Dark and singular solitons in optical metamaterials with anti-cubic nonlinearity by modified simple equation approach

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This paper employs the modified simple equation method to obtain soliton solutions in optical metamaterials. Dark and singular soliton solutions are retrieved and the existence criteria for these solitons are also presented.

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1. Introduction

While solitons in optical fibers have been extensively studied [1-22], it is about time to change gears to a different form of waveguide that has proved to be advantageous over optical fibers. These waveguides are optical metamaterials. The best way these metamaterials have an edge over optical fibers is the avoidance of polarization mode dispersion. These metamaterials do not undergo differential group delay and thus completely avoids splitting of pulses which consequently eliminates the formation of birefringent fibers. This paper is thus going to focus on metamaterials that is studied with a newly proposed nonlinear law. It is the anti-cubic nonlinearity. The integration algorithm adopted in this paper is the modified simple equation method that reveals solitons and other solutions to the model. It is only dark and singular optical solitons that are retrievable by the aid of this scheme. The existence criteria of these solitons are also presented.

2. Governing equation

In this work, we investigate a cubic-quintic NLSE in optical metamaterials with an additional anti-cubic nonlinear term of the form [3, 15]

$$\begin{aligned} &iq_t + aq_{xx} + \left(b_1|q|^{-4} + b_2|q|^2 + b_3|q|^4\right)q = \\ &i\left\{\alpha q_x + \beta\left(|q|^2 q\right)_x + \nu\left(|q|^2\right)_x q\right\} + \\ &\theta_1\left(|q|^2 q\right)_{xx} + \theta_2|q|^2 q_{xx} + \theta_3 q^2 q^*_{xx}. \end{aligned} \tag{1}$$

where a, b_1, b_2 and b_3 are real-valued constants. The spatial and temporal variables are x and t respectively while the complex-valued dependent variable is $q(x,t)$. The coefficient of a is group velocity dispersion, while the three nonlinearterms are the coefficients of b_j for $j = 1,2,3$. When b_1 vanishes, the model represents parabolic law of nonlinearity. With $b_1 \neq 0$, equation (1) is with anti-cubic nonlinearity. Now, on the right hand side of (1), α gives the inter-modal dispersion, β is self-steepening while ν represents nonlinear dispersion. The remaining terms on the right side of (1) are accounted for optical metamaterials.

In order to solve Eq. (1) by the traveling wave hypothesis, the starting hypothesis is

$$q(x,t) = g(\xi)\, e^{i\Phi(x,t)}, \tag{2}$$

where $g(\xi)$ represents the shape of the pulse and

$$\xi = k(x - vt), \tag{3}$$

$$\Phi(x,t) = -\kappa x + \omega t + \theta. \tag{4}$$

In (2), the function $\Phi(x,t)$ is the phase component of the solitons. Then in (4), κ is the soliton frequency, while ω is the wave number of the soliton and θ is the phase constant. Finally in Eq. (3), v is the velocity of the soliton.

Substituting (2) in (1) leads to a pair of relations that stem from real and imaginary parts. The imaginary part gives

$$v = -2\kappa a - \alpha, \tag{5}$$

and

$$3\beta + 2v - 2\kappa(3\theta_1 + \theta_2 - \theta_3) = 0, \tag{6}$$

while the real part yields

$$ak^2 g'' - \left(\omega + a\kappa^2 + \alpha\kappa\right) g + b_1 g^{-3} + \left(b_2 - \beta\kappa + \kappa^2\theta_1 + \kappa^2\theta_2 + \kappa^2\theta_3\right) g^3 + b_3 g^5 - \left(3k^2\theta_1 + k^2\theta_2 + k^2\theta_3\right) g^2 g'' - 6k^2\theta_1 g g'^2 = 0.$$

$$ak^2 g'' - \left(\omega + a\kappa^2 + \alpha\kappa\right) g + b_1 g^{-3} + \left(b_2 - \beta\kappa + \kappa^2\theta_1 + \kappa^2\theta_2 + \kappa^2\theta_3\right) g^3 + b_3 g^5 - \left(3k^2\theta_1 + k^2\theta_2 + k^2\theta_3\right) g^2 g'' - 6k^2\theta_1 g g'^2 = 0. \tag{7}$$

