Design of mixing color optical system based on a novel composite lens array

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The development of optics of mixed white light illumination is a challenging problem to provide high uniformity and high-power color mixing and meet many other specific requirements. This study proposes an optical design method of high-power color mixing illumination based on free-form composite lens arrays. The designed composite lens array has a high convergence capability to converge light from different locations to a single point, eliminating the sensitivity to the position of the light source. The system utilizes a freeform mixer to reduce the normalized standard deviation of the chromatic aberration values and the correlated color temperatures. In the design example, the system produces white light with a color different value of 0.008 and a normalized standard deviation (NSD) value of 0.000624 at the lens distance of 25 mm and the focal length of 180 mm. The proposed optical system design method can effectively eliminate the problems of white point shift and color difference in hybrid white light illumination.

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1. Introduction

LED light sources have a wide range of applications due to their advantages of high efficiency, energy saving, and rich colors. LED plays an important role in daily life. Scene illumination is the most common color-mixing application. When used for scene illumination, multi-colored LEDs are mixed to render a color-cast illumination effect that is suitable for a special occasion with the help of optical elements, such as diffusers, light pipes, and batteries of lenses [1-3]. Among them, High-quality white light with high color rendering and uniform spectral distribution provides bright, clear, and natural illumination techniques, which are pursued in various fields. Among the methods of generating white light using LEDs, the combination of multiple chips attracts widespread attention due to its high luminous efficiency and flexible adjustment [4]. However, there are currently deficiencies in the color rendering and uniformity of mixed white light. Therefore, improving the quality and performance of mixed white light through optical system design methods is of great significance.

At this point, non-imaging optical design plays only an important role. Shaping light from incoherent sources, such as light-emitting diodes, has important applications in indoor and outdoor illumination design. This is traditionally achieved by freeform refractive lenses using design principles from non-imaging optics. Non-imaging optics focuses on the optimal transfer of energy from a light source onto a target without requiring the formation of an image of the source [5].

The method of achieving high-quality white light through RGB mixing requires a meticulous optical design to ensure uniform distribution and color balance of the mixed light [6] and optical design is still necessary to achieve high power. Freeform surface shapes have a great degree of freedom to control color mixing more accurately [7-8]. Chen et al. developed an ultra-thin freeform micro-lens array module that offers high flexibility without constraints on the boundaries and is capable of dividing incident light into uniform small segments, enabling uniform color mixing [9]. LEE et al. propose a hemispherical lens with a serrated structure that generates a composite light field by superimposing multiple small light fields, resulting in a uniform compound light field. Compared to lenses without microstructures, this lens significantly improves color mixing uniformity and greatly reduces average chromatic aberration [10]. Zhao et al. propose a freeform TIR lens consisting of a collimating lens and a freeform group composed of several small freeform panels. The freeform panels cancel out the color non-uniformity of the collimated light, improving the uniformity of color mixing and reducing the normalized standard deviation value of mixed white light [11]. These micro-scale composite structures alter the light path multiple times and superimpose the light field, greatly improving the uniformity of color mixing but there are still unresolved issues related to color mixing [12-13].

The different propagation directions of light can cause color shadows and color difference to occur. J. Chaves et al. proposes an optical component called "Shell Mixer" to tackle this problem. This optical component consists of numerous Köhler channels covering the surface, which mix the light emitted in both spatial and angular dimensions through Köhler integration, forming a highly uniform virtual light source on the surface of the light source [14]. This improves color uniformity in color mixing and eliminates defects such as color shadows and color shifts. Afterward, Sorgato et al. propose a "Freeform Shell-Mixer" based on the "Shell-Mixer" to improve the Köhler channels, allowing for more precise control of light and creating virtual light sources that closely resemble real light sources. It achieves a volume reduction to one-third of the "Shell-Mixer" while maintaining good color mixing performance [15].

High-power mixing light often requires multiple light sources, which also leads to problems of color mixing. Sun et al. proposed a high-power collimation system based on Fresnel lenses to form arrays of LEDs, using a concentrator and extender to achieve high-power illumination in the near and far fields. The design of the LED arrays is worth pondering [16]. Liu et al. propose a high-power color mixing method for LED by combining LED with lenses to form lens units. This method utilizes several lens units of different colors to create a rectangular array, resulting in excellent color-mixing performance [17]. However, the cost of this method is relatively high. Chen et al. propose a multi-channel optical system consisting of a multi-channel elliptical reflector and a color mixing component to achieve high-power color mixing more effectively. The multi-channel reflector converges multiple beams of different colors within each channel to a point, creating a virtual light source [18]. This method allows for the generation of high-power and highly uniform mixed white light.

