

# Discrete arbitrary polygon electromagnetic cloak based on tensor transmission-line metamaterials elements

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Existing discrete electromagnetic (EM) cloaks are often limited to circular shape and always based on one dimensional discretization of radius variable in cylindrical coordinates. In this paper, we proposed a two dimensional (2D) discrete arbitrary polygon invisible cloak, not limited to symmetrical shapes. A tensor transmission-line (TL) unit cell method is brought in to realize the anisotropic element in the proposed cloak, and the relative formulas between the medium parameters and the lumped circuit parameters are deduced. Furthermore, numerical simulations of typical polygon cloaks are performed, verifying the validity and feasibility of the proposed approach. In summary, the proposed physically realizable cloak could provide a practical approach to an experimental demonstration of electromagnetic cloaking.

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## 1. Introduction

After Pendry invented the transformation optics (TO) [1] which can control electromagnetic waves and tailor material properties to obtain desired and unprecedented electromagnetic behaviours in 2006, ‘invisible cloak’, as an interesting application of TO, has become a hotspot in electromagnetic research. In EM cloaking, a material is employed to render a volume effectively invisible to incident EM radiation. The design process for the cloak involves a coordinate transformation that squeezes space from a volume into a shell surrounding the concealment volume. Maxwell’s equations are form-invariant to coordinate transformations, so that only the components of the permittivity  $\epsilon$  and the permeability  $\mu$  are affected by the transformation [2]. The possibility of the ‘invisibility’ was lately studied intensively by analytical and numerical methods [3-7]. In addition to invisibility, the mirage effects of an invisible cloak have been numerically demonstrated in [8] as well. However, these cloaks often require both continuously varying and anisotropic media parameters which always aren’t able to be implemented in practice. Thus, designing a physically realizable cloak has become an important and challenging work.

## 2. Theory

As a realization approach, the concept of discrete cloak was first proposed in [8] where a discrete circular cloaking shell was implemented in a stepwise homogeneous eight-layer approximation of the continuous material parameters, and similar invisibility was achieved compared to the ideal continuous circular cloak. Such discrete cloak has also been presented in [9], where a concentric layered structure was proposed to substitute the continuous anisotropic media and the low-reflection and power-flow bending properties was obtained. However, most studies on discrete invisible cloaks have been limited to circular shape. Lots of discussions have focused on the spherical or circular cylindrical cloaks and were always based on cylindrical coordinates which only can deal with such rotational symmetry shapes. The discretization are often performed in the radial direction of the symmetrical structures. Nevertheless, in practice, it is often desirable to have the shapes of invisible cloaks be conformally tailored with the objects to be concealed. Most recently, design of cloaks with arbitrary shapes was proposed in several papers [11-15]. However, most of them only deal with cloak structures with continuous media parameters which generally can’t be physically realizable.

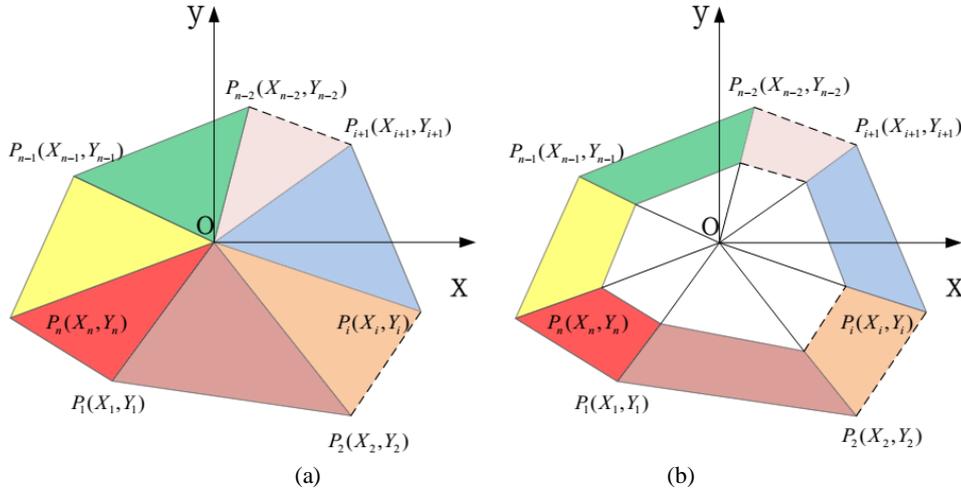


Fig. 1. The geometry transformation of an arbitrary polygon shape cloak 16. (a) Original space; (b) transformed space. Sub-section with same color in sub-figure (a) maps into corresponding sub-sections in sub-figure (b)

