

# Detection probability estimation method and calculation model of photoelectric detection target in complex background

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The detection probability of photoelectric detection target (PDT) is greatly influenced in complex backgrounds, it is very difficult to evaluate and calculate its detection performance and detection probability effectively, to scientific describe the detection probability and give the scientific estimation, this paper sets up the target detection probability calculation model of PDT based on the definition of detection distance and signal to noise ratio (SNR), derives target detection probability functions under different background radiation luminance by using target gray value and complex background gray function, establishes the new SNR calculation model and give the target detection probability calculation method and calculation function, analyses the relationships between the target detection probability and signal to noise ratio(SNR). Through calculation and experiment analysis, the results show that the method of target detection probability can effectively assess target detection probability of PDT in complex backgrounds.

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## 1. Introduction

Photoelectric detection target (PDT) is a kind of photoelectric detection equipment that is used to detect and gather the information of flight target in weapon testing field. PDT mainly composes of optical lens, slit diaphragm, photoelectric detection receiving sensor and processing circuit, and so on; it belongs to the application research of photoelectric detection and optical technology. To gain more the parameters of flight target, multiple photoelectric detection targets (PDTs) were used to form four screens intersection testing system or six screens intersection testing system based on a certain geometry relationship, reference [1] and [2] give the geometry relationship of four and six screens intersection testing system, which can obtain the velocity, fire coordinates and azimuth angle of flight target.

Because of the role of the slit diaphragm of PDT, it maps out a detection screen with a certain thickness [3]. When the target passes through this detection screen, the photoelectric detection receiving sensor can obtain an instant changed signal, this changed signal is amplified and converted, we can gain the time between two unit photoelectric detection screen by timer in four screens intersection testing system or six screens intersection testing system, at last, calculate the parameters of position and velocity of flying target based on the gathered time and spatial geometry of multi-screen intersection sensor testing system[4-6].

From the detection principle of PDT, it uses sky as background light source to detect target information,

because of the changed in environmental illumination, its detection ability will be influenced, especially in low illumination condition, PDT often appears the phenomenon that cannot capture target, and in high illumination condition, PDT often appears saturation or self-excitation phenomenon, which weaken the detection probability of PDT, so, the changed of illumination is an important factor affecting PDT's detection performance. It is very necessary to establish a scientific mathematics description the detection capability evaluation mechanization, which can improve PDT's design effectively in future.

To further improve the detection performance of PDT, reference [7] proposes a method that uses mosaic photoelectric array column devices to increase working field, and uses the instrument amplifying method to design detection circuit, which can improve the SNR and detection performance effectively. Reference [8] proposes a method that uses adjustable light aperture to weaken background light, this measure can effectively improve photoelectric detection performance under certain conditions, but it still can not solve the detection performance of low illumination. Those documents also don't give the scientific mathematics description the detection ability evaluation mechanization.

To solve the question that the detection ability of PDT is not high in low illumination condition, we use the infrared photoelectric detection sensor to design PDT; it can compensate the shortcoming that PDT cannot detect target information in low illumination condition. Based on new photoelectric detection method of PDT, this paper

studies the detection probability estimation method and scientific calculation model of PDT in complex background, and give scientific mathematics description its detection ability.

## 2. Detection probability estimation and calculation of photoelectric detection target

### 2.1. Definition of target detection probability and the detection distance model of PDT

In the detection system of PDT, the detection capability is related to SNR and detection distance, we use the relation between SNR and detection distance to define its detection probability, the formula (1) gives the definition function.

$$P = f(SNR, R) \quad (1)$$

According to the detection principle and design structure of PDT, the effective detection distance function can be obtained by formula (2).

$$R = \sqrt{\frac{\delta(L_1 - L_b)A_1A_o\tau_o\tau_a(R)D^*}{(A_d\Delta f)^{1/2} \cdot SNR}} \quad (2)$$

In (2),  $L_1$  is the target radiation luminance,  $L_b$  is the background radiation luminance,  $A_o$  is the effective photosensitive area of photoelectric detection system,  $A_1$  is the target projective area in detection screen,  $\tau_a(R)$  is the atmospheric transmittance rate,  $R$  is the detection distance between the target and photoelectric detection receiving sensor,  $A_d$  is the area of detector total photosurface,  $\Delta f$  is an equivalent noise bandwidth,  $\tau_o$  is the transmittance of the optical system,  $D^*$  is detection star degree[9].