The paper proposes a new optical system design method for high-power RGB LED color mixing illumination based on the above analysis. The optical system combines a collimating lens with a converging lens to form a new composite lens array, which precisely controls the light beam to achieve high-quality and high-power white light.

2. Optical system design

2.1. Design of color-mixing optical system

A mixed-color optical system composed of a composite lens array and a freeform mixer is proposed in this study to address issues such as color difference and correlated color temperature deviation. The integrated feature of the composite lens array allows for precise shaping of the light beam and achieves highly focused light rays [19]. Additionally, the array structure facilitates the realization of high-power mixing. Therefore, this mixed-color optical system can achieve higher quality mixing of light. The color mixing technique of RGB LEDs is effectively improved by the optical system, and the combination of multiple light sources into a high-power virtual light source for high-power mixing is allowed by the array design. The schematic diagram of the color-mixing optical system is shown in Fig. 1.

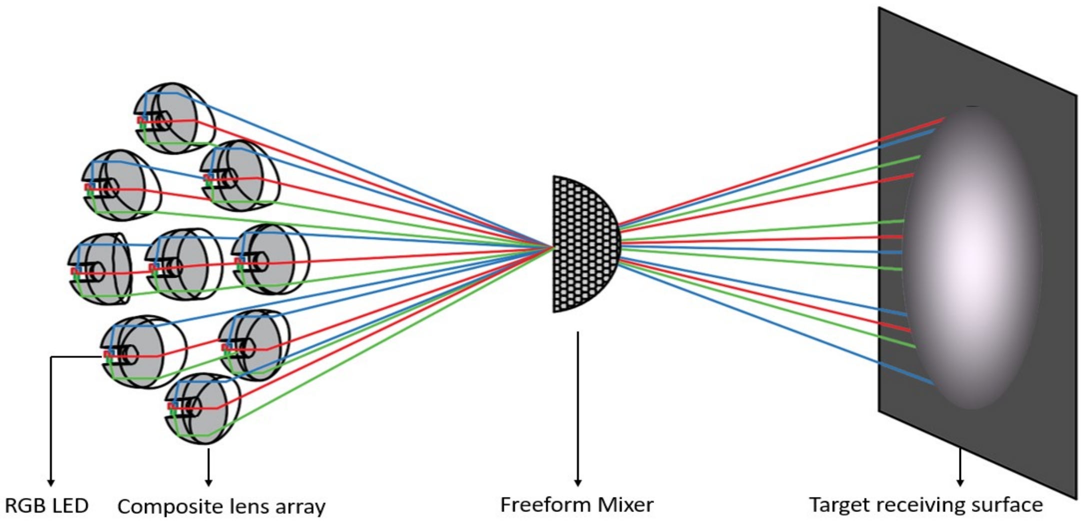

Fig. 1. Diagram of a color-mixing optical system (color online)

The light emitted by light sources of different colors converges into a single point after undergoing refraction and reflection through a composite lens array. The light then undergoes refraction through a freeform mixer, causing the light to scatter and form a uniformly mixed color spot.

2.2. Design of composite lens

In the study of optical color mixing, the direct mixing of multiple different color LEDs often results in significant color difference. Therefore, optical design is commonly employed to improve the color mixing quality. This study uses a collimating lens to collimate different color lights and reduce light divergence. and use the convergent ability of a convergent lens to converge the collimated beam, the convergence of light emitted from different positions into a single light source point reduces the sensitivity of color mixing to the position of the light source and improves color mixing quality and energy efficiency.

The composite lens is composed of a collimating lens and a convergent lens, which converges the collimated light rays onto the same focal point. Considering the energy loss caused by the propagation of the rays, it is possible to simplify the system by placing the convergent lens directly on the surface of the collimating lens and removing the unused portion of it. This saves a significant amount of material and reduces energy loss. The schematic diagram of the composite lens structure is shown in Fig. 2.