To obtain the analytic solution, the transformations $\theta_1 = 0,\ \theta_2 = -\theta_3$ are applied in Eq. (7), that implies

$$a k^2 g'' - \left(\omega + a\kappa^2 + \alpha\kappa\right) g + b_1 g^{-3} + (b_2 - \kappa\beta) g^3 + b_3 g^5 = 0, \tag{8}$$

where

$$3\beta + 2v + 4\kappa\theta_3 = 0. \tag{9}$$

3. Overview of modified simple equation method

We start with a nonlinear evolution equation of the form

$$P(u, u_t, u_x, u_{xx}, u_{tt}, u_{tx}, \ldots) = 0, \tag{10}$$

where P is a polynomial in $u(x,t)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. The following steps outlines the integraytion scheme as follows [4, 5, 10]:

Step-1: The transformation given by

$$u(x,t) = u(\xi),\ \xi = x - ct, \tag{11}$$

where c is a constant to be determined, reduces (10) to the ordinary differential equation given by

$$Q(u, u', u'', \ldots) = 0, \tag{12}$$

where Q is a polynomial in $u(\xi)$ along with its total derivatives, while the operator is defined as $' = \frac{d}{d\xi}$.

Step-2: Next, assume Eq. (12) has the formal solution.

$$u(\xi) = \sum_{l=0}^{N} a_l \left(\frac{\psi'(\xi)}{\psi(\xi)}\right)^l, \tag{13}$$

where a_l are constants to be determined, such that $a_N \neq 0$, and $\psi(\xi)$ is an unknown function that needs to be found later.

Step-3: We locate the positive integer N in Eq. (13) by considering the homogeneous balance between the highest order derivatives and the nonlinear terms in Eq. (12).

Step-4: Next, substitute (13) into (12), and calculate all the necessary derivatives $u', u'', \ldots$ of the unknown function $u(\xi)$ and we account the function $\psi(\xi)$. As a result of this substitution, we get a polynomial of $\psi'(\xi)/\psi(\xi)$ and its derivatives. In this polynomial, collect all terms of like power of $\psi^{-j}(\xi), j = 0,1,2,\ldots$ and its derivatives, and we equate with zero all coefficients of this polynomial. This implies a system of equations which solves for a_k and $\psi(\xi)$. Consequently, we recover exact solutions of Eq. (10).

4. Application to optical metamaterials

In this section, we construct dark and singular soliton solutions to the governing equation with the help of the modified simple equation method. To this end, we set

$$g = U^{\frac{1}{2}}, \tag{14}$$

so that (8) transforms to

$$ak^2\left(2UU''-U'^2\right)-4\left(\omega+a\kappa^2+\alpha\kappa\right)U^2+ \\ 4b_1+4(b_2-\kappa\beta)U^3+4b_3U^4=0. \tag{15}$$

Balancing UU'' with U^4 in Eq. (15), then we get $N=1$. Consequently we reach

$$U(\xi)=a_0+a_1\left(\frac{\psi'(\xi)}{\psi(\xi)}\right),\ a_1\neq 0. \tag{16}$$

Substituting Eq. (16) in Eq. (15) and then setting the coefficients of $\psi^{-j}(\xi),\ j=0,1,2,3,4$ to zero, then we obtain a set of algebraic equations involving $a_0, a_1, k, a, \kappa, \alpha, \beta b_l, b_2, b_3$ and ω as follows:

ψ^{-4} coeff.:

$$a_1^2\left(4a_1^2b_3+3ak^2\right)\psi'^4=0, \tag{17}$$

ψ^{-3} coeff.:

$$4a_1\begin{pmatrix} a_0\psi'\left(4a_1^2b_3+ak^2\right)- \\ a_1\left(a_1\psi'\left(\beta\kappa-b_2\right)+ak^2\psi''\right)\end{pmatrix}\psi'^2=0, \tag{18}$$

ψ^{-2} coeff.:

$$a_1^2(4\psi'^2\left(3a_0\left(\beta\kappa-b_2\right)-6a_0^2b_3+a\kappa^2+\alpha\kappa+\omega\right) \\ +ak^2\psi''^2-2ak^2\psi'''\psi')+6aa_0a_1k^2\psi'\psi''=0, \tag{19}$$

ψ^{-1} coeff.:

$$2a_0a_1\begin{pmatrix} ak^2\psi'''-2\psi' \\ \begin{pmatrix} 2\left(a\kappa^2+\alpha\kappa+\omega\right) \\ +3a_0\left(\beta\kappa-b_2\right)-4a_0^2b_3\end{pmatrix}\end{pmatrix}=0, \tag{20}$$