Usually, it is always convenient to deal with arbitrary shapes in Cartesian coordinates [16]. But it could lead to anisotropic discrete elements when we perform discretization in the two directions ( $x$  and  $y$ ) of Cartesian coordinates. Therefore, to make a physically realizable EM cloak, we should design a physically realizable anisotropic element at first. Fortunately, in [16], the authors introduced and analyzed a transmission-line metamaterial with arbitrary full tensors, furthermore, they proposed a 2D discrete circular EM cloak with prominent performances. On the basis of [16], a 2D discrete arbitrary polygon invisible cloak is proposed in this letter. The proposed invisible cloak consists of several discrete elements. Each element is assigned with a proper anisotropic EM parameters. And we bring in a physically realizable approach to these anisotropic element through a tensor transmission-line unit cell. Furthermore, we deduce the relationship between the media parameters and the lumped circuit parameters. Full wave simulations are performed to typical discrete polygon cloaks, verifying the proposed method. In summary, this novel cloaking structure could be possibly realized, and could provide a practical approach to an experimental demonstration of electromagnetic cloaking.

Following the approach in Ref [16], an arbitrary polygon shape cloak with  $n$  vertices  $P(X, Y), P(X, Y), \dots, P_n(X_n, Y_n)$  is shown in Fig. 1. This polygon cloak is decomposed into  $n$  sections and the transformed permittivity and permeability tensors of each section could be derived as

$$\varepsilon'_{xx} = \frac{(1-\tau^2)x'^2 + \tau^2 U_i^2 V_i^2 x'^2 + 2\tau U_i V_i x' y' + y'^2}{(x'^2 + y'^2)(1-\tau U_i)} \quad (1)$$

$$\varepsilon'_{xy} = \frac{-\tau^2 U_i (2-\tau U_i) x' y' + \tau^2 U_i^2 V_i^2 x'^2 - \tau U_i V_i (x'^2 - y'^2)}{(x'^2 + y'^2)(1-\tau U_i)} \quad (2)$$

$$\varepsilon'_{yy} = \frac{(1-\tau^2)y'^2 + \tau^2 U_i^2 V_i^2 y'^2 - 2\tau U_i V_i x' y' + x'^2}{(x'^2 + y'^2)(1-\tau U_i)} \quad (3)$$

$$\varepsilon'_{zz} = \frac{1-\tau U_i}{(1-\tau)^2} \quad (4)$$

with

$$U_i = \frac{Y_i X_{i+1} - X_i Y_{i+1}}{y'(X_{i+1} - X_i) - x'(Y_{i+1} - Y_i)} \quad (5)$$

$$V_i = \frac{x'(X_{i+1} - X_i) + y'(Y_{i+1} - Y_i)}{y'(X_{i+1} - X_i) - x'(Y_{i+1} - Y_i)} \quad (6)$$

where  $\tau$  which is smaller than 1 represents the linear compression ratio. From equation (1)-(4), the obtained media parameters are continuous spatial functions with two dimensional variables  $x$  and  $y$ . obviously, such continuous inhomogeneous and anisotropy material is often physically unrealizable.

Our idea here is to conduct discretization respectively along the  $x$  and  $y$  direction. Then the original continuous cloak transferred to a discrete cloak consists of several homogeneous elements, which can be looked as an approximation of the original cloak. As shown in Fig. 2, the discretization is performed in each section divided by the lines from the origin to the outer vertices. In the  $i$ th section, we make  $x'_{im} = X_i + mdx'$  and  $y'_{in} = Y_i + ndy'$  respectively, where  $m$  and  $n$  denote the  $m$ th and  $n$ th discrete point in the  $x$  and  $y$  direction,  $dx'$  and  $dy'$  denotes the discrete space.

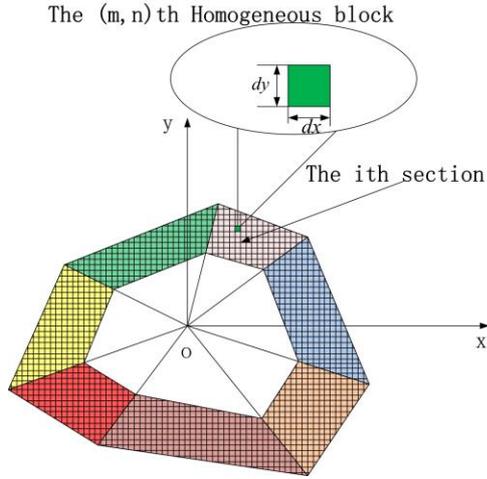


Fig. 2. Discretization an arbitrary polygon shape cloak. Different colors represent different transformed sub-sections