According to formula (2), the detection distance of PDT can be obtained by a certain detection probability, in reality, the continuous variation complex backgrounds affect the detection performance of PDT, and the detection probability of PDT is very low.

If we want to improve the detection probability of PDT, it is necessary to consider the optical system of PDT, including the focal length of optical lens, the caliber size of optical lens, the responsivity of photoelectric detection receiving sensor, the SNR of detection circuit and the characteristics of target, in those factor, the target characteristics are key, it will effect the detection performance of whole system.

Target optics characteristics come from the environment illumination, radiant energy of target itself and the target size, those factors affect the output signal, and thus, we can make full use of the information of target radiation to establish the detection probability model.

### 2.2. Detection probability estimation calculation and analysis

There are a lots of noise sources in the detection system of PDT, the main noises come from the photoelectric detection receiving sensor, which conforms to the gaussian distribution. When the target signal and the noise signal both exist, the output signal of PDT still conforms to the gaussian distribution [10], and then, the probability density can be describes by formula (3) in photoelectric detection system.

$$P(V) = \frac{1}{\sqrt{2\pi}V_n} e^{-\frac{(V-V_a)^2}{2V_n^2}} \quad (3)$$

In (3),  $V$  is the maximum effective amplitude of target,  $V_a$  is the average signal of target,  $V_n$  is the mean square root of noise. The detection probability can be written as formula (4) when the formula (3) was integrated.

$$P(SNR) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{SNR-TNR} e^{-\frac{t^2}{2}} dt \quad (4)$$

In (4),  $SNR$  is the signal to noise ratio of detection system,  $TNR = (V - V_a)/V_n$  is the threshold SNR of photoelectric detection receiving sensor [11].  $TNR$  is main influencing factor of detection probability, it can be ensured by using the ratio between the one half of peak value of target signal and the average noise of the detection system. When  $TNR$  is determined, the target detection probability of PDT is monotone increasing function of input signal to noise ratio. The lower the threshold of signal to noise ratio is, the higher the detection probability is, but, the false probability of PDT will increase.

In the detection system of PDT, the signal to noise ratio have a relationship with the infrared radiation of target, background radiation and the detection distance, therefore, the relationship between the input signal to noise ratio and the detection distance can be derived by formula (5).

$$SNR(L_1, L_b, R) = \frac{\delta(L_1 - L_b)A_1\tau_a(R)}{NR^2} \quad (5)$$

In (5),  $N$  is the noise equivalent difference temperature. It can be expressed by formula (6).

$$N = \frac{\sqrt{A_d\Delta f}}{A_o\tau_oD^*} \quad (6)$$

From formula (6), if the infrared imaging system of PDT, target radiation and detection distance are determined, the input signal to noise ratio of the system is only related to the background radiation, and then, their function turns into  $SNR(L_b)$ , the target detection probability

can be derived by formula (7) under the condition of single background.

$$P_{\text{single}}(\text{SNR}(L_b)) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\text{SNR}(L_b) - \text{TNR}} e^{-\frac{t^2}{2}} dt \quad (7)$$

Generally, the detection probability is calculated in a certain false alarm probability, it is necessary to mention the calculation method of false alarm probability. False alarm probability in infrared imaging system of PDT refers to the probability that the targets actually do not exist, but the system wrongly detects the targets. We regard the false alarm probability as the probability when the noise output voltage of photoelectric detection receiving sensor is over the threshold voltage, it can describe by formula (8).

$$P_{\text{single}}(\text{TNR}) = e^{-\frac{\text{TNR}^2}{2}} \quad (8)$$

Clearly, the false alarm probability of PDT is only related to the threshold signal to noise ratio ( $\text{TNR}$ ), the false alarm probability will change in continuous variation complex backgrounds.