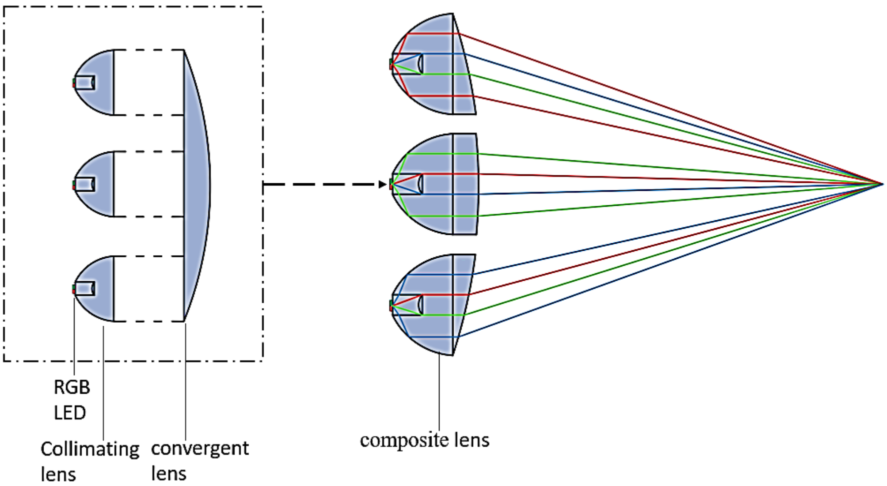

Fig. 2. Schematic diagram of a composite lens structure (color online)

In this structure, light sources of different colors emit light rays that are refracted and reflected in the composite lens, causing the convergence of light of various colors at a focal point. Snell's law is used in the design process. The design principal diagram of the composite lens is shown in Fig. 3.

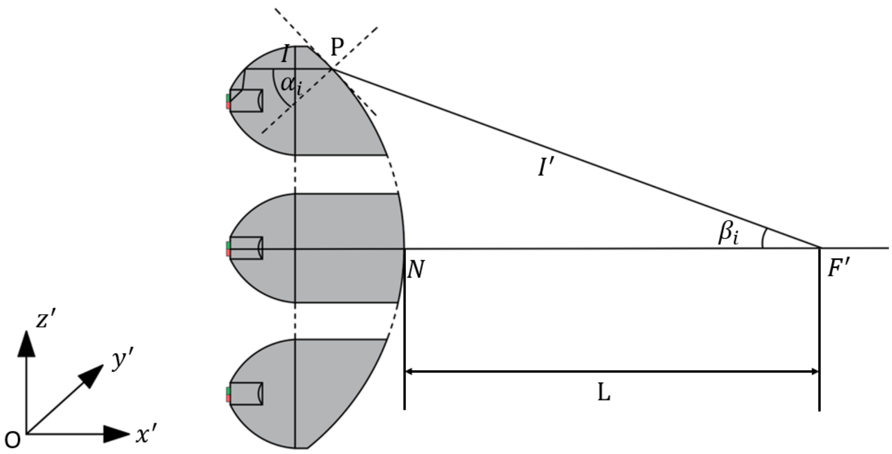

Fig. 3. Design schematic of composite lens

The light emitted from the light source is refracted twice on the inner surface of the collimating lens and then emitted collimated, intersecting with the surface of the converging lens at the point P, where refection occurs, and the coordinates of the point P are (x'_i, y'_i). According to Snell's law, the slope k_i of the surfaces can be obtained.

$$n(I \times N) = n'(I' \times N) \tag{1}$$

$$k_i = \frac{I \cdot N}{\sqrt{(|\vec{I}||\vec{N}|)^2 - (I \cdot N)^2}} \tag{2}$$

where α_i is the incident angle, β_i is the angle between the refracted light and the coordinate system. n is the refractive index of the lens. The difference in the ordinates of each point on a free surface is $\Delta y'$, the slope on the free surface can be expressed as

$$k_{i+1} = \frac{y'_{i+1} - y'_i}{x'_{i+1} - x'_i} \quad (3)$$

The iterative formula for coordinates of points on a free surface can be expressed as

$$\begin{cases} x'_{i+1} = x'_i + \frac{\Delta y'}{k_i} \\ y'_{i+1} = y'_i - \Delta y' \end{cases} \quad (4)$$

The iterative formula for the coordinates of points on a free surface is determined by the slope k_{i+1} can be expressed as

$$\begin{cases} x'_{i+1} = x'_i - \frac{\Delta y'}{k_{i+1}} \\ y'_{i+1} = y'_i + \frac{\Delta y' k_{i+1}}{k_i} \end{cases} \quad (5)$$

The surface shape of a convergent lens can be obtained through the above formula. A composite lens is made of a collimating lens and a convergent lens, which converges the light emitted by a light source.

2.3. Design of freeform mixer

In the process of light mixing, insufficient mixing of light in terms of spatial and angular distribution leads to problems such as color difference and correlated color temperature deviation. To achieve high-quality white light by fully mixing the light beam converged by a composite lens array, a freeform mixer is used to finely control the light beam through several freeform Köhler channels [15]. The working principle is shown in Fig. 4.

