# Determination of field profiles in multiple superconducting quantum well optical planar waveguides

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This paper reports a new generalized dispersion equation and a intersection between the real parts of effective propagation constants for  $TE_0$  and  $TM_0$  and  $TE_1$  and  $TM_1$  modes as a function of number of glass layers in a multiple superconducting quantum well optical waveguide where the lossless glass layers have refractive index lower than that of the lossy superconducting layers. For the imaginary parts of the effective propagation constants, the intersections appear between  $TE_0$ ,  $TE_1$ ,  $TE_2$  modes or between  $TM_0$ ,  $TM_1$ ,  $TM_2$  modes. An increase in number of the quantum well, which is equivalent with an increase in refractive index, leads to a tighter confinement of the TE and TM modes to the core region.

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#### 1. Introduction

Numerical simulation of the optical and electronic properties of a structure containing multiple quantum wells presents difficulties since the refractive index and the effective mass vary significantly and the widths of wells and barriers are very small [1-2]. The bandgap discontinuities of the quantum wells and barriers confine the carries (electrons and holes) to the active core region, forcing them to occupy a planar region and these particles can only have discrete energy values. Furthermore, the waveguide structure confines the optical field to the same region.



Fig. 1. The real part of the refractive index profile for a multiple superconducting well structure. The inset is a structure with a single glass layer.

The TE and TM modes for a superconducting multiple quantum well structure (Fig. 1) satisfy the equations [2]

$$\frac{d^2\psi(x)}{dx^2} + k^2 n^2(x)\psi(x) = \beta^2\psi(x)$$
(1)

where

$$n(x) = \begin{cases} n_1, & \text{for } (n-1)\Lambda + b < x < n\Lambda, n = 1, 2, ..., N, \\ n_2, & \text{for } (n-1)\Lambda < x < (n-1)\Lambda + b, n = 1, 2, ..., N+1, \\ n_3, & \text{for } x < 0, x > n\Lambda + b. \end{cases}$$
(2)

*N* is number of the glass layers, *N* +1 is number of the superconducting layers, *a* and *b* are the thicknesses of the alternating layers, A = a + b is a periodicity length, L = NA + b is the total thickness of the waveguide,  $n_1$  is the refractive index of the glass layers,  $n_2$  is the refractive index of the superconducting (YBCO) quantum wells,  $n_3$  is the refractive index of the claddings (air),  $\beta$  is the propagation constant, and *k* is the free space wave number. For the lowest confined (guided) state propagation constant ( $kn_3 < \text{Re} (\beta) < kn_1 < \text{Re} (kn_2)$ ) we have a sinusoidal behaviour of the field in well and an exponential decay in claddings. We apply the boundary conditions for the TE (TM) modes [ $\Psi = E_y$  and  $d\Psi/dx$ , ( $\Psi = H_y$  and  $(1/n^2) (d\Psi/dx)$ ) are continuous at each interface].



Fig. 2. The real part and imaginary parts of the fundamental field profiles  $(-E_{y}, --H_{y})$  of a multiple superconducting well waveguide with 55 glass layers (a = 15nm, b = 20nm) as a function of the depth distance.

In this paper we give a new generalized dispersion equation and determine the effective propagation constants for a superconducting multiple quantum well structure.

#### 2. Multiple superconducting quantum well optical waveguide

Our dispersion equations [2], for determining  $\beta$  of TE modes (similar with the numerical Runge-Kutta method), can be generalized and extended to the TM modes:

$$A_{2(N+1)} = -\frac{\alpha_1}{n_3^{2\xi}},\tag{3}$$

where

$$\alpha_1 = \sqrt{\beta^2 - (n_3 k)^2}, \gamma_1 = \sqrt{(n_2 k)^2 - \beta^2}, \gamma_2 = \sqrt{(n_1 k)^2 - \beta^2} \quad (4)$$

$$A_1 = \frac{\alpha_1}{n_3^{2\xi}},$$
 (5)

$$A_{n} = \begin{cases} -\frac{\gamma_{1}}{n_{2}^{2\xi}} \tan\left[\gamma_{1}b - \arctan\left[\frac{A_{n-1}}{\gamma_{1}}n_{2}^{2\xi}\right]\right], n = 2, 4, \dots, 2N+2, \\ -\frac{\gamma_{2}}{n_{1}^{2\xi}} \tan\left[\gamma_{2}a - \arctan\left[\frac{A_{n-1}}{\gamma_{2}}n_{1}^{2\xi}\right]\right], n = 3, 5, \dots, 2N+1, \end{cases}$$
(6)

where  $\xi$  reads as 0 for TE modes and 1 for TM modes.

Also, we have solved the wave equations and Schrödinger equations for the given boundary conditions (the Dirichlet boundary condition at the ends of the interval where the wave function can be approximated with 0 and the Neuman boundary condition for each coordinate of the interfaces) by using the Galerkin's variant of the finite element method, with triangular grid and variable step [3].

## 3. Numerical results and conclusion

The investigated multiple superconducting quantum well optical waveguide structure (Fig.1) at wavelength  $\lambda = 1.55 \mu m$  consist of 56 lossy superconducting layers, each of thickness b = 20 nm, interspersed with 55 lossless glass layers of width a = 15 nm. The refractive indices of the barriers (glass), superconducting quantum wells (YBCO), and cladding materials (air) are  $n_1 = \sqrt{2.07}$ ,  $n_2 = \sqrt{2.32 - 1.536j}$  [4] and  $n_3 = 1$ , respectively.



