OPTOELECTRONICS AND ADVANCED MATERIALS – RAPID COMMUNICATIONS Vol. 16, No. 11-12, November-December 2022, p. 529-535

Cubic–quartic optical solitons in fiber Bragg gratings with Fokas–Lenells equation and two algorithms

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This paper presents cubic–quartic optical soliton solutions that come from Fokas–Lenells equation having dispersive reflectivity. Two integration algorithms granted success with this retrieval. They are G'/G–expansion and the extended simplest equation schemes. These expose the bright and singular soliton solutions to the model along with their existence criterion.

(Received April 27, 2022; accepted December 5, 2022)

Keywords: Bragg gratings, Solitons, Fokas–Lenells equation

1. Introduction

One of the most innovative ways to supplement the depletion of chromatic dispersion (CD) is to replenish with third–order dispersion (3OD) and fourth–order dispersion (4OD) effects which are collectively known to yield cubic–quartic (CQ) solitons [1–35]. The second procedure, to compromise for this low count of CD, is to introduce Bragg gratings [5–9, 22, 25] which is an engineering marvel, that has been implemented to a wide variety of models in optical engineering, which come from the nonlinear Schrödinger's equation (NLSE). The current paper is a combination of both of these technological splendors applied to an optical fiber with Fokas–Lenells equation (FLE) [1–4, 10–16, 20, 21, 23, 24, 26] as its platform as opposed to NLSE as its governing model. Thus, FLE will be addressed with Bragg gratings for the first time in this paper with CQ solitons.

Two integration architectures will be implemented to catch out the soliton solutions from the model. They are G'/G–expansion and the extended simplest methods. These couple of schemes would lead to a spectrum of CQ soliton solutions that are enumerated in the work. The parametric restrictions for the existence of such solitons are also enlisted along with these solutions. The details of the two mathematical approaches along with the solution extraction methodologies are exhibited in the rest of the paper.

1.1. Governing model

The cubic-quartic perturbed Fokas-Lenells equation in polarization preserving fibers is written as [27]

$$iu_t + iau_{xxx} + bu_{xxxx} + |u|^2(cu + idu_x) = i[\alpha u_x + \lambda(|u|^2u)_x + \mu(|u|^2)_x u], \quad (1)$$

where a, b, c, d, α, λ and μ are real–valued constants. a and b are the coefficients of 3OD and 4OD sequentially and $i = \sqrt{-1}$. $u(x,t)$ is a complex-valued function which denotes the wave profile. d, c, α, μ and λ are the coefficients of nonlinear dispersion, Kerr law nonlinearity, inter–modal dispersion, higher–order dispersion and self–steepening, sequentially.

FLE as given by (1), with Bragg gratings gets restructured as

$$iq_t + ia_1 r_{xxx} + b_1 r_{xxxx}$$
$$+(c_1|q|^2 + d_1|r|^2)(e_1 q + if_1 q_x)$$
$$+qr^*(\gamma_1 r + i\eta_1 r_x) + i\alpha_1 q_x + \beta_1 r + \sigma_1 q^* r^2$$
$$= i\begin{bmatrix} \lambda_1(|q|^2 q)_x + \upsilon_1(|r|^2 q)_x \\ +\mu_1(|q|^2)_x q + \theta_1(|r|^2)_x q \end{bmatrix}, \quad (2)$$

and

$$ir_t + ia_2 q_{xxx} + b_2 q_{xxxx}$$
$$+(c_2|r|^2 + d_2|q|^2)(e_2 r + if_2 r_x)$$
$$+rq^*(\gamma_2 q + i\eta_2 q_x) + i\alpha_2 r_x + \beta_2 q + \sigma_2 r^* q^2$$
$$= i\begin{bmatrix} \lambda_2(|r|^2 r)_x + \upsilon_2(|q|^2 r)_x \\ +\mu_2(|r|^2)_x r + \theta_2(|q|^2)_x r \end{bmatrix}, \quad (3)$$