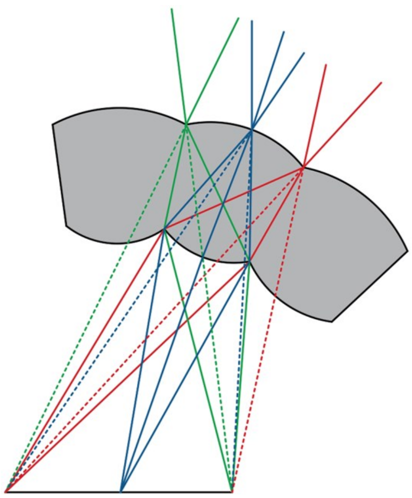

Fig. 4. Schematic diagram of freeform mixer (color online)

The edge rays of the light source are projected onto the first surface of the Freeform Mixer and focus through refraction on the edge of the second surface. The second surface refracts the focused rays to form a virtual light source focused on the edge of the light source. The real light source and the virtual light source are thoroughly mixed, forming a well-blended light source that is emitted through the mixer. This allows for good color mixing.

3. Color mixing mechanisms of chromaticity coordinates

In the CIE 1931 chromaticity diagram, when two or more colors are mixed, the chromaticity coordinates of the resulting mixed color can be traced on the chromaticity diagram. The coordinates of a mixed color obtained by blending two colors in different proportions lie on the line connecting the two colors. The coordinates of a mixed color obtained by blending multiple colors in different proportions always lie within the polygon formed by the multiple colors. The schematic diagram of chromaticity mixing is shown in Fig. 5.

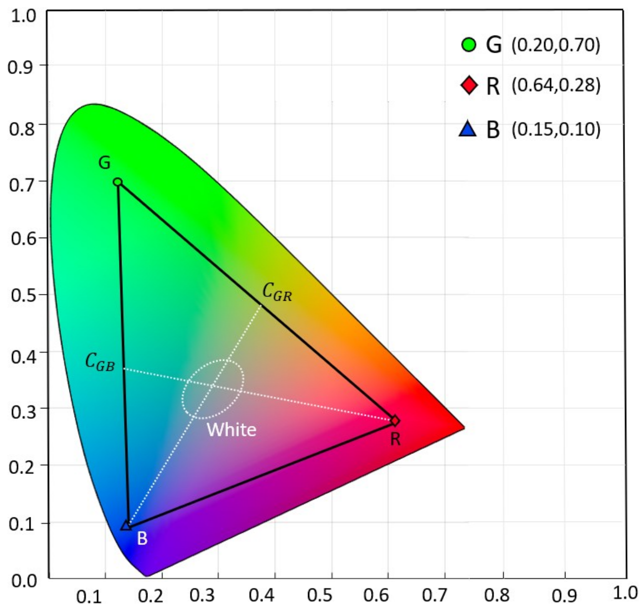

Fig. 5. Schematic diagram of chroma mixing (color online)

In a chromaticity diagram, the mixing of RGB creates a triangle representing white light, and the chromatic coordinates of the mixed white light are dependent on the chromatic coordinates and luminance of the three light sources to mixing white light requires combining several RGB light sources with lenses in each array unit in a certain proportion. After determining the target color, the proportions of the three primary colors can be calculated. The color blending ratio is usually represented by the color's tristimulus values. In Fig. 7, C_{GR} is the color formed by mixing R and G in a certain proportion, and

C_{GB} is the color formed by mixing G and B in a certain proportion. Any of these mixed colors can be mixed with another color in the RGB system to obtain white.

Assuming that the three stimulus values of a color are $X, Y, Z,$ the color coordinate is $\left(x\ ,\ y\right)$, and the luminance is u, the relationship is expressed as

$$\begin{cases} X = \frac{x}{y}u \\ Y = u \\ Z = \frac{z}{y} = \frac{1-x-y}{y}u \end{cases} \quad (6)$$

where the relationship among the three stimulus values and the color coordinates is:

$$\begin{cases} \frac{X}{X+Y+Z} = x \\ \frac{Y}{X+Y+Z} = y \\ \frac{Z}{X+Y+Z} = z \end{cases} \quad (7)$$

When mixing white light using RGB LED, it is assumed that the wavelengths and brightness of the RGB LED are known. The color coordinates of red, green, and blue are represented by(x_R, y_R), (x_G, y_G) and (x_B, y_B) respectively, corresponding to their three stimulus values (X_R, Y_R, Z_R), (X_G, Y_G, Z_G), (X_B, Y_B, Z_B). The relationship between the tristimulus values (X_0, Y_0, Z_0) of white light and the tristimulus values of the three primary colors, red, green, and blue, is expressed as follows:

$$\begin{cases} X_0 = X_R + X_G + X_B \\ Y_0 = Y_R + Y_G + Y_B \\ Z_0 = Z_R + Z_G + Z_B \end{cases} \quad (8)$$