ψ^0 coeff.:

$$-4\left(a_0^2\left(a\kappa^2+\alpha\kappa+\omega\right)+a_0^3\left(\beta\kappa-b_2\right)-a_0^4b_3-b_1\right)=0. \tag{21}$$

Solving this system, we obtain

$$a_0=-\frac{\sqrt{3\left(32b_3\left(a\kappa^2+\alpha\kappa+\omega\right)+9\left(b_2-\beta\kappa\right)^2\right)}}{8b_3} \\ -\frac{3\left(b_2-\beta\kappa\right)}{8b_3},\ a_1=\mp\sqrt{-\frac{3ak^2}{4b_3}}, \\ b_1=-\frac{3\left(16b_3\left(a\kappa^2+\alpha\kappa+\omega\right)+3\left(b_2-\beta\kappa\right)^2\right)^2}{1024b_3^3}, \tag{22}$$

and

$$\psi''=\pm\sqrt{-\frac{32b_3\left(a\kappa^2+\alpha\kappa+\omega\right)+9\left(b_2-\beta\kappa\right)^2}{4ab_3k^2}}\psi', \tag{23}$$

$$\psi'''=-\frac{32b_3\left(a\kappa^2+\alpha\kappa+\omega\right)+9\left(b_2-\beta\kappa\right)^2}{4ab_3k^2}\psi'. \tag{24}$$

From Eqs. (23) and (24), we can deduce that

$$\psi'(\xi)=\pm\sqrt{-\frac{4ab_3k^2}{32b_3\left(a\kappa^2+\alpha\kappa+\omega\right)+9\left(b_2-\beta\kappa\right)^2}}c_1 \\ e^{\pm\sqrt{-\frac{32b_3\left(a\kappa^2+\alpha\kappa+\omega\right)+9\left(b_2-\beta\kappa\right)^2}{4ab_3k^2}}\xi}, \tag{25}$$

$$\psi(\xi)=-\frac{4ab_3k^2}{32b_3\left(a\kappa^2+\alpha\kappa+\omega\right)+9\left(b_2-\beta\kappa\right)^2}c_1 \\ e^{\pm\sqrt{-\frac{32b_3\left(a\kappa^2+\alpha\kappa+\omega\right)+9\left(b_2-\beta\kappa\right)^2}{4ab_3k^2}}\xi}+c_2, \tag{26}$$

where c_1 and c_2 are constants of integration. Substituting Eq. (25) and Eq. (26) into Eq. (16),

$$q(x,t)=\left\{\begin{matrix} -\frac{\sqrt{3\left(32b_3\left(a\kappa^2+\alpha\kappa+\omega\right)+9\left(b_2-\beta\kappa\right)^2\right)}}{8b_3} \\ -\frac{3\left(b_2-\beta\kappa\right)}{8b_3}\mp\sqrt{-\frac{3ak^2}{4b_3}} \\ \frac{\pm\sqrt{-\frac{4ab_3k^2}{32b_3\left(a\kappa^2+\alpha\kappa+\omega\right)+9\left(b_2-\beta\kappa\right)^2}}c_1}{-\frac{4ab_3k^2}{32b_3\left(a\kappa^2+\alpha\kappa+\omega\right)+9\left(b_2-\beta\kappa\right)^2}c_1} \\ \frac{e^{\pm\sqrt{-\frac{32b_3\left(a\kappa^2+\alpha\kappa+\omega\right)+9\left(b_2-\beta\kappa\right)^2}{4ab_3k^2}}\xi}}{e^{\pm\sqrt{-\frac{32b_3\left(a\kappa^2+\alpha\kappa+\omega\right)+9\left(b_2-\beta\kappa\right)^2}{4ab_3k^2}}\xi}+c_2}\end{matrix}\right\}^{\frac{1}{2}} e^{i(-\kappa x+\omega t+\theta)} \tag{27}$$