where a_j, b_j, c_j, d_j, e_j, f_j, η_j, γ_j, β_j, α_j, λ_j, σ_j, υ_j, μ_j and θ_j for $(j = 1,2)$ are real-valued constants. $q(x,t)$ and $r(x,t)$ are complex–valued functions which signify the wave profiles. c_j coupled with e_j are the self-phase modulation. a_j and b_j represent the coefficients of 3OD and 4OD, respectively. f_j represent nonlinear dispersion. d_j coupled with e_j are the cross–phase modulation. Also, α_j are the inter-modal dispersions, β_j represent the difference between the propagation constants, σ_j are from four–wave mixing, υ_j and λ_j are self–steepening effects. Then, μ_j are from self–frequency shift and θ_j are nonlinear dispersive reflectivity.

2. Mathematical set–up

In this section, the kick off is with the solutions hypothesis structure

$$q(x,t) = P_1(\xi)\exp[i\psi(x,t)],$$
$$r(x,t) = P_2(\xi)\exp[i\psi(x,t)], \quad (4)$$

and

$$\psi(x,t) = -\kappa x + \omega t + \theta_0, \quad \xi = x - Vt, \quad (5)$$

where κ, ω, θ_0 and V are real constants, in which κ is the frequency, ω is the wave number, θ_0 is the phase parameter, while V is the velocity. Next, the real functions $P_1(\xi)$, $P_2(\xi)$ and $\psi(x,t)$ depict respectively the amplitude and phase components of the wave profile.

Inserting (4) and (5) into (2) and (3), the real parts are

$$b_1 P_2^{(4)} + 3\kappa(a_1 - 2b_1\kappa)P_2''$$
$$+[d_1(e_1 + f_1\kappa) + \gamma_1 + \eta_1\kappa + \sigma_1 - \upsilon_1\kappa]P_1 P_2^2$$
$$+[c_1(e_1 + f_1\kappa) - \lambda_1\kappa]P_1^3 + (\beta_1 + b_1\kappa^4 - a_1\kappa^3)P_2$$
$$+(\kappa\alpha_1 - \omega)P_1 = 0, \quad (6)$$

$$b_2 P_1^{(4)} + 3\kappa(a_2 - 2b_2\kappa)P_1''$$
$$+[d_2(e_2 + f_2\kappa) + \gamma_2 + \eta_2\kappa + \sigma_2 - \upsilon_2\kappa]P_2 P_1^2$$
$$+[c_2(e_2 + f_2\kappa) - \lambda_2\kappa]P_2^3 + (\beta_2 + b_2\kappa^4 - a_2\kappa^3)P_1$$
$$+(\kappa\alpha_2 - \omega)P_2 = 0, \quad (7)$$

while the imaginary parts are

$$(a_1 - 4b_1\kappa)P_2''' + (f_1 c_1 - 3\lambda_1 - 2\mu_1)P_1^2 P_1'$$
$$+(\eta_1 - 2\upsilon_1 - 2\theta_1)P_1 P_2 P_2' + (d_1 f_1 - \upsilon_1)P_2^2 P_1'$$
$$+(\alpha_1 - V)P_1' + \kappa^2(4b_1\kappa - 3a_1)P_2' = 0, \quad (8)$$

$$(a_2 - 4b_2\kappa)P_1''' + (f_2 c_2 - 3\lambda_2 - 2\mu_2)P_2^2 P_2'$$
$$+(\eta_2 - 2\upsilon_2 - 2\theta_2)P_2 P_1 P_1' + (d_2 f_2 - \upsilon_2)P_1^2 P_2'$$
$$+(\alpha_2 - V)P_2' + \kappa^2(4b_2\kappa - 3a_2)P_1' = 0. \quad (9)$$