The chromaticity coordinates of white light can be obtained by analogy from equation (7) as follows:

$$\begin{cases} x_0 = \frac{X_0}{X_0+Y_0+Z_0} \\ y_0 = \frac{Y_0}{X_0+Y_0+Z_0} \\ z_0 = \frac{Z_0}{X_0+Y_0+Z_0} \end{cases} \quad (9)$$

When the tristimulus values of any two LEDs are known (such as known red and green), the tristimulus values of the third light source can be determined using equations (8) and (9):

$$\begin{cases} X_B = 2Y_G - X_R - X_G \\ Y_B = Y_G - Y_R \\ Z_B = 2Y_G - Z_R - Z_G \end{cases} \quad (10)$$

Assuming the brightness ratio of red, green, and blue is $L_R : L_G : L_B$. From equations (6), (8), and (9), it can be concluded that:

$$\begin{pmatrix} L_R \\ L_G \\ L_B \end{pmatrix} = \begin{pmatrix} \frac{x_R}{y_R} & \frac{x_G}{y_G} & \frac{x_B}{y_B} \\ \frac{1}{y_R} & \frac{1}{y_G} & \frac{1}{y_B} \\ 1 & 1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} \frac{x_0}{y_0} \\ \frac{1}{y_0} \\ L_0 \end{pmatrix} \quad (11)$$

The relationship between brightness L and power P is:

$$L = K_m \cdot V_{(\lambda)} \cdot P \quad (12)$$

where, K_m is the spectral luminous efficiency, $V_{(\lambda)}$ is an observable function. For the selected three primary colors, the power ratio is:

$$P_R : P_G : P_B = \frac{L_R}{V_{R(\lambda)}} : \frac{L_G}{V_{G(\lambda)}} : \frac{L_B}{V_{B(\lambda)}} \quad (13)$$

The power ratio of RGB required for mixing white light can be obtained from the analysis above. The quantity ratio of RGB can be calculated using the power ratio:

$$n_R : n_G : n_B = \frac{P_R}{P_r} : \frac{P_G}{P_g} : \frac{P_B}{P_b} \quad (14)$$

where, n_R, n_G, n_B is the number of each light source, P_r, P_g, P_b is the power of each light source.

4. Simulation and analysis

The light source utilized in the system is the OSRAM RGBW four-color light source (model LERTDUW S2WN), which has dimensions of 4.68 mm by 5.75 mm, as illustrated in Fig. 6.

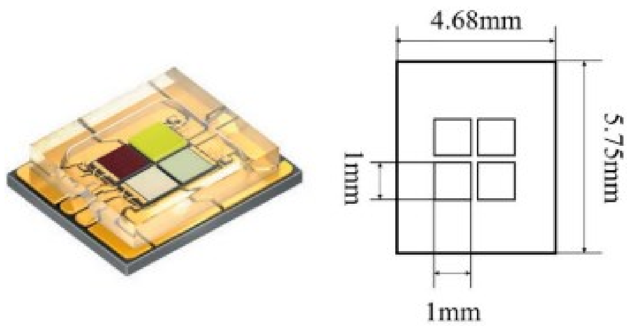

Fig. 6. The shape and geometric dimension of the light source (color online)

This LED features distinct wavelengths for each color: red at 625 nm, green at 525 nm, and blue at 453 nm. The spectral and intensity distributions for these light sources are depicted in Fig. 7.

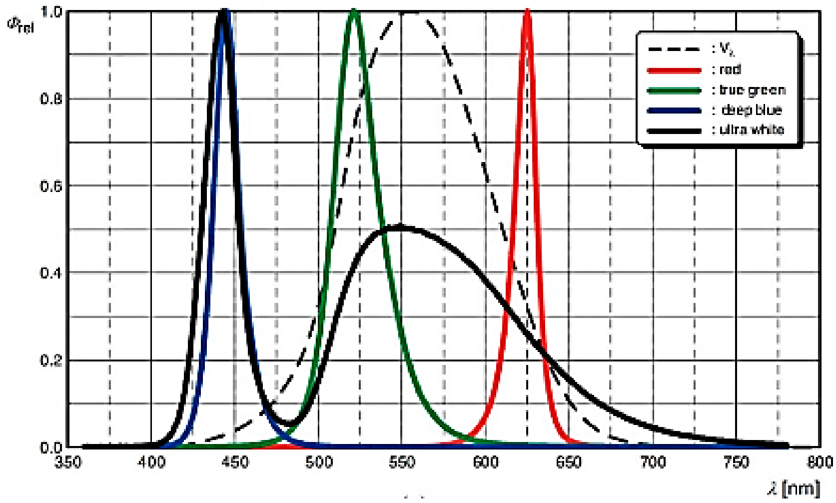

Fig. 7. Spectrum of a light source (color online)

The system uses 9 sets of RGB LED light sources, and the main light source parameters of the system are shown in the table below.