Then we substitute  $x'_{im}$  and  $y'_{in}$  into equation (1)-(6), and obtain the following media parameters of the discrete cloak in each homogeneous element,

$$\begin{aligned} \varepsilon'_{ixx}(mn) &= \frac{1}{(x'^2_{im} + y'^2_{in}(mn))(1 - \tau U_i(mn))} \\ &\times [(1 - \tau^2)x'^2_{im} + \tau^2 U_i(mn)^2 V_i(mn)^2 x'^2_{im} \\ &+ 2\tau U_i(mn) V_i(mn) x'_{im} y'_{in} + y'^2_{in}] \end{aligned} \quad (7)$$

$$\begin{aligned} \varepsilon'_{ixy}(mn) &= \frac{1}{(x'^2_{im} + y'^2_{in}(mn))(1 - \tau U_i(mn))} \\ &\times [-\tau U_i(mn)(2 - \tau U_i(mn)) x'_{im} y'_{in} x'^2_{im} \\ &+ \tau^2 U_i^2(mn) V_i^2(mn) x'^2_{im} \\ &- \tau U_i(mn) V_i(mn) (x'^2_{im} - y'^2_{in})] \end{aligned} \quad (8)$$

$$\begin{aligned} \varepsilon'_{iyy}(mn) &= \frac{1}{(x'^2_{im} + y'^2_{in}(mn))(1 - \tau U_i(mn))} \\ &\times [(1 - \tau^2)y'^2_{in} + \tau^2 U_i(mn)^2 V_i(mn)^2 y'^2_{in} \\ &- 2\tau U_i(mn) V_i(mn) x'_{im} y'_{in} + x'^2_{im}] \end{aligned} \quad (9)$$

$$\varepsilon'_{izz} = \frac{1 - \tau U_i(mn)}{(1 - \tau)^2} \quad (10)$$

with

$$U_i(mn) = \frac{Y_i X_{i+1} - X_i Y_{i+1}}{y'_{in}(X_{i+1} - X_i) - x'_{im}(Y_{i+1} - Y_i)} \quad (11)$$

$$V_i(mn) = \frac{x'_{im}(X_{i+1} - X_i) + y'_{in}(Y_{i+1} - Y_i)}{y'_{in}(X_{i+1} - X_i) - x'_{im}(Y_{i+1} - Y_i)} \quad (12)$$

where  $\tau$  which is smaller than 1 represents the linear compression ratio, and the  $m$  and  $n$  index denote the  $m$ th and  $n$ th homogeneous element.

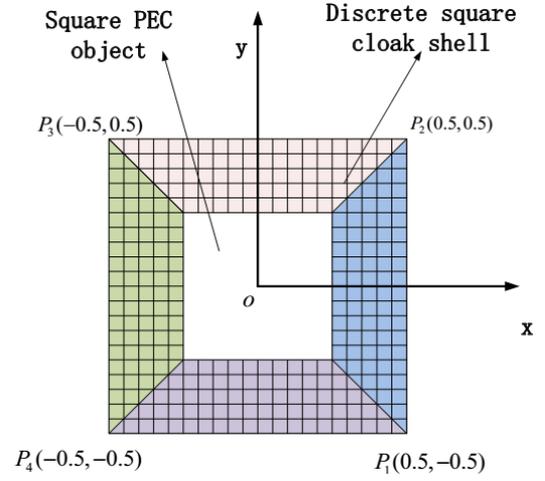


Fig. 3. Discrete square shape cloak. Different colors represent different transformed sub-sections

For validation of the above discretization, the COMSOL multiphysics finite element-based EM solver is used for simulations because of its flexibility in specifying material anisotropy. For simplicity, and as a special case in polygon shapes, here we demonstrate a 2D discrete square shaped cloak which applies the derived parameters in equation (7)-(10) as design material parameters. As shown in Fig. 3, the square shaped cloak is a shell between two square. The outer square has a side length of 1m, and the inner square has a side length of 0.5 m, respectively. The four vertices of the outer square are  $P_1(0.5, -0.5)$ ,  $P_2(0.5, 0.5)$ ,  $P_3(-0.5, 0.5)$ , and  $P_4(-0.5, -0.5)$ . The linear compression ratio  $\tau$  is 0.5 here. According to the TO method in Ref [16], we divided the shell cloak into four sections by the four lines between the origin and the four vertices. In each section of the shell, discretization is performed. Both the  $x$  and  $y$  directional discrete space are chosen to be 0.1  $m(\lambda/3)$ , 0.05  $m(\lambda/6)$ , 0.025  $m(\lambda/12)$ , respectively, where  $\lambda$  is the free space wavelength. We assume there exist a perfect conductor whose edges are closely coincide with the inner square. After setting the frequency of the TE background plane waves to 1 GHz, the simulation is performed and the results are shown in Fig. 4. It is seen in the figure intuitively that the more discrete elements with the better cloaking performance. And in Fig. 4(c) and Fig. 4(d), we can find that similar invisibility can be achieved between discretization and the ideal case. It means that this 2D discrete cloak would be a possible approximation to the original continuous cloak after performing proper discretization.