In fact, the working environment of PDT is complex, it means the background of target is not single. If we only consider single background, then the target detection probability cannot be obtained fully, the detection imaging signal have multiple gray values in complex background, the spatial distribution of gray pixels are no regular at all, the gray value of background reflects the capacity of infrared radiation, the bigger the gray value is, the stronger the capacity of infrared radiation is, that is to say, the background noise of the detection system is too high to capture the information of target. Because the imaging position is uncertain that target appears in field of view of PDT, the background radiation luminance is likely to be any one in the current background brightness value. Thus, we can divide the output signal from the PDT into different gray values, and calculate the target detection probability in accordance with different grades of gray value.

Assume that detection information of target has  $m$  different signal gray values; the formula (5) can be changed into (9).

$$\text{SNR}(P_i) = \frac{\delta(L_1 - L_b(P_i))A_1\tau_a(R)}{N \cdot R^2} \quad (9)$$

In (9),  $P_i$  is the corresponding probability of  $m$  gray values, and  $i=1,2,\dots,m$ ,  $\sum_{i=1}^m P_i = 1$ ,  $L_b(P_i)$  is the background infrared radiation luminance,  $\text{SNR}(P_i)$  is the input signal to noise ratio, its probability can be described by formula (10).

$$P(\text{SNR}(P_i)) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\text{SNR}(P_i) - \text{TNR}} e^{-\frac{t^2}{2}} dt \quad (10)$$

Formula (10) gives the target detection probability of different background radiation luminance.

From above the analysis, the target detection probability of PDT is related to signal to noise ratio of detection system and detection distance closely. To the target detection probability of PDT, the main measures are to use telephoto lenses to design the detection system that reduce the influence of incident light and improve the gain of the detection circuit, at the same time, make the threshold signal to noise ratio ( $\text{TNR}$ ) can adaptive adjustment.

In complex background, the gray values of target are different, and the output signals also are different, this is reflected in the target signal peak and average noise in different background illumination, so, we make the  $\text{TNR}$  is adaptive adjustable according to background illumination to stabilize detection performance.

### 3. The SNR calculation model of PDT

The signal to noise ratio ( $\text{SNR}$ ) of PDT represents the relationship between the received target signals of photoelectric detection receiving sensor and the total noise. According to the calculation function of target detection probability, if we want to get target detection probability under a certain distance, need to calculate the  $\text{SNR}$  under this condition. Signal to noise ratio ( $\text{SNR}$ ) is described by equation (11).

$$\text{SNR} = S / N \quad (11)$$

Formula (12) describes the function of photo electrons number of target.

$$S = \frac{t \cdot A_d \cos \theta \int_{\lambda_1}^{\lambda_2} L_1(\lambda) \tau_a(\lambda) \tau_o(\lambda) \eta(\lambda) \lambda d\lambda}{hcR^2} \quad (12)$$

In (12),  $\eta(\lambda)$  is the spectral quantum efficiency [12],  $\theta$  is the angle between the normal vectors of the photoelectric detection receiving sensor and the target,  $h$  is the Planck's constant,  $c$  is the speed of light,  $t$  is the total time that target pass through the detection screen of PDT.

For the detection system of PDT, the noise mainly consists of shot noise of target radiation, background noise and the noise of photoelectric detection receiving sensor, it can be expressed by formula (13).

$$N = \sqrt{N_s^2 + N_B^2 + N_d^2} \quad (13)$$

The noise of photoelectric detection receiving sensor is mainly the thermal noise, it can be expressed by formula (14).

$$N_s = \sqrt{4kT_0 t / q^2 R_i} \quad (14)$$

In (14),  $T_0$  is absolute temperature of photoelectric

detection receiving sensor,  $k$  is Boltzmann's constant,  $q$  is the electric charge that photoelectric detection receiving sensor can response,  $R_i$  is the equivalent resistance [13].