Table 1. The light source parameters

wavelength (nm)	Chromatic coordinates	V_λ	Power/W
0.625	(0.64,0.33)	0.265	3
0.527	(0.30,0.60)	0.862	10.4
0.453	(0.15,0.06)	0.033	0.4

These groups of RGB LED light sources are arranged in a circular shape, with a group of light sources in the center and 8 groups of light sources arranged around them. Each group of light sources deflects light through a composite lens, and the distance between the lenses is called the lens distance. The optical system is simulated and the light from the light source is traced as shown in Fig. 8.

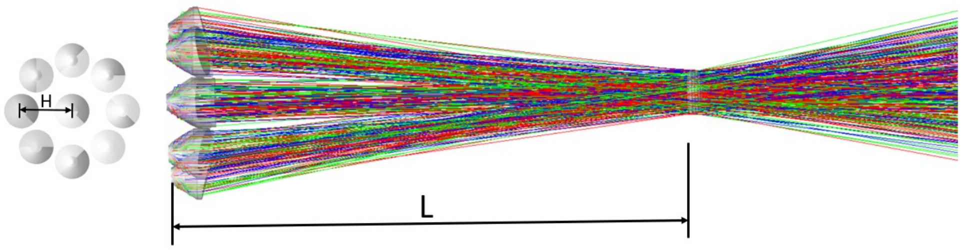

Fig. 8. System simulation diagram (color online)

The rays emitted by the light source groups are converged on a receiving surface through a composite lens array, and then mixed by a freeform mixer with the focal length L of 180 mm. Calculated analysis shows that at this focal length, the refractive part of the energy utilization is highest at a lens distance of 25 mm. To analyze the effect of lens distance on color mixing multiple lens distances are set. The receiving surface is located at the distance of 1 m and the illumination diagram of mixed light with the change of lens distance is shown in Fig. 9.

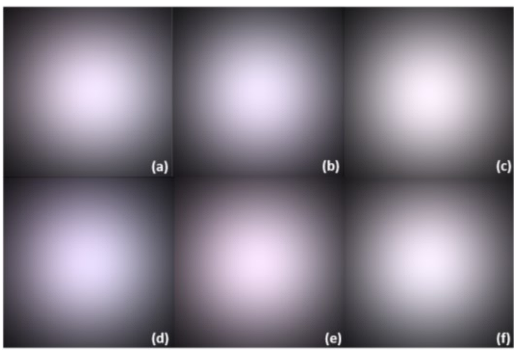

Fig. 9. The pattern of color mixing under different lens distance (a) H=23 mm (b) H=24 mm (c) H=25 mm (d) H=26 mm (e) H=27 mm (f) H=28 mm

It becomes apparent that there is a varying color deviation in the mixed color spots for lens distances of 23 mm, 24 mm, 26 mm, 27 mm, and 28 mm by comparing the simulation results shown in Fig. 9. The presence of these deviations necessitates specific parameters for evaluation. To assess the colorimetric performance of white light, color difference and correlated color temperature were primarily chosen to evaluate important colorimetric characteristics.

4.1. Color difference analysis

The simulation results at difference lens distance are shown in Fig. 9, the color difference formula proposed by CIE1976 is used to analyze and calculate the target plane to quantitatively describe the color uniformity of the target plane and to analyze the trend of the system's color mixing performance with parameter changes. The formula is as follows:

$$\Delta u'v'_{rms} = \sqrt{\frac{1}{M}\sum_{i=1}^{M}\left[\left(u'_i - u'_{avg}\right)^2 + \left(v'_i - v'_{avg}\right)^2\right]} \quad (15)$$

where M is the number of sampling points, u'_i and v'_i is the color coordinate of the sampling point in the CIE1976 uniform color space, u'_{avg} and v'_{avg} is the average value of the color coordinates of all sampling points.