Set

$$P_2(\xi) = AP_1(\xi), \; A \neq 0, \; A \neq 1. \quad (10)$$

Thus, the real parts change to

$$b_1 A P_1^{(4)} + 3\kappa A(a_1 - 2b_1\kappa)P_1''$$
$$+[A(\beta_1 + b_1\kappa^4 - a_1\kappa^3) + \kappa\alpha_1 - \omega]P_1$$
$$+\begin{bmatrix} A^2 d_1(e_1 + f_1\kappa) \\ +A^2(\gamma_1 + \eta_1\kappa + \sigma_1 - \upsilon_1\kappa) \\ +c_1(e_1 + f_1\kappa) - \lambda_1\kappa \end{bmatrix} P_1^3 = 0, \quad (11)$$

and

$$b_2 P_1^{(4)} + 3\kappa(a_2 - 2b_2\kappa)P_1''$$
$$+[\beta_2 + b_2\kappa^4 - a_2\kappa^3 + A(\kappa\alpha_2 - \omega)]P_1$$
$$+A\begin{bmatrix} d_2(e_2 + f_2\kappa) \\ +(\gamma_2 + \eta_2\kappa + \sigma_2 - \upsilon_2\kappa) \\ +A^2 c_2(e_2 + f_2\kappa) - A^2\lambda_2\kappa \end{bmatrix} P_1^3 = 0, \quad (12)$$

while the imaginary parts become

$$A(a_1-4b_1\kappa)P_1'''$$
$$+[\alpha_1-V+A\kappa^2(4b_1\kappa-3a_1)]P_1'$$
$$+\begin{bmatrix} f_1c_1-3\lambda_1-2\mu_1 \\ +A^2(\eta_1-2\upsilon_1-2\theta_1) \\ +A^2(d_1f_1-\upsilon_1)\end{bmatrix}P_1^2P_1'=0, \quad (13)$$

and

$$(a_2-4b_2\kappa)P_1'''$$
$$+[A(\alpha_2-V)+\kappa^2(4b_2\kappa-3a_2)]P_1'$$
$$+A\begin{bmatrix} A^2(f_2c_2-3\lambda_2-2\mu_2) \\ +\eta_2-2\upsilon_2-2\theta_2 \\ +d_2f_2-\upsilon_2\end{bmatrix}P_1^2P_1'=0. \quad (14)$$

Integrating Eqs. (13) and (14), one retrieves

$$A(a_1-4b_1\kappa)P_1''$$
$$+[\alpha_1-V+A\kappa^2(4b_1\kappa-3a_1)]P_1$$
$$+\frac{1}{3}\begin{bmatrix} f_1c_1-3\lambda_1-2\mu_1 \\ +A^2(\eta_1-2\upsilon_1-2\theta_1) \\ +A^2(d_1f_1-\upsilon_1)\end{bmatrix}P_1^3=0, \quad (15)$$

and

$$(a_2-4b_2\kappa)P_1''$$
$$+[A(\alpha_2-V)+\kappa^2(4b_2\kappa-3a_2)]P_1$$
$$+\frac{1}{3}A\begin{bmatrix} A^2(f_2c_2-3\lambda_2-2\mu_2) \\ +\eta_2-2\upsilon_2-2\theta_2 \\ +d_2f_2-\upsilon_2\end{bmatrix}P_1^3=0. \quad (16)$$

Eqs. (15) and (16) give the frequency

$$\kappa=\frac{a_l}{4b_l}, \quad l=1,2, \ a_1b_2=a_2b_1, \quad (17)$$

and the velocity

$$V=\alpha_1+A\kappa^2(4b_1\kappa-3a_1), \quad (18)$$

$$AV=A\alpha_2+\kappa^2(4b_2\kappa-3a_2), \quad (19)$$

by virtue of the constraint conditions

$$f_1c_1-3\lambda_1-2\mu_1+A^2(\eta_1-2\upsilon_1-2\theta_1)$$
$$+A^2(d_1f_1-\upsilon_1)=0, \quad (20)$$

$$A^2(f_2c_2-3\lambda_2-2\mu_2)+\eta_2-2\upsilon_2-2\theta_2$$
$$+d_2f_2-\upsilon_2=0. \quad (21)$$