If we set

$$c_1 = -\frac{32b_3\left(a\kappa^2+\alpha\kappa+\omega\right)+9\left(b_2-\beta\kappa\right)^2}{4ab_3k^2}$$

$$c_1 e^{\pm\sqrt{-\frac{32b_3\left(a\kappa^2+\alpha\kappa+\omega\right)+9\left(b_2-\beta\kappa\right)^2}{4ab_3k^2}}\xi_0}, \quad c_2 = \pm 1,$$

we obtain:

$$q(x,t) = \left\{ -\frac{3\left(b_2-\beta\kappa\right)}{8b_3} \pm \frac{\sqrt{3}\sqrt{9\left(b_2-\beta\kappa\right)^2+32b_3\left(a\kappa^2+\alpha\kappa+\omega\right)}}{8b_3} \right\}^{\frac{1}{2}}$$

$$\left\{ \tanh\left[\sqrt{-\frac{9\left(b_2-\beta\kappa\right)^2+32b_3\left(a\kappa^2+\alpha\kappa+\omega\right)}{16ab_3k^2}} (k(x+(2\kappa a+\alpha)t)+\xi_0) \right] \right\}^{\frac{1}{2}}$$

$$\times e^{i(-\kappa x+\omega t+\theta)}. \tag{28}$$

or

$$q(x,t) = \left\{ -\frac{3\left(b_2-\beta\kappa\right)}{8b_3} \pm \frac{\sqrt{3}\sqrt{9\left(b_2-\beta\kappa\right)^2+32b_3\left(a\kappa^2+\alpha\kappa+\omega\right)}}{8b_3} \right\}^{\frac{1}{2}}$$

$$\left\{ \coth\left[\sqrt{-\frac{9\left(b_2-\beta\kappa\right)^2+32b_3\left(a\kappa^2+\alpha\kappa+\omega\right)}{16ab_3k^2}} (k(x+(2\kappa a+\alpha)t)+\xi_0) \right] \right\}^{\frac{1}{2}}$$

$$\times e^{i(-\kappa x+\omega t+\theta)}. \tag{29}$$

These are dark and singular soliton solutions respectively and are valid for

$$9\left(b_2-\beta\kappa\right)^2+32b_3\left(a\kappa^2+\alpha\kappa+\omega\right) > 0, \quad ab_3 < 0. \tag{30}$$

Next, singular periodic solutions are:

$$q(x,t) = \left\{ -\frac{3\left(b_2-\beta\kappa\right)}{8b_3} \pm \frac{\sqrt{3}\sqrt{-9\left(b_2-\beta\kappa\right)^2-32b_3\left(a\kappa^2+\alpha\kappa+\omega\right)}}{8b_3} \right\}^{\frac{1}{2}}$$

$$\left\{ \tan\left[\sqrt{\frac{9\left(b_2-\beta\kappa\right)^2+32b_3\left(a\kappa^2+\alpha\kappa+\omega\right)}{16ab_3k^2}} (k(x+(2\kappa a+\alpha)t)+\xi_0) \right] \right\}^{\frac{1}{2}}$$

$$\times e^{i(-\kappa x+\omega t+\theta)}. \tag{31}$$

or

$$q(x,t) = \left\{ -\frac{3\left(b_2-\beta\kappa\right)}{8b_3} \mp \frac{\sqrt{3}\sqrt{-9\left(b_2-\beta\kappa\right)^2-32b_3\left(a\kappa^2+\alpha\kappa+\omega\right)}}{8b_3} \right\}^{\frac{1}{2}}$$

$$\left\{ \cot\left[\sqrt{\frac{9\left(b_2-\beta\kappa\right)^2+32b_3\left(a\kappa^2+\alpha\kappa+\omega\right)}{16ab_3k^2}} (k(x+(2\kappa a+\alpha)t)+\xi_0) \right] \right\}^{\frac{1}{2}}$$

$$\times e^{i(-\kappa x+\omega t+\theta)}. \tag{32}$$

and these exist when

$$9\left(b_2-\beta\kappa\right)^2+32b_3\left(a\kappa^2+\alpha\kappa+\omega\right) < 0, \quad ab_3 < 0. \tag{33}$$

4. Conclusions

This paper secured dark and singular soliton solutions in optical metamaterials by the aid of modified simple equation approach. The type of nonlinearity that is considered in this paper is anti-cubic. The results appear with several constraints that are necessary for the existence of the soliton solutions. In addition to these soliton solutions, there are a couple of other solutions that show up as a byproduct of this integration scheme. These solutions, although not applicable in optics or optoelectronics, are nevertheless listed to gain a complete spectrum of solutions. The results of this paper carry a lot of promise. This algorithm can be applied to other laws of nonlinearity in the context of metamaterials. Additionally, this scheme can also reveal results in other optical devices such as optical couplers, PCF, DWDM systems, magneto-optic waveguides and several others. The results of those research activities are going to be sequentially reported.

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