The density of color coordinate distribution determines the magnitude of color difference in mixed light, and the confidence ellipses are used to represent the system's color mixing performance. The distribution of color coordinates on the CIE1976 chromaticity diagram at different combinations of focal length L and lens distances H.

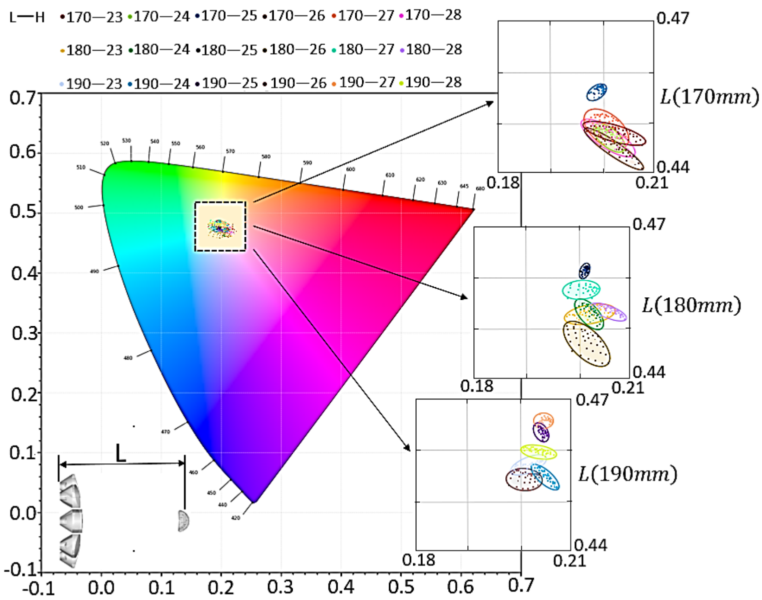

Fig. 10. Color coordinate distribution in CIE1976 color mixing area (color online)

All color coordinates in the figure are distributed around the position (0.198, 0.462) and around. The colored ellipses in the figure represent the confidence ellipses of the color coordinate data points, which indicate the color blending performance of the model under the combined influence of focal length and the lens distance. The label "170-23" in the figure represents the data model of the focal length is 170 mm and the lens distance is 23 mm. The larger the range of the ellipse, the lower the color consistency in that region. To further analyze the relationship between distance and color difference within the optical system, the variation of color difference with distance was plotted in Fig. 11.

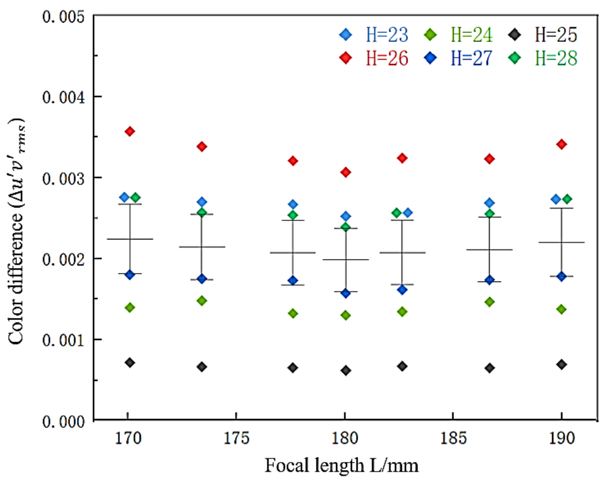

Fig. 11. Color difference analysis chart (color online)

The optical system described utilizes composite lenses and mixers to achieve exceptionally low color deviation, ensuring precise color accuracy. Specifically, under optimal design conditions, the system produces a remarkably small color deviation—less than 0.003. This level of precision is crucial because, theoretically, high-quality mixed white light is considered to be achieved when chromatic aberration is kept below a threshold of 0.1. Therefore, the system's ability to maintain a color deviation significantly lower than this threshold underscores its effectiveness in producing accurate white light. The color difference is sensitive to the lens distance, and the impact of focal length on it can be ignored. This is because composite lenses are designed for specific spacing, which requires them to be maintained at a certain lens spacing. The mixer can determine and change its position based on the aperture, without causing color deviation to deteriorate.

4.2. Correlated color temperature analysis

The color difference is one of the most important performance parameters for the variation of the correlated color temperature brought about by white light. The average color temperature and color temperature range at the receiving surface is sampled and counted as shown in Fig. 12.