The background radiation noise is caused by the incident photo electrons. The background radiation contains sky background radiation and the background radiation of sun, moon and other stars [14]. Sky background radiation is the reflection and scattering of solar radiation by the atmosphere, and atmospheric emission itself. Background noise can be calculated by formula (15).

$$N_B = \sqrt{\frac{t \cdot A_d \cdot \Omega \cdot \int_{\lambda_1}^{\lambda_2} L_b(\lambda) \tau_a(\lambda) \tau_o(\lambda) \eta(\lambda) \lambda d\lambda}{hc}} \quad (15)$$

In (15),  $\Omega$  is detection field angle of photoelectric detection system of PDT.

The shot noise caused by the radiation of flight target, it is the random producing that the photosurface of photoelectric detection receiving sensor absorb the photons [15]. In a certain incident light, the number of photo electron is not the same in the instantaneous interval and any light sensitive surface, but it ups and downs over an average value, which produce shot noise.

Shot noise is proportional to the square root of the total number of signals, and it is an inherent noise of photo electronic components and can not be suppressed by the subsequent circuit or offset [16]. The shot noise is mainly determined by the number of electrons that come from the target radiation, and the mean value of shot noise can be calculated by formula (16).

$$N_d = \sqrt{\frac{t \cdot A_d \cdot \cos \theta \cdot \int_{\lambda_1}^{\lambda_2} L_t(\lambda) \tau_a(\lambda) \tau_o(\lambda) \eta(\lambda) \lambda d\lambda}{hcR^2}} \quad (16)$$

According to the formula (14), (15) and (16),  $N$  can be calculated, and then, using formula (11), the total signal to noise ratio of PDT can be calculated by formula (17).

$$SNR = \frac{t \cdot A_d \cos \theta \int_{\lambda_1}^{\lambda_2} L_t(\lambda) \tau_a(\lambda) \tau_o(\lambda) \eta(\lambda) \lambda d\lambda}{hcR^2 \sqrt{N_s^2 + N_B^2 + N_d^2}} \quad (17)$$

The function of  $SNR$  is substituted into the target detection probability model; the total detection probability of PDT can be gained.

Assume  $SNR_{min}$  is the minimum  $SNR$ , the target detection probability can be expressed by formula (18).

$$P = \frac{SNR - SNR_{min}}{SNR} \cdot SNR(P_i) \cdot 100\% \quad (18)$$

Based on the analysis of the above, the target detection probability is closely related to the  $SNR$ , but,  $SNR$  is related to the detection distance. From formula (18), we find the total target detection probability is the function

of detection distance, optical aperture and  $SNR$ .

## 4. Calculation and experiment analysis

### 4.1. Calculation analysis

Photoelectric detection target (PDT) uses the medium wave infrared imaging sensor method to design its optical system, based on the characteristic and principle of PDT, combined with the above theory and analysis, we use the projectile that the diameter of projectile is 76 mm to calculate and verify its detection probability that the velocity about is 750 m/s, the length of projectile about is 241 mm, suppose, the focal length of lens is 105 mm, the lens's effective aperture about is 62 mm, the detection screen's thickness about is 40 mm to 246 mm when the detection distance is from 12 m to 75 m. Under those conditions, we calculate and analyze the target detection probability of PDT in blue sky and cloudy background.

From the relationship between the target detection probability and the input signal to noise ratio in the single background in formula (3) and (4), Fig. 1 gives the changed curve between the detection probability and the input signal noise ratio in a single background of blue sky.

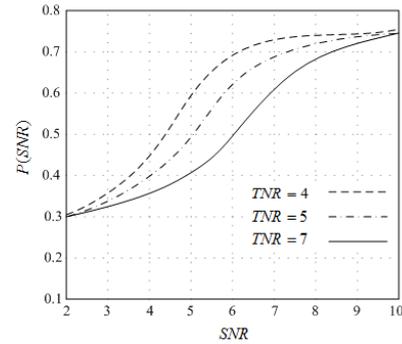


Fig. 1. The changed curve between the detection probability and the input signal noise ratio in a single background of blue sky

As shown in Fig. 1, it can be seen that the detection probability is a single increasing function with the input signal to noise ratio, when the input signal to noise ratio ( $SNR$ ) is determined, the lower the  $TNR$  is, the higher the detection probability is.