Eqs. (11) and (12) are equivalent provided

$$b_2=b_1A, \quad (22)$$

$$A(a_1-2b_1\kappa)=a_2-2b_2\kappa, \quad (23)$$

$$A(\beta_1-a_1\kappa^3+b_1\kappa^4)+\kappa\alpha_1-\omega$$
$$=\beta_2-a_2\kappa^3+b_2\kappa^4+A(\kappa\alpha_2-\omega), \quad (24)$$

$$A^2d_1(e_1+f_1\kappa)+A^2(\gamma_1+\eta_1\kappa+\sigma_1-\upsilon_1\kappa)$$
$$+c_1(e_1+f_1\kappa)-\lambda_1\kappa$$
$$=A\begin{bmatrix} d_2(e_2+f_2\kappa) \\ +(\gamma_2+\eta_2\kappa+\sigma_2-\upsilon_2\kappa) \\ +A^2c_2(e_2+f_2\kappa)-A^2\lambda_2\kappa\end{bmatrix}. \quad (25)$$

From (17), (22) and (24), one reveals the wave number

$$\omega=\frac{A\beta_1-\beta_2+4b_1\kappa^4(1-A)+\kappa(\alpha_1-A\alpha_2)}{(1-A)}. \quad (26)$$

Eq. (11) can be structured in the form

$$P_1^{(4)}+L_1P_1''+L_2P_1+L_3P_1^3=0, \quad (27)$$

where

$$L_1=\frac{3\kappa A(a_1-2b_1\kappa)}{b_1A}, b_1A\neq 0,$$

$$L_2=\frac{A(\beta_1+b_1\kappa^4-a_1\kappa^3)+\kappa\alpha_1-\omega}{b_1A},$$

$$L_3=\frac{A^2d_1(e_1+f_1\kappa)+A^2(\gamma_1+\eta_1\kappa+\sigma_1-\upsilon_1\kappa)+c_1(e_1+f_1\kappa)-\lambda_1\kappa}{b_1A}. \quad (28)$$

3. G'/G–expansion

Eq. (27) permits the exact solution [28-31]

$$P_1(\xi)=\Pi_0+\Pi_1\left(\frac{G'(\xi)}{G(\xi)}\right)+\Pi_2\left(\frac{G'(\xi)}{G(\xi)}\right)^2, \ \Pi_2\neq 0, \quad (29)$$

where Π_0, Π_1 and Π_2 are constants, while the function $G(\xi)$ satisfies the auxiliary equation

$$G''(\xi)+h_1G'(\xi)+h_2G(\xi)=0, \quad (30)$$

where h_1 and h_2 are constants and (G'/G) admits the analytical solutions

$$\frac{G'(\xi)}{G(\xi)}=\frac{-h_1}{2}+\frac{1}{2}\sqrt{\Delta}\begin{bmatrix} \dfrac{C_1\sinh\left(\frac{1}{2}\xi\sqrt{\Delta}\right)+C_2\cosh\left(\frac{1}{2}\xi\sqrt{\Delta}\right)}{C_1\cosh\left(\frac{1}{2}\xi\sqrt{\Delta}\right)+C_2\sinh\left(\frac{1}{2}\xi\sqrt{\Delta}\right)}\end{bmatrix}, \ \Delta>0, \quad (31)$$

$$\frac{G'(\xi)}{G(\xi)}=\frac{-h_1}{2}+\frac{1}{2}\sqrt{-\Delta}\left[\frac{-C_1\sin\left(\frac{1}{2}\xi\sqrt{-\Delta}\right)+C_2\cos\left(\frac{1}{2}\xi\sqrt{-\Delta}\right)}{C_1\cos\left(\frac{1}{2}\xi\sqrt{-\Delta}\right)+C_2\sin\left(\frac{1}{2}\xi\sqrt{-\Delta}\right)}\right],\ \Delta<0, \tag{32}$$