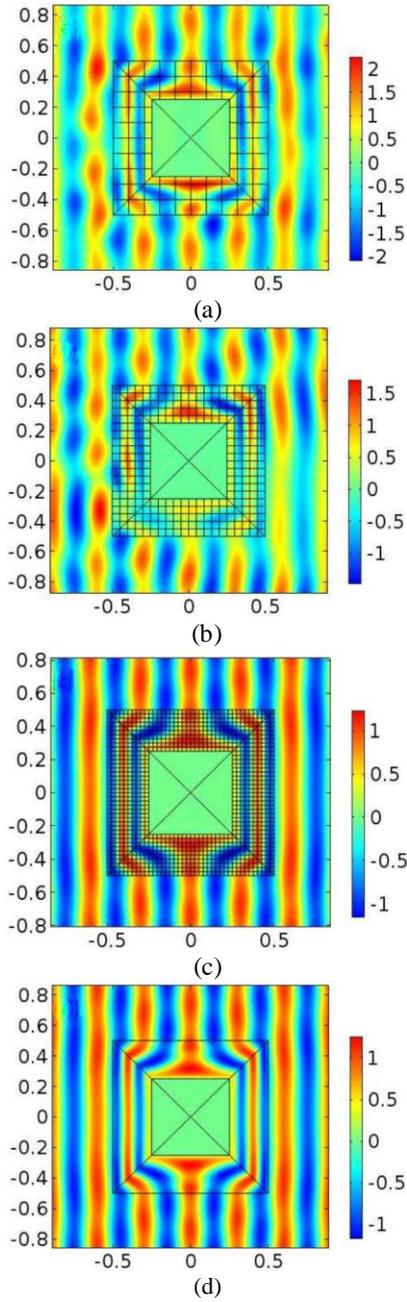


Fig. 4. Comparison of electric fields distribution with different numbers of discrete elements. (a)  $dx=dy=0.1m$ ; (b)  $dx=dy=0.05 m$ ; (c)  $dx=dy=0.025 m$ ; (d) the original ideal continuous cloak

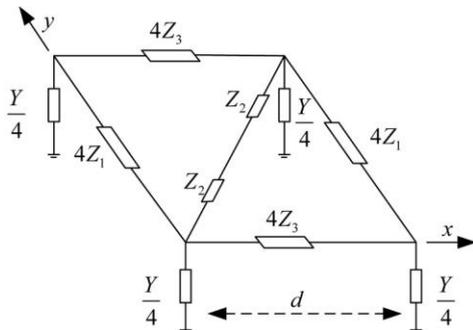


Fig. 5. Equivalent circuit of a transmission line metamaterial unit cell [16]

Here, we show how an anisotropic element can be realized. In above sections, we have obtained the tensors for each discrete homogeneous elements, and then the design mission changes to find a physically realizable anisotropic element with a desired permittivity and permeability tensor. Fortunately, Ref [16] proposed a TL metamaterial cell which can be used to realize an anisotropic element as we required. As shown in [16], a TL metamaterial that can be represented by a full tensor is demonstrated in Fig. 5. This 3-branch structure is in general anisotropic due to the different impedances of  $Z_1$ ,  $Z_2$ ,  $Z_3$  and admittance of  $Y$ . Assume the permittivity, the permeability tensor and the impedance tensor are  $\bar{\bar{\epsilon}}$ ,  $\bar{\bar{\mu}}$ ,  $\bar{\bar{Z}}$ , respectively. Then,

$$\bar{\bar{\epsilon}} = \begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{xz} & \epsilon_{yz} & \epsilon_{zz} \end{pmatrix} \quad (13)$$

$$\bar{\bar{\mu}} = \begin{pmatrix} \mu_{xx} & \mu_{xy} & \mu_{xz} \\ \mu_{yx} & \mu_{yy} & \mu_{yz} \\ \mu_{xz} & \mu_{yz} & \mu_{zz} \end{pmatrix} \quad (14)$$

$$\bar{\bar{Z}} = \begin{pmatrix} z_{xx} & z_{xy} & z_{xz} \\ z_{yx} & z_{yy} & z_{yz} \\ z_{xz} & z_{yz} & z_{zz} \end{pmatrix} = \begin{pmatrix} \frac{2z_3(z_1+z_2)}{z_1+z_2+z_3} & \frac{-2z_1z_3}{z_1+z_2+z_3} & 0 \\ \frac{-2z_1z_3}{z_1+z_2+z_3} & \frac{2z_1(z_2+z_3)}{z_1+z_2+z_3} & 0 \\ 0 & 0 & \frac{1}{Y} \end{pmatrix} \quad (15)$$