Fig. 2 gives the changed curve between the detection probability and the input signal to noise ratio in cloudy background, the target detection probability is significantly improved under the same input signal to noise ratio ( $SNR$ ), it indicates that the target's features are very obvious and the contrast of detection system is very high in cloudy background, it reflects the target radiation has great contribution to the photoelectric detection receiving sensor, the influence of background noise is relatively small, so, the detection sensitivity of PDT is high. Especially, when the  $SNR$  is more than 6 and  $TNR$  is 3, the detection probability of PDT is over 94%.

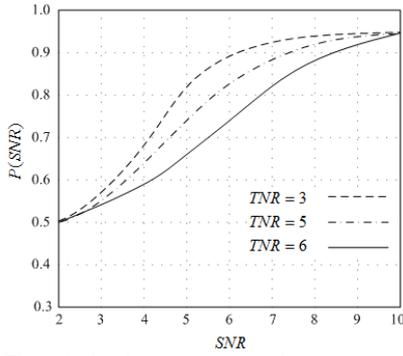


Fig. 2. The relation between target detection probability and SNR under different TNR in cloudy background

In order to assess the target detection probability of PDT, under different background conditions, we gather the target infrared radiation luminance and the gray level data of the target output signal in detection system, and calculate the target detection probability corresponding to 0-250 gray scale value, Fig. 3 is the changed relation between image gray and target radiance luminance, and Fig. 4 are the changed relation between image gray and detection probability.

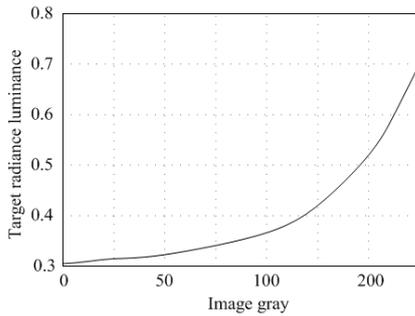


Fig. 3. The changed relation between image gray and target radiance luminance

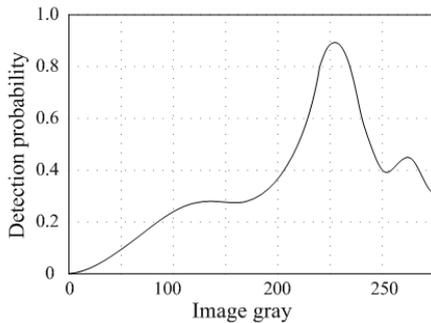


Fig. 4. The changed relation between image gray and detection probability

It can be seen from the results under the complex background, due to the differences of environment, the target detection probability have many possible values, but sky background contrast is quite obvious, under the same detection distance, the target detection probability have improved, and the signal to noise ratio of the photoelectric detection system is obviously increased in PDT. From Fig.

4, we find that when the target gray increases, the detection probability is also improved, but, the image gray value is not direct proportional to target detection probability, because photoelectric detection target (PDT) has not only target gray value but also background gray, the gray differences reflect the true detection probability.

## 4.2. Experiment analysis

In order to observe the information of PDT in different background, the gray value information of target detected convert the analog signal, we use the acquisition instrument to obtain the signal waveform of flight target, the projectile that the diameter of projectile is 76 mm that the velocity about is 750 m/s in one experiment.

Fig. 5 is the output signal in dark blue background when the detection range about is 800 times of target diameter, namely, the detection distance is 60.8 m; Fig. 6 is output signal in cloudy background under the same detection distance.

The output signal amplitude has obviously different in two kinds of environment, it illustrates that the target's features and the contrast of detection system are very obvious, and the target radiation has great contribution to the photoelectric detection receiving sensor in cloudy background, so, the detection probability also have improved significantly.