$$\frac{G'(\xi)}{G(\xi)}=\frac{-h_1}{2}+\frac{C_2}{C_1+C_2\xi},\ \Delta=0, \tag{33}$$

where $\Delta=h_1^2-4h_2$ and C_1 and C_2 are real–valued constant parameters. Putting (29) together with (30) into (27), one recovers

$$\Pi_0=2h_2\sqrt{-\frac{30}{L_3}},\Pi_1=2\sqrt{-\frac{30}{L_3}}\sqrt{\frac{L_1+20h_2}{5}},$$

$$\Pi_2=2\sqrt{-\frac{30}{L_3}},h_1=\sqrt{\frac{L_1+20h_2}{5}},h_2=h_2, \tag{34}$$

and

$$L_1=L_1,L_2=\frac{4}{25}L_1^2,L_3=L_3, \tag{35}$$

provided $L_1+20h_2>0$ and $L_3<0$. Now, Eq. (29) becomes

$$P_1(\xi)=2\sqrt{-\frac{30}{L_3}}\left[h_2+\sqrt{\frac{L_1+20h_2}{5}}\left(\frac{G'(\xi)}{G(\xi)}\right)+\left(\frac{G'(\xi)}{G(\xi)}\right)^2\right]. \tag{36}$$

For $L_1>0$, Eq. (31) evolves as

$$q(x,t)=\sqrt{-\frac{3L_1^2}{10L_3}}\times\left[1-\left(\frac{C_1\sinh\left[\sqrt{\frac{L_1}{20}}(x-Vt)\right]+C_2\cosh\left[\sqrt{\frac{L_1}{20}}(x-Vt)\right]}{C_1\cosh\left[\sqrt{\frac{L_1}{20}}(x-Vt)\right]+C_2\sinh\left[\sqrt{\frac{L_1}{20}}(x-Vt)\right]}\right)^2\right]\times e^{i(-\kappa x+\omega t+\theta_0)}, \tag{37}$$

and

$$r(x,t)=A\sqrt{-\frac{3L_1^2}{10L_3}}\times\left[1-\left(\frac{C_1\sinh\left[\sqrt{\frac{L_1}{20}}(x-Vt)\right]+C_2\cosh\left[\sqrt{\frac{L_1}{20}}(x-Vt)\right]}{C_1\cosh\left[\sqrt{\frac{L_1}{20}}(x-Vt)\right]+C_2\sinh\left[\sqrt{\frac{L_1}{20}}(x-Vt)\right]}\right)^2\right]\times e^{i(-\kappa x+\omega t+\theta_0)}. \tag{38}$$

If $C_1=0$ and $C_2\neq 0$, then one retrieves the singular solitons

$$q(x,t)=\sqrt{-\frac{3L_1^2}{10L_3}}\operatorname{csch}^2\left[\sqrt{\frac{L_1}{20}}(x-Vt)\right]\times e^{i(-\kappa x+\omega t+\theta_0)}, \tag{39}$$

and

$$r(x,t)=A\sqrt{-\frac{3L_1^2}{10L_3}}\operatorname{csch}^2\left[\sqrt{\frac{L_1}{20}}(x-Vt)\right]\times e^{i(-\kappa x+\omega t+\theta_0)}, \tag{40}$$

while if $C_1\neq 0$ and $C_2=0$, then we recover the bright solitons

$$q(x,t)=\sqrt{-\frac{3L_1^2}{10L_3}}\operatorname{sech}^2\left[\sqrt{\frac{L_1}{20}}(x-Vt)\right]\times e^{i(-\kappa x+\omega t+\theta_0)}, \tag{41}$$

and

$$r(x,t)=A\sqrt{-\frac{3L_1^2}{10L_3}}\operatorname{sech}^2\left[\sqrt{\frac{L_1}{20}}(x-Vt)\right]\times e^{i(-\kappa x+\omega t+\theta_0)}. \tag{42}$$

The two cases where $L_1<0$ and $L_1=0$ are not included, since they lead to periodic solutions and plane waves. Figs. 1 and 2 represent the surface plots of bright solitons (41) and (42). The selected parameter values are $\kappa=1$, $a_1=3$, $b_1=1$, $A=2$, $\gamma_1=1$, $\eta_1=1$, $\sigma_1=1$, $\upsilon_1=1$, $c_1=1$, $e_1=1$, $f_1=1$, $d_1=-1$ and $\lambda_1=3$.