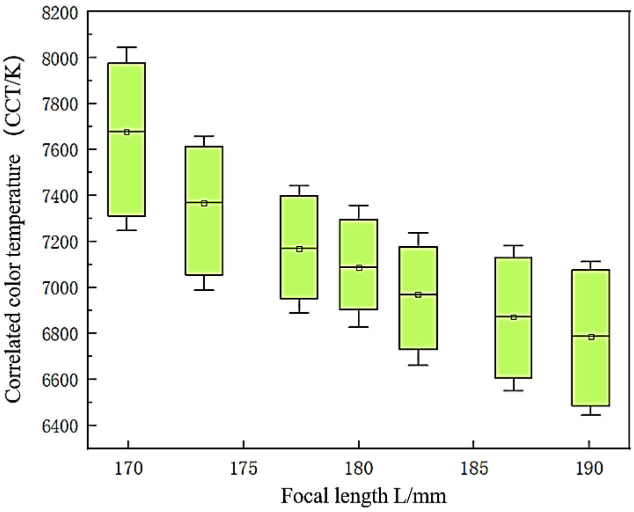

Fig. 12. Correlation color temperature analysis chart (color online)

In the scenario depicted in Fig. 12, the average correlated color temperature (CCT) of the system exhibits a noticeable decrease as the distance between the mixer and the system is adjusted. Specifically, when the distance is increased from 170 mm to 190 mm, the CCT drops from 7700 K to 6800 K. This indicates that the placement of the mixer relative to the system plays a significant role in determining the overall color temperature. The distribution of the correlated color temperature is observed to be most uniform when the mixer is positioned precisely at the focal point of the mixing lens. At this optimal location, the system achieves a more consistent color output, minimizing variations in the CCT across different areas of the beam.

4.3. White point shift analysis

White point drift refers to the inconsistency in the direction of light emitted from light sources, resulting in uneven energy distribution on the cross-section of the light rays. As a result, the intensity distribution within the mixed region becomes non-uniform, leading to a chaotic distribution of correlated color temperature [20]. The normalized standard deviation (NSD) of the associated correlated color temperature was used to assess this phenomenon to analyze the effect of distance from the optical system on the NSD, calculated as follows.

$$NSD = \frac{\sigma\left(CCT_{(x,y)}\right)}{E_{CCT}} \tag{16}$$

where, E_{CCT} represents the average correlated color temperature within the mixing range, and $\sigma\left(CCT_{(x,y)}\right)$ represents the standard deviation of the correlated color temperature.

The normalized standard deviation of correlated color temperature is analyzed, and the results are shown in Fig. 11.

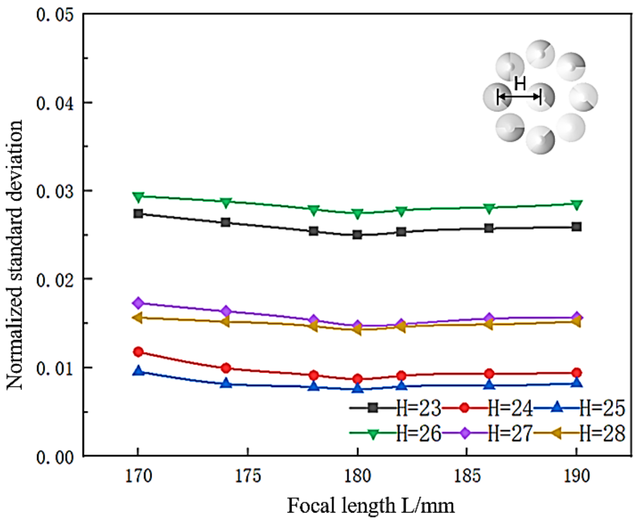

Fig. 13. Normalized standard deviation analysis plot (color online)

As shown in Fig. 13, the convergence of the light source and mixer is perfectly aligned. At this specific distance, the system reaches a minimum normalized standard deviation (NSD) value of 0.008, indicating highly effective color mixing with minimal variation. Deviation of lens distance causes the misalignment between the converged light source and color mixer, resulting in insufficient mixing of light and increasing of NSD. In summary, this color-mixing optical system solves color difference and white point drift at a lens distance of 25 mm and a focal length of 180 mm.

5. Results and discussion

The paper proposes an optical system based on a composite lens array to improve the quality of mixed white light. This color mixing system consists of a composite lens array and a freeform mixer. The composite lens array enables the convergence of light energy from multiple RGB LED arrays to enhance the power output of light energy. The freeform mixer shapes the converged light rays, thereby improving the uniformity of the mixed light. The white point shift and color difference of the system are analyzed through simulation experiments, and the results show that this method substantially reduces both in the mixed light. The proposed optical system can be used to generate high-power, high-quality white light.

Disclosures

The authors declare no conflicts of interest.

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