_{3,2}\in V_3$; …; $u_{n-1,1},u_{n-1,2}\in V_{n-1}$ and $u_n\in V_n$ are given. We also assume that
$$B=B(G_1,G_2,...,G_n,u_1,u_{2,1}u_{2,2},u_{3,1},u_{3,2},...,u_{n-1,1},u_{n-1,2},u_n)$$
is the bridge graph of these n graphs with respect to these given vertices. If $\varepsilon_i(u)$ be the eccentricity of u, as a vertex of G_i, for $i=1,2,...,n$, then the eccentric connectivity index of the bridge graph B, $\xi(B)$ is given by

$$\xi(B)=\sum_{u\in V_1}\deg_B(u)Max\{d(u,u_1)+(\sum_{i=2}^{n-1}d_i)+\varepsilon_n(u_n)+n-1;\varepsilon_1(u)\}+$$
$$\sum_{j=2}^{n-1}\sum_{u\in V_j}\deg_B(u)Max\{\varepsilon_1(u_1)+(\sum_{i=2,i\neq j}^{n-1}d_i)+d(u,u_{j,1})+d(u,u_{j,2})+\varepsilon_n(u_n)+n-1;\varepsilon_j(u)\}+$$
$$\sum_{u\in V_n}\deg_B(u)Max\{\varepsilon_1(u_1)+(\sum_{i=2}^{n-1}d_i)+d(u,u_n)+n-1;\varepsilon_n(u)\},$$
where $d_i=d(u_{i,1},u_{i,2})$.

Proof. By the definition of the eccentricity of a vertex in a graph, we can see that for any vertex $u\in V(B)$, $\varepsilon(u)$ the eccentricity of u, as a vertex of B is given by $\varepsilon(u)=Max\{d(u,u_1)+(\sum_{i=2}^{n-1}d_i)+\varepsilon_n(u_n)+n-1;\varepsilon_1(u)\}$, if $u\in V_1$ and also
$\varepsilon(u)=$
$Max\{\varepsilon_1(u_1)+(\sum_{i=2,i\neq j}^{n-1}d_i)+d(u,u_{j,1})+d(u,u_{j,2})+\varepsilon_n(u_n)+n-1;\varepsilon_j(u)\}$,
if $u\in V_j$ for $j=2,3,...,n-1$. We can see similarly that $\varepsilon(u)=Max\{\varepsilon_1(u_1)+(\sum_{i=2}^{n-1}d_i)+d(u,u_n)+n-1;\varepsilon_n(u)\}$, if $u\in V_n$. This completes our proof.

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