4. Extended simplest equation

Eq. (27) assumes the explicit solution [32-35]

$$P_1(\xi) = \chi_0 + \chi_1\left[\frac{\phi'(\xi)}{\phi(\xi)}\right] + \chi_2\left[\frac{\phi'(\xi)}{\phi(\xi)}\right]^2$$
$$+B_0\left[\frac{1}{\phi(\xi)}\right] + B_1\left[\frac{\phi'(\xi)}{\phi(\xi)}\right]\left[\frac{1}{\phi(\xi)}\right], \qquad (43)$$

where χ_0, χ_1, χ_2, B_0 and B_1 are constants, $\chi_2^2 + B_1^2 \neq 0$, and the function $\phi(\xi)$ presumes the auxiliary equation

$$\phi''(\xi) + \delta\phi(\xi) = \upsilon, \qquad (44)$$

where δ and υ are constants. The cases where $\delta > 0$ and $\delta = 0$ are ignored since these two situations yield periodic solutions and plane waves both of which have no place in optics. Therefore, looking at $\delta < 0$ yields optical solitons.

Substituting (43) into (27) and use Eq. (44) together with the relation

$$\left(\frac{\phi'(\xi)}{\phi(\xi)}\right)^2 = \Delta_1\left(\frac{1}{\phi(\xi)}\right)^2 - \delta + \frac{2\upsilon}{\phi(\xi)}, \qquad (45)$$

where $\Delta_1 = \delta(A_1^2 - A_2^2) - \frac{\upsilon^2}{\delta}$, while A_1 and A_2 are constants, then one gets the results:

Result–1:

$$\chi_0 = \delta\sqrt{-\frac{30}{L_3}}, \chi_1 = 0, \chi_2 = \sqrt{-\frac{30}{L_3}},$$
$$B_0 = -\upsilon\sqrt{-\frac{30}{L_3}}, B_1 = \sqrt{-\frac{30\Delta_1}{L_3}}, \qquad (46)$$

and

$$L_1 = 5\delta, L_2 = 4\delta^2, L_3 = L_3, \qquad (47)$$

provided $L_3 < 0$ and $\Delta_1 > 0$. Hence, we retrieve

$$q(x,t) = \sqrt{-\frac{30}{L_3}}\begin{bmatrix}\delta - \delta\Theta_1^2(\xi) - \upsilon\Theta_2(\xi)\\ +\sqrt{-\delta\Delta_1}\Theta_1(\xi)\Theta_2(\xi)\end{bmatrix}$$
$$\times e^{i(-\kappa x + \omega t + \theta_0)}, \qquad (48)$$

and

$$r(x,t) = A\sqrt{-\frac{30}{L_3}}\begin{bmatrix}\delta - \delta\Theta_1^2(\xi) - \upsilon\Theta_2(\xi)\\ +\sqrt{-\delta\Delta_1}\Theta_1(\xi)\Theta_2(\xi)\end{bmatrix}$$
$$\times e^{i(-\kappa x + \omega t + \theta_0)}, \qquad (49)$$

where

$$\Theta_1(\xi) = \frac{\begin{pmatrix}A_1\sinh\left[\sqrt{-\delta}(x - Vt)\right]\\ +A_2\cosh\left[\sqrt{-\delta}(x - Vt)\right]\end{pmatrix}}{\begin{pmatrix}A_1\cosh\left[\sqrt{-\delta}(x - Vt)\right]\\ +A_2\sinh\left[\sqrt{-\delta}(x - Vt)\right] + \frac{\upsilon}{\delta}\end{pmatrix}},$$
$$\Theta_2(\xi) = \frac{1}{\begin{pmatrix}A_1\cosh\left[\sqrt{-\delta}(x - Vt)\right]\\ +A_2\sinh\left[\sqrt{-\delta}(x - Vt)\right] + \frac{\upsilon}{\delta}\end{pmatrix}}. \qquad (50)$$