Fig. 3. The real part of the exact effective propagation constants as a function of number of glass layers for the modes  $TE_0$ ,  $TM_0$ ,  $TE_1$ ,  $TM_1$ ,  $TE_2$  and  $TM_2$  of the multiple superconducting quantum well structure (a = 15nm, b = 20nm).

Our calculated value of the effective propagation constant for  $TE_0$ ,  $TM_0$ ,  $TE_1$ ,  $TM_1$ ,  $TE_2$  and  $TM_2$  modes are 1.47965 – 0.29278j, 1.52824 – 0.23763j, 1.36852 – 0.30252j, 1.38329 – 0.25685j, 1.16682 – 0.31820j and 1.10835 – 0.26473j, respectively. Fig. 2. shows the real part and imaginary parts of the fundamental field profiles (-  $E_y$ , - -  $H_y$ ) of this multiple superconducting well waveguide as a function of the depth distance.



Fig. 4. The imaginary part of the exact effective propagation constants as a function of number of glass layers for the modes  $TE_0$ ,  $TM_0$ ,  $TE_1$ ,  $TM_1$ ,  $TE_2$  and  $TM_2$ 

# of the multiple superconducting quantum well structure (a = 15 nm, b = 20 nm).

The field amplitude has been normalized to a maximum value of unity. The maximum real values of the field intensity for TE<sub>0</sub> and TM<sub>0</sub> modes are in the middle of the waveguide structure where the lossless glass layer has refractive index lower than that of the lossy superconducting layers. This phenomen of confining and guiding light in low index material can be explained by total internal reflection when the glass gap is narrower than the characteristic decay length of the evanescent tail of the modes inside the glass layer and the tails merge into a high intensity. A similar situation appears in photonic crystal slotted slab waveguides [5]. Figs. 3 - 4. show the real and imaginary parts of the exact effective propagation constants as a function of number of glass layers for the TE<sub>0</sub>, TM<sub>0</sub>, TE<sub>1</sub>, TM<sub>1</sub>, TE<sub>2</sub> and TM<sub>2</sub> modes.

As another example, we have calculated the exact value of the effective index  $\beta/k$  for a superconducting quantum well waveguide with the same thickness (1945nm) at wavelength  $\lambda = 1.55 \mu m$  which consist of 11 lossy superconducting layers, each of thickness b = 95nm, interspersed with 10 lossless glass layers of width a =90nm. Our calculated value of the effective propagation constant for TE<sub>0</sub>, TM<sub>0</sub>, TE<sub>1</sub>, TM<sub>1</sub>, TE<sub>2</sub> and TM<sub>2</sub> modes are 1.46857 - 0.26682j, 1.51710 - 0.20957j, 1.35984 -0.27845j, 1.37590 - 0.22957j, 1.16302 - 0.29715j and 1.10922 - 0.23954j, respectively. In contrast with the first example, now the maximum real values of the field intensity for TE<sub>0</sub> and TM<sub>0</sub> modes are in the middle of the waveguide structure where the lossy superconducting layer has refractive index higher than that of the lossless glass layers. Figs. 5 - 6. show the real and imaginary parts of the exact effective propagation constants as a function of number of glass layers for the TE<sub>0</sub>, TM<sub>0</sub>, TE<sub>1</sub>, TM<sub>1</sub>, TE<sub>2</sub> and TM2 modes.



Fig. 5. The real part of the exact effective propagation constants as a function of number of glass layers for the modes  $TE_0$ ,  $TM_0$ ,  $TE_1$ ,  $TM_1$ ,  $TE_2$  and  $TM_2$  of the multiple superconducting quantum well structure (a = 90nm, b = 95nm).



Fig. 6. The imaginary part of the exact effective propagation constants as a function of number of glass layers for the modes  $TE_0$ ,  $TM_0$ ,  $TE_1$ ,  $TM_1$ ,  $TE_2$  and  $TM_2$ of the multiple superconducting quantum well structure (a = 90nm, b = 95nm).

For the real parts (Fig. 3 and Fig. 5) of effective propagation constants there is a intersection between the  $TE_0$  and  $TM_0$ ,  $TE_1$  and  $TM_1$ ,  $TE_2$  and  $TM_2$  modes as a function of number of glass layers in a multiple superconducting quantum well optical waveguide where the lossless glass layers have refractive index lower than that of the lossy superconducting layers. For the imaginary parts (Fig. 4 and Fig. 6) of the effective propagation constants, the intersections appear between  $TE_0$ ,  $TE_1$ ,  $TE_2$ modes or between  $TM_0$ ,  $TM_1$ ,  $TM_2$  modes. This behaviour can be used to superconductive traveling wave photodetectors which are insensitive to two of polarizations [4,6].

An increase in number of the quantum wells (equivalent with an increase in refractive index) leads to a tighter confinement of the  $TE_0[2]$  and  $TM_0$  modes to the core region of the waveguide. For the same thickness of the multiple superconducting waveguide structure, the values of the effective propagation constants are strongly dependent on the number and distribution of the superconductor and glass layers.

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