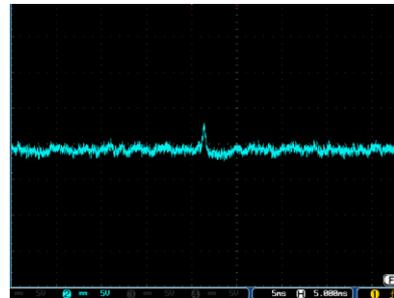


Fig. 5. The output signal in dark blue background when the detection range about is 800 times of target diameter

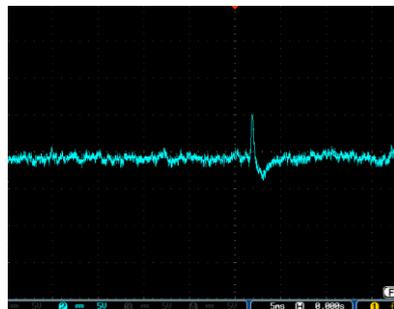


Fig. 6. The output signal in cloudy background under the detection range is 800 times of target diameter

From Fig. 5 and Fig. 6, in same detection distance, the target radiant energy can enter the photosurface of

photoelectric detection receiving sensor is different in blue and cloudy background, the main reasons is the reflection of target is poor in blue background, but, in cloudy background, the luminance of cloudy contribute to the target surface is very strong, which increase target radiant energy, namely, the contrast is very high.

Fig. 7 is the three output signal when photoelectric detection target (PDT) uses the photoelectric detection receiving sensors with three kinds' wavelength, and the testing background is cloudy. The wavelength of photoelectric detection receiving sensors of the channel 1 is 890 nm to 1100 nm; the wavelength of photoelectric detection receiving sensors of the channel 2 is 560 nm to 910 nm, and the wavelength of photoelectric detection receiving sensors of the channel 3 about is 1800 nm.

According to the output signal, we find when the wavelength of photoelectric detection receiving sensors is different; the amplitude of output signal also has obvious different, it reflects the contribution ability of target radiation characteristics to the detection sensor is different under the different wavelengths. Therefore, the target detection probability of PDT is also related to the wavelength of photoelectric detection receiving sensors. From the model of  $SNR$  and the detection principle of PDT, photoelectric detection receiving sensor is the most core, usually, it can respond different wavelength in PDT, to improve the target detection capability, we choose the photoelectric detection receiving sensor with medium wave-band to design its detection system based on target's radiation characteristic and environment illuminance.

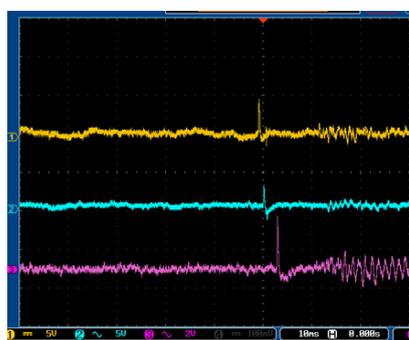


Fig. 7. The three output signal when PDT uses the photoelectric detection receiving sensors with three kinds' wavelength

According to formula (18), the signal to noise ratio ( $SNR$ ) is about 4.0 in Fig. 5, the signal to noise ratio is about 5.6 in Fig. 6, if  $SNR_{min}$  is 3.5, we can calculate target detection probability about are 12.5% and 37.5%. The results show that the target detection probability is less than 50% in the limit detection distance for two different environmental backgrounds (blue background and cloudy background), especially, in the dark blue background, the detection probability of PDT is less than 15%, which will seriously affect the detection ability.

According to above the analysis, we find that the detection probability of PDT is related to detection distance, and the signal to noise ratio ( $SNR$ ) is related to

the optical aperture size, to improve the detection capability of PDT, it is necessary to increase the optical aperture size and add auxiliary light source to design the optics detection system of PDT. The theory model of detection probability estimation provides a more concrete design idea for new photoelectric detection target.

## 5. Conclusions

Based on the detection principle of PDT, this paper studies the detection probability estimation method of PDT, deduces the target detection probability calculation function in complex background, look for the related parameters that affect the detection ability, from the  $SNR$ , detection distance, noise and optical parameters of PDT, and give a scientific detection probability estimation function. Through the calculation and experiment analysis, the results show that target detection probability of PDT is very different in different background; it is mainly reflected in the poor gray value of the target imaging, which caused the detection probability difference. The experiment verifies the model of target detection probability is scientific and reasonable, it can provide a more concrete improved design for PDT in future.

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