Setting $A_1 = 0$, $A_2 \neq 0$, $\upsilon = 0$ in (48) and (49), the combo singular solitons are indicated below

$$q(x,t) = \delta\sqrt{-\frac{30}{L_3}}\operatorname{csch}\left[\sqrt{-\delta}(x - Vt)\right]$$
$$\times\left\{\coth\left[\sqrt{-\delta}(x - Vt)\right] - \operatorname{csch}\left[\sqrt{-\delta}(x - Vt)\right]\right\}$$
$$\times e^{i(-\kappa x + \omega t + \theta_0)}, \qquad (51)$$

and

$$r(x,t) = A\delta\sqrt{-\frac{30}{L_3}}\operatorname{csch}\left[\sqrt{-\delta}(x - Vt)\right]$$
$$\times\left\{\coth\left[\sqrt{-\delta}(x - Vt)\right] - \operatorname{csch}\left[\sqrt{-\delta}(x - Vt)\right]\right\}$$
$$\times e^{i(-\kappa x + \omega t + \theta_0)}. \qquad (52)$$

Result–2:

$$\chi_0 = \chi_0, \chi_1 = 0, \chi_2 = \frac{L_3\chi_0 + \sqrt{-L_2L_3}}{L_3\delta}, B_1 = 0,$$
$$B_0 = -\frac{2\upsilon\left(L_3\chi_0 + \sqrt{-L_2L_3}\right)}{L_3\delta}, A_1 = \sqrt{A_2^2 + \frac{\upsilon^2}{\delta^2}}, \qquad (53)$$

provided $L_2L_3 < 0$ and $A_2^2 + \frac{\upsilon^2}{\delta^2} > 0$. Hence, we reveal

$$q(x,t) = \left[\chi_0 - \left(\chi_0 + \sqrt{-\frac{L_2}{L_3}}\right)\left[\rho_1^2(\xi) - \frac{2\upsilon}{\delta}\rho_2(\xi)\right]\right]$$
$$\times e^{i(-\kappa x + \omega t + \theta_0)}, \qquad (54)$$

and

$$r(x,t) = A\left[\chi_0 - \left(\chi_0 + \sqrt{-\frac{L_2}{L_3}}\right)\left[\rho_1^2(\xi) - \frac{2\upsilon}{\delta}\rho_2(\xi)\right]\right]$$
$$\times e^{i(-\kappa x + \omega t + \theta_0)}, \qquad (55)$$

where

$$\rho_1(\xi) = \frac{\left(\begin{array}{c}\sqrt{A_2^2+\frac{\upsilon^2}{\delta^2}}\sinh\left[\sqrt{-\delta}(x-Vt)\right]\\ +A_2\cosh\left[\sqrt{-\delta}(x-Vt)\right]\end{array}\right)}{\left(\begin{array}{c}\sqrt{A_2^2+\frac{\upsilon^2}{\delta^2}}\cosh\left[\sqrt{-\delta}(x-Vt)\right]\\ +A_2\sinh\left[\sqrt{-\delta}(x-Vt)\right]+\frac{\upsilon}{\delta}\end{array}\right)},$$

$$\rho_2(\xi) = \frac{1}{\left(\begin{array}{c}\sqrt{A_2^2+\frac{\upsilon^2}{\delta^2}}\cosh\left[\sqrt{-\delta}(x-Vt)\right]\\ +A_2\sinh\left[\sqrt{-\delta}(x-Vt)\right]+\frac{\upsilon}{\delta}\end{array}\right)}. \tag{56}$$