For TE planar wave case, the deduced relationship between the media parameters and the circuit impedance parameters are

$$Y = j\omega\epsilon_{zz}d \quad (16)$$

$$z_{xx} = j\omega\mu_{xx}d \quad (17)$$

$$z_{xy} = -j\omega\mu_{xy}d \quad (18)$$

$$z_{yx} = -j\omega\mu_{yx}d \quad (19)$$

$$z_{yy} = j\omega\mu_{yy}d \quad (20)$$

where  $d$  represents the cell length,  $\omega$  represents the radian frequency.

In order to build the relationship between the media parameters and the lumped circuit parameters, we define

$$Z_1 = j\omega L_1 \quad (21)$$

$$Z_2 = j\omega L_2 \quad (22)$$

$$Z_3 = j\omega L_3 \quad (23)$$

$$Y = j\omega C \quad (24)$$

where  $L_1, L_2, L_3$ , and  $C$  represent lumped inductances and capacitance, respectively. And we substitute them into equation (15)-(20), then we can obtain the following lumped parameters

$$L_1 = \frac{4\mu_{xy}^2 d - \mu_{xx}\mu_{yy}d}{4\mu_{xy} - 2\mu_{yy}} \quad (25)$$

$$L_2 = \frac{\mu_{xx}\mu_{yy}d - 4\mu_{xy}^2 d}{4\mu_{xy}} \quad (26)$$

$$L_3 = \frac{4\mu_{xy}^2 d - \mu_{xx}\mu_{yy}d}{4\mu_{xy} - 2\mu_{xx}} \quad (27)$$

$$C = \varepsilon_{zz}d \quad (28)$$

### 3. Results and discussion

We propose a 2D discrete arbitrary polygon EM cloak based on TL metamaterials, and validate its EM performance through full wave simulation. In the simulation model, the six vertices of the outer square are  $P_1(0.5, -0.1)$ ,  $P_2(0.3, 0.5)$ ,  $P_3(-0.4, 0.5)$ ,  $P_4(-0.5, 0)$ ,  $P_5(-0.2, -0.5)$ ,  $P_6(0.3, -0.5)$ , respectively, and the linear compression ratio  $\tau$  is 0.5. Here, we set the discrete space to 0.025m ( $\lambda/12$ ) in both  $x$  and  $y$  directions. Then we calculated the circuit parameters in each unit cell as mentioned in above section. We assume a 1GHz planar EM wave propagates from the left to the right and execute simulation. As shown in Fig. 6, the electric fields distribution and power flow bending properties are quite prominent (the black arrow in the figure represent the power flow of the TE wave), it means that the proposed discrete cloak has an excellent EM cloaking ability.

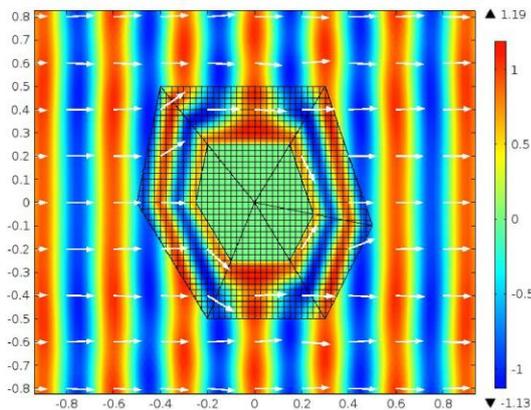


Fig. 6. The calculated electric-field distribution and power flow around the arbitrary polygon conducting cylinder with the proposed cloak

### 4. Conclusions

In summary, we apply a 2D discretization approach

and a tensor TL method to solve the challenging non-homogeneous and anisotropic problems in metamaterial realization, respectively. A novel 2D discrete arbitrary cloak is proposed in this letter. The simulated EM performances such as electric field distribution and power flow bending properties show that the proposed cloak can be chosen to be a suitable candidate for physically realizable cloak. Furthermore, this technology proposed in this letter will also find use in the future design of microwave devices based on transformation optics.

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