Dark and singular solitons of Kundu-Eckhaus equation for optical fibers

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This paper obtains dark and singular soliton solutions of Kundu-Eckhaus equation by the ansatz method. There are constraint conditions that fall out and these conditions guarantee the existence of such solitons.

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1. Introduction

Optical solitons is a very prominent research focus in the field of nonlinear fiber optics. There are a few models that describe this dynamics. The most popular one that is very commonly visible is nonlinear Schrödinger's equation (NLSE). There are several other models that describe the dynamics of optical solitons. A few of them are Schödinger-Hirota equation, Biswas-Milovic equation and others. This paper studies another model that is frequently visible. This is the Kundu-Eckhaus (KE) equation that falls in the class of NLSE and describes the propagation of waves in a dispersive medium. This paper will obtain dark and singular soliton solutions to KE equation. The integration tool that will be implemented in this paper is the ansatz scheme. These soliton solutions will be derived along with their constraint conditions, that guarantee the existence of these solitons, will also be given.

2. Overview of the method

In this work, the KE equation to be considered is given by

$$iq_t + aq_{xx} + b|q|^4 q + c\left(|q|^2\right)_x q = 0 \qquad (1)$$

In (1), the two independent variables are x and t that represent spatial and temporal variables respectively. The dependent variable $q(x,t)$ is the soliton pulse profile. The first term in equation (1) take care of the evolution of the nonlinear wave, while the real-valued constants a, b and c represents group velocity dispersion (GVD), quintic nonlinearity, and nonlinear dispersion respectively.

As mentioned that (1) falls in the category of NLSE. However, NLSE is commonly studied with Kerr law nonlinearity or other forms nonlinearity such as power law, parabolic law, dual-power law and log law. It is pointed out earlier that NLSE with quintic nonlinearity leads to self-focusing singularity [1]. Therefore, bright solitons do not exist in this case. In fact this discussion was carried out in the context of power law nonlinearity when the power law nonlinearity parameter is such that NLSE condenses to quintic nonlinearity. These discussions are detailed during 2008 [1]. Hence, this paper stays focused with the derivation of dark and singular solitons for KE.

3. Soliton solutions

This section will focus on the derivation of dark and singular soliton solutions to KE. A proper ansatz or guess will be selected that will be substituted in KE equation which will lead to appropriate parameter connections and dynamics. Furthermore, balancing principle will lead to the value of the unknown exponent. This will lead to a complete final picture of the soliton solutions. This study will be split into two subsections that discuss the two forms of solitons.

3.1 Dark solutions

For dark solitons, the starting ansatz is [8]

$$q(x,t) = \left(A + B\tanh\tau\right)^p e^{i\phi} \tag{2}$$

where

$$\tau = \mu(x - vt) \tag{3}$$

$$\phi = -\kappa x + \omega t + \theta \tag{4}$$

In (2)-(4) A and B are free parameters to be determined, while μ, υ, κ and ω are, respectively, slope of the connector between the two stable states of the soliton solution, soliton speed, frequency and wave number of the soliton. Also, θ represents the phase constant of the soliton. The exponent p is also unknown and it will be determined in the next few lines.

By substituting (2) into (1) and splitting into real and imaginary parts one obtains for the imaginary portion

$$\begin{aligned}&\mu p(v + 2a\kappa)\{(A^2 - B^2)[A + B\tanh\tau]^{p-1}\\&-2A[A + B\tanh\tau]^p + [A + B\tanh\tau]^{p+1}\} = 0\end{aligned} \tag{5}$$

and for the real part one gets

$$\begin{aligned}&\left\{-\left(\omega + a\kappa^2\right)B^2 - 2ap^2\mu^2\left(B^2 - 3A^2\right)\right\}[A + B\tanh\tau]^p\\&-2ap(2p-1)\mu^2 A\left(A^2 - B^2\right)[A + B\tanh\tau]^{p-1}\\&+ap(p-1)\mu^2\left(A^2 - B^2\right)^2[A + B\tanh\tau]^{p-2}\\&-2cp\mu\left(A^2 - B^2\right)B[A + B\tanh\tau]^{3p-1}\\&-2ap(2p+1)\mu^2 A[A + B\tanh\tau]^{p+1}\\&+ap(p+1)\mu^2[A + B\tanh\tau]^{p+2}\\&-2cp\mu B[A + B\tanh\tau]^{3p+1}\\&+4cp\mu AB[A + B\tanh\tau]^{3p}\\&+bB^2[A + B\tanh\tau]^{5p} = 0\end{aligned} \tag{6}$$

From equation (5), one clearly sees that the soliton speed is determined by

$$v = -2a\kappa \tag{7}$$

By equating the exponents of $[A + B\tanh\tau]^{p+1}$ and $[A + B\tanh\tau]^{3p}$ in (6) one get

$$p = \frac{1}{2} \tag{8}$$

It is worth to mention that the same value of p can be retrieve from the coefficient of $[A + B\tanh\tau]^{p-1}$. Notice also that from coefficients of the stand-alone elements $[A + B\tanh\tau]^{p-2}$ and $[A + B\tanh\tau]^{3p-1}$ it is possible to write

$$A = \pm B \tag{9}$$

For the other parameters it is needed to collect the coefficients of the functions of the same exponents from $[A + B\tanh\tau]^{rp+j}$ for r =1, 3, 5 and j = 0, 1, 2 in equation (6), respectively, where each has to vanish. In fact, one obtain the following system of algebraic equations

$$[-(\omega + a\kappa^2) + a\mu^2]A^2 = 0 \tag{10}$$

$$2\mu(cA - a\mu)A = 0 \tag{11}$$

$$3a\mu^2 + 4bA^2 - 4c\mu A = 0 \tag{12}$$

whose solution is given by

$$\omega = a(\mu^2 - \kappa^2) \tag{13}$$

$$\mu = \frac{cA}{a} \tag{14}$$

$$c^2 = 4ab \tag{15}$$

where the last identity (15) was obtained after inserting (9) and (14) into (12). Equation (14) implies the condition

$$a \neq 0 \tag{16}$$

which serves as a constraint for existence of these solitons. An additional constraint condition for the existence of these solitons is given by (15) that connect the coefficients of GVD, nonlinearity and dispersion.

Thus, dark soliton solution for the KE equation (1) is given by

$$q(x,t) = \sqrt{A\left[1 \pm \tanh\{\mu(x - vt)\}\right]}e^{i(-\kappa x + \omega t + \theta)} \tag{17}$$

where the free parameters are related by (9), the parameter μ is given in (14), while the soliton wavenumber in (13). The soliton speed was retrieved from the imaginary part, leading to (7). Notice that all the coefficients of the source equation (1) have to follow the condition (15).

3.2 Singular solutions

To extract singular solitons from the KE equation (1), the starting ansatz is

$$q(x,t) = \left(A + B\coth\tau\right)^p e^{i\phi} \tag{18}$$

Where τ is as in defined in (3). Once again, A and B are free parameters. The remaining parameters have the same meaning as in the previous section. By substituting the ansatz (18) into (1) and splitting into the real and imaginary parts one can retrieve from the imaginary portion the same expression for the soliton speed as in (7). For the real part one arrives at

$$\begin{aligned}
&\left\{-\left(\omega+a\kappa^2\right)B^2-2ap^2\mu^2\left(B^2-3A^2\right)\right\}[A+B\coth\tau]^p\\
&-2ap(2p-1)\mu^2A\left(A^2-B^2\right)[A+B\coth\tau]^{p-1}\\
&+ap(p-1)\mu^2\left(A^2-B^2\right)^2[A+B\coth\tau]^{p-2}\\
&-2cp\mu\left(A^2-B^2\right)B[A+B\coth\tau]^{3p-1}\\
&-2ap(2p+1)\mu^2A[A+B\coth\tau]^{p+1}\\
&+ap(p+1)\mu^2[A+B\coth\tau]^{p+2}\\
&-2cp\mu B[A+B\coth\tau]^{3p+1}\\
&+4cp\mu AB[A+B\coth\tau]^{3p}\\
&+bB^2[A+B\coth\tau]^{5p}=0
\end{aligned} \tag{19}$$

Balancing principle allows one to retrieve not only the same value of p as in (8), but also the relation (9). Thus, results (10)-(15) follow. Finally, the singular soliton solution for (1), following the ansatz approach, is given by

$$q(x,t)=\sqrt{A\left[1\pm\coth\{\mu(x-vt)\}\right]}e^{i(-\kappa x+\omega t+\theta)} \tag{20}$$

where the free parameters A and B are related by (9), the free parameter μ is given by (14), while the soliton wavenumber in (13). The soliton speed falls out from the imaginary part and is given by (7).

The coefficients of KE equation (1) must follow the condition (15) in order for the solitons to exist.

4. Conclusions

This paper retrieved dark and singular soliton solutions to KE model that governs the dynamics of dispersive waves. The quintic nonlinearity does not permit the existence of bright solitons because self-focusing singularity comes into play. The results of this paper form a pillar of strength for additional novel future work. Later, the focus will be geared towards the inclusion of spatio-temporal dispersion. Additionally, time-dependent coefficients will be considered. Moreover, there will be perturbation terms that will be taken into account. This will lead to an extended version of the equation who’s dark and singular soliton solutions will also be retrieved. The result of those researches will be reported soon.

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