When $A_2 = 0$, Eqs. (54) and (55) simplify to the bright solitons

$$q(x,t) = \left[\begin{array}{c}\chi_0-\left(\chi_0+\sqrt{-\frac{L_2}{L_3}}\right)\\ \times\left(\frac{1-3\operatorname{sech}\left[\sqrt{-\delta}(x-Vt)\right]}{1+\operatorname{sech}\left[\sqrt{-\delta}(x-Vt)\right]}\right)\end{array}\right] \times e^{i(-\kappa x+\omega t+\theta_0)}, \tag{57}$$

and

$$r(x,t) = A\left[\begin{array}{c}\chi_0-\left(\chi_0+\sqrt{-\frac{L_2}{L_3}}\right)\\ \times\left(\frac{1-3\operatorname{sech}\left[\sqrt{-\delta}(x-Vt)\right]}{1+\operatorname{sech}\left[\sqrt{-\delta}(x-Vt)\right]}\right)\end{array}\right] \times e^{i(-\kappa x+\omega t+\theta_0)}. \tag{58}$$

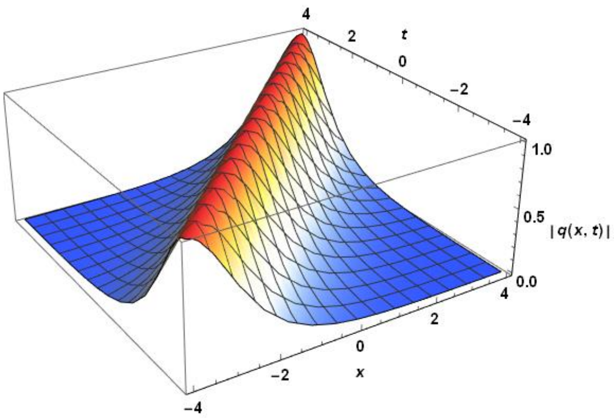

Fig. 1. A surface plot of a bright solitons (41) (color online)

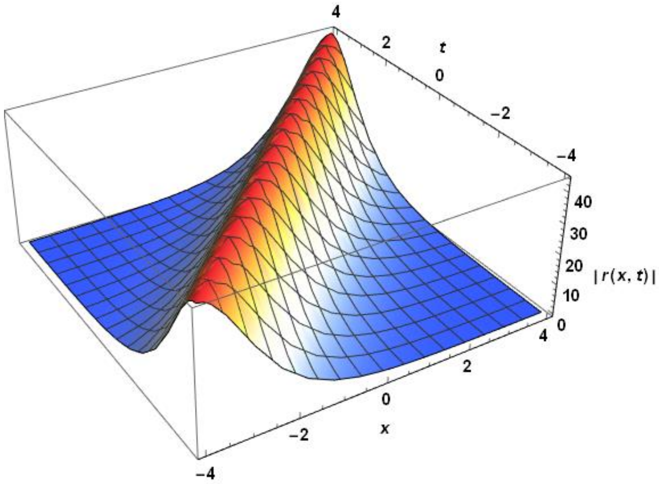

Fig. 2. A surface plot of a bright solitons (42) (color online)

5. Conclusions

For the first time, CQ solitons in fiber Bragg gratings are obtained with FLE as its platform than from the usual NLSE. Two integration schemes have made this retrieval possible that yielded a spectrum of such solitons. The solitons appear with parameter restrictions that are listed as constraints. The possibilities are endless that would give way to avenues to venture. The starting point would be to get the conservation laws in place which would also come with their respective fluxes. Later, these CQ solitons in Bragg gratings would be considered with additional laws of nonlinear refractive index that are now plentifully many. Thus, there is work up ahead that will keep us occupied!

Disclosure

The authors claim that they have no conflicts of